



# 基于双幂次滑模的多机器人编队控制

## 摘要

本文研究了基于领航跟随法的多机器人系统编队控制问题.首先,基于队形约束,给出跟随者期望的轨迹,将编队问题转化为单个跟随者的轨迹跟踪问题.在此基础上,基于双幂次滑模趋近律,设计了跟随者的线速度和角速度控制器,保证了跟踪误差能够快速收敛到零,从而保证了编队队形的稳定.最后,通过仿真验证了所提方法的有效性.

## 关键词

多机器人编队;领航跟随;双幂次趋近律

中图分类号 TP13

文献标志码 A

收稿日期 2020-08-31

资助项目 国家自然科学基金(61773146,61973102,61703132)

## 作者简介

李天龙,男,硕士,工程师,研究方向为自动化控制系统.284367458@qq.com

柏建军(通信作者),男,博士,副教授,研究方向为多智能体系统、鲁棒控制.baijianjun@hdu.edu.cn

1 浙江机电职业技术学院 现代信息技术学院,杭州,310053

2 杭州电子科技大学 自动化(人工智能)学院,杭州,310018

## 0 引言

从 20 世纪 70 年代到现在,机器人技术作为一个新兴的研究热点,得到了广大学者的关注<sup>[1]</sup>.轮式移动机器人由于其移动灵活、适应性强,具有强大的自身承载能力等优点,已在航空航天、工业生产、物流运输等领域得到了广泛的应用<sup>[2]</sup>.然而,随着需要机器人解决的问题越来越复杂,对机器人的性能提出了更高的要求,甚至出现了某些任务靠单个机器人是无法完成的情况.为了解决该问题,人们开始考虑用多个廉价、功能简单的机器人合作去完成一项复杂繁重的任务.多机器人编队控制作为机器人协同控制的一个重要研究方向,得到了广泛的研究并取得了许多有意义的成果<sup>[3-6]</sup>.

根据控制策略的不同,现有的编队控制方法主要有领航跟随法、虚拟结构法、基于行为法<sup>[7]</sup>等.相比于其他方法,领航跟随法具有鲁棒性强,能够很好地对系统稳定性进行理论分析等优点,并因此得到了广泛的关注<sup>[8]</sup>.文献[9]基于领航跟随法研究了基于视觉的编队算法,并对队形变换问题进行了探讨;文献[10]研究了具有输入饱和约束下的编队控制问题;文献[11]给出了沿直线编队行进的控制算法,去除了角速度不能为零的限制;文献[12]通过构造新的状态观测器,给出了一种基于视觉的自适应编队控制算法;文献[13]则将滑模控制应用到了船舶的编队控制中,有效地解决了模型不确定与外界干扰对系统性能的影响.

随着滑模控制理论不断发展,许多学者提出了基于幂次趋近律的滑模控制算法<sup>[14-17]</sup>,有效地解决了传统滑模控制中收敛速度慢、抖震等问题.然而,该方法在编队控制中还少有研究.因此,本文基于双幂次趋近律的滑模控制与领航跟随编队方法相结合,给出了一种能够快速收敛的多机器人编队控制算法.

## 1 领航-跟随编队模型

移动机器人的结构模型如图 1 所示.以地面为参考系建立全局坐标系  $XOY$ ,以移动机器人作为参考系建立移动机器人坐标系  $xoy$ .图 1 中  $2l$  为  $y$  轴方向两个驱动轮之间的距离,  $2r$  为驱动轮直径,  $a$  为驱动轮轴线中点,  $d$  为移动机器人质心  $o$  与  $a$  在  $x$  轴方向的距离,  $\theta$  为移动机器人行驶方向与水平方向的夹角,因此移动机器人的位姿坐标可表示为  $[x, y, \theta]^T$ .

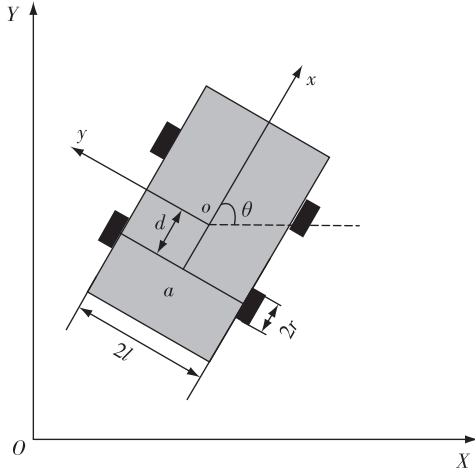


图1 移动机器人的结构模型

Fig. 1 Structure of the wheeled mobile robot

该移动机器人的运动学方程为

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos \theta & -d \sin \theta \\ \sin \theta & d \cos \theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}. \quad (1)$$

领航-跟随编队结构模型如图2所示.

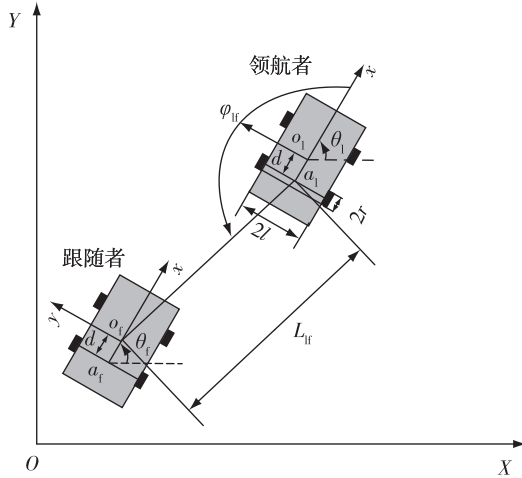


图2 领航-跟随编队结构模型示意

Fig. 2 Structure of the leader-follower formation

在全局坐标系  $XOY$  中, 定义领航者的位姿为  $[x_1, y_1, \theta_1]^T$ , 即质心  $o_1$  的坐标, 第  $i$  ( $i$  为跟随者的编号,  $i = 1, 2, \dots, n$ ) 个跟随者的位姿为  $[x_{fi}, y_{fi}, \theta_{fi}]^T$ , 即质心  $o_{fi}$  的坐标. 其中,  $\theta_1, \theta_{fi}$  分别为领航者和第  $i$  个跟随者的航向角. 以移动机器人的质心  $o_1, o_{fi}$  作为参考点,  $L_{fi}$  为第  $i$  个跟随者参考点  $o_{fi}$  与领航者驱动轮轴中心点  $a_1$  之间的距离, 即编队之间应该保持的距离.  $\varphi_{fi}$  为领航者航向角与两机器人之间约束距离连线的夹角, 称为相对相角. 以领航者与第  $i$  个跟随者

之间的相对相角  $\varphi_{fi}$  和位置距离  $L_{fi}$  来约束编队. 每个机器人通过自身携带的传感器以及借助外界传感器获取自身的位姿, 跟随者通过无线通信获取领航者的位姿并根据领航-跟随编队控制算法计算出期望跟随者的位姿, 第  $i$  个跟随者在控制律的作用下跟踪期望的跟随位置  $[x_{fi}, y_{fi}, \theta_{fi}]^T$ , 最终第  $i$  个跟随者的实际位置与期望跟随位置之间的误差缩减到零, 形成期望的编队. 综上可计算第  $i$  个跟随者的位置为

$$\begin{cases} x_{fi} = x_1 - d \cos \theta_1 + L_{fi} \cos(\varphi_{fi} + \theta_1), \\ y_{fi} = y_1 - d \sin \theta_1 + L_{fi} \sin(\varphi_{fi} + \theta_1), \\ \theta_{fi} = \theta_{fi}, \end{cases} \quad (2)$$

其中,

$$L_{fi} = \sqrt{(x_1 - d \cos \theta_1 - x_{fi})^2 + (y_1 - d \sin \theta_1 - y_{fi})^2}. \quad (3)$$

令

$$x_{fi} = x_1 - d \cos \theta_1 - x_{fi} = -L_{fi} \cos(\varphi_{fi} + \theta_1), \quad (4)$$

$$y_{fi} = y_1 - d \sin \theta_1 - y_{fi} = -L_{fi} \sin(\varphi_{fi} + \theta_1), \quad (5)$$

并且分别对式(4)、(5)求导得:

$$\begin{aligned} \dot{x}_{fi} &= \dot{x}_1 + d \sin \theta_1 \dot{\theta}_1 - \dot{x}_{fi} = \\ & \nu_1 \cos \theta_1 + d \omega_1 \sin \theta_1 - \nu_{fi} \cos \theta_{fi}, \end{aligned} \quad (6)$$

其中,  $\nu_1$  为领航者的前进速度,  $\nu_{fi}$  为第  $i$  个跟随者的前进速度,  $\omega_{fi}$  为第  $i$  个跟随者的角速度.

$$\begin{aligned} \dot{y}_{fi} &= \dot{y}_1 - d \cos \theta_1 \dot{\theta}_1 - \dot{y}_{fi} = \\ & \nu_1 \sin \theta_1 - d \omega_1 \cos \theta_1 - \nu_{fi} \sin \theta_{fi}. \end{aligned} \quad (7)$$

领航者与第  $i$  个跟随者之间的相对相角

$$\varphi_{fi} = \pi + \arctan\left(\frac{y_{fi}}{x_{fi}}\right) - \theta_1, \quad (8)$$

对式(8)求导可得:

$$\begin{aligned} \dot{\varphi}_{fi} &= \frac{1}{L_{fi}} (\nu_1 \sin \varphi_{fi} - \nu_{fi} \sin(\varphi_{fi} + \theta_1 - \theta_{fi}) + \\ & d \omega_1 \cos(\varphi_{fi} + \theta_1 - \theta_{fi})) - \omega_1. \end{aligned} \quad (9)$$

对式(3)求导并且代入式(4)~(8)得:

$$\begin{aligned} \dot{L}_{fi} &= \nu_{fi} \cos(\varphi_{fi} + \theta_1 - \theta_{fi}) - \nu_1 \cos \varphi_{fi} + \\ & d \omega_1 \sin(\varphi_{fi} + \theta_1 - \theta_{fi}). \end{aligned} \quad (10)$$

因此可以得出领航-跟随编队控制模型为

$$\begin{cases} \dot{L}_{fi} = \nu_{fi} \cos(\varphi_{fi} + \theta_1 - \theta_{fi}) - \nu_1 \cos \varphi_{fi} + \\ \quad d \omega_1 \sin(\varphi_{fi} + \theta_1 - \theta_{fi}), \\ \dot{\varphi}_{fi} = \frac{1}{L_{fi}} (\nu_1 \sin \varphi_{fi} - \nu_{fi} \sin(\varphi_{fi} + \theta_1 - \theta_{fi}) + \\ \quad d \omega_1 \cos(\varphi_{fi} + \theta_1 - \theta_{fi})) - \omega_1, \\ \dot{\theta}_{fi} = \omega_1 - \omega_{fi}. \end{cases} \quad (11)$$

第  $i$  个跟随者实际位姿与期望参考位姿之间的

误差方程<sup>[18]</sup> 定义为

$$\begin{bmatrix} x_{efi} \\ y_{efi} \\ \theta_{efi} \end{bmatrix} = \begin{bmatrix} \cos \theta_{fi} & \sin \theta_{fi} & 0 \\ -\sin \theta_{fi} & \cos \theta_{fi} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{rfi} - x_{fi} \\ y_{rfi} - y_{fi} \\ \theta_{rfi} - \theta_{fi} \end{bmatrix}. \quad (12)$$

对式(12)求导得:

$$\begin{cases} \dot{x}_{efi} = v_1 \cos(\theta_1 - \theta_{fi}) - v_{fi} + y_{efi} \omega_{fi} - \\ \quad \omega_1 L_{rfi} \sin(\varphi_{rfi} + \theta_1 - \theta_{fi}), \\ \dot{y}_{efi} = v_1 \sin(\theta_1 - \theta_{fi}) - x_{efi} \omega_{fi} - d \omega_{fi} + \\ \quad \omega_1 L_{rfi} \cos(\varphi_{rfi} + \theta_1 - \theta_{fi}), \\ \dot{\theta}_{efi} = \omega_{rfi} - \omega_{fi}, \end{cases} \quad (13)$$

其中,  $L_{rfi}$  为第  $i$  个跟随者参考点  $o_{fi}$  与领航者驱动轮轴中心点之间期望的参考距离,  $\varphi_{rfi}$  为领航者与第  $i$  个跟随者之间期望的参考相对相角.

## 2 多机器人编队滑模控制器设计

基于双幂次滑模趋近律,本节设计了编队控制算法,保证了编队队形的稳定.主要结果如下:

**定理 1** 对于第  $i$  个跟随者的实际位姿与期望参考位姿之间的误差方程(13),线速度控制器(14)和角速度控制器(15)能够保证跟踪误差收敛到零.

$$\begin{aligned} v_{fi} &= v_1 \cos(\theta_1 - \theta_{fi}) + y_{efi} \omega_{fi} - \\ &\quad \omega_1 L_{rfi} \sin(\varphi_{rfi} + \theta_1 - \theta_{fi}) + \\ &\quad \alpha_{f11} |s_{f1}|^{\frac{p_{f11}}{q_{f11}}} \text{sign}(s_{f1}) + \alpha_{f12} |s_{f1}|^{\frac{p_{f12}}{q_{f12}}} \text{sign}(s_{f1}), \quad (14) \\ \omega_{fi} &= \frac{1 + (v_1 y_{efi})^2}{1 + (v_1 y_{efi})^2 + v_1(x_{efi} + d)} \left( \omega_1 + \frac{\dot{v}_1 y_{efi}}{1 + (v_1 y_{efi})^2} + \right. \\ &\quad \left. \frac{v_1}{1 + (v_1 y_{efi})^2} (v_1 \sin(\theta_1 - \theta_{fi}) + \omega_1 L_{rfi} \cos(\varphi_{rfi} + \right. \\ &\quad \left. \theta_1 - \theta_{fi})) + \alpha_{f21} |s_{f2}|^{\frac{p_{f21}}{q_{f21}}} \text{sign}(s_{f2}) + \right. \\ &\quad \left. \alpha_{f22} |s_{f2}|^{\frac{p_{f22}}{q_{f22}}} \text{sign}(s_{f2}) \right), \quad (15) \end{aligned}$$

其中:  $\alpha_{f11}, \alpha_{f12}, \alpha_{f21}, \alpha_{f22} > 0; p_{f11} < q_{f11}, p_{f12} > q_{f12}, p_{f21} < q_{f21}, p_{f22} > q_{f22}$  均为正奇数;  $\text{sign}(x)$  为符号函数.

**证明** 选取如下滑模面:

$$s_{f1} = x_{efi}, \quad (16)$$

$$s_{f2} = \theta_{efi} + \arctan(v_1 y_{efi}), \quad (17)$$

分别对式(16)、(17)求导并将式(14)、(15)代入得:

$$\begin{aligned} \dot{s}_{f1} &= \dot{x}_{efi} = v_1 \cos(\theta_1 - \theta_{fi}) - v_{fi} + \\ &\quad y_{efi} \omega_{fi} - \omega_1 L_{rfi} \sin(\varphi_{rfi} + \theta_1 - \theta_{fi}) = \\ &\quad - \alpha_{f11} |s_{f1}|^{\frac{p_{f11}}{q_{f11}}} \text{sign}(s_{f1}) - \alpha_{f12} |s_{f1}|^{\frac{p_{f12}}{q_{f12}}} \text{sign}(s_{f1}), \quad (18) \end{aligned}$$

$$\dot{s}_{f2} = \dot{\theta}_{efi} + \arctan(v_1 y_{efi}) = \dot{\theta}_{efi} + \frac{v_1 y_{efi} + v_1 \dot{y}_{efi}}{1 + (v_1 y_{efi})^2} =$$

$$- \alpha_{f21} |s_{f2}|^{\frac{p_{f21}}{q_{f21}}} \text{sign}(s_{f2}) - \alpha_{f22} |s_{f2}|^{\frac{p_{f22}}{q_{f22}}} \text{sign}(s_{f2}), \quad (19)$$

满足如下双幂次趋近律:

$$\dot{s}_{f1} = - \alpha_{f11} |s_{f1}|^{\frac{p_{f11}}{q_{f11}}} \text{sign}(s_{f1}) - \alpha_{f12} |s_{f1}|^{\frac{p_{f12}}{q_{f12}}} \text{sign}(s_{f1}), \quad (20)$$

$$\dot{s}_{f2} = - \alpha_{f21} |s_{f2}|^{\frac{p_{f21}}{q_{f21}}} \text{sign}(s_{f2}) - \alpha_{f22} |s_{f2}|^{\frac{p_{f22}}{q_{f22}}} \text{sign}(s_{f2}). \quad (21)$$

选取李雅普诺夫函数

$$V = \frac{1}{2} s_{f1}^2 + \frac{1}{2} s_{f2}^2, \quad (22)$$

求导并代入式(20)、(21)可得:

$$\begin{aligned} \dot{V} &= s_{f1} \dot{s}_{f1} + s_{f2} \dot{s}_{f2} = - \alpha_{f11} |s_{f1}|^{1 + \frac{p_{f11}}{q_{f11}}} - \\ &\quad \alpha_{f12} |s_{f1}|^{1 + \frac{p_{f12}}{q_{f12}}} - \alpha_{f21} |s_{f2}|^{1 + \frac{p_{f21}}{q_{f21}}} - \\ &\quad \alpha_{f22} |s_{f2}|^{1 + \frac{p_{f22}}{q_{f22}}} \text{sign}(s_{f2}) \leq 0. \quad (23) \end{aligned}$$

由李雅普诺夫稳定性理论可知,跟踪误差收敛到零,从而保证了编队控制系统渐近稳定.

## 3 仿真结果及分析

本节通过仿真来验证本文给出的编队控制算法的有效性.不失一般性,考虑两个移动机器人进行编队的情况.领航者初始位姿为  $[3, 4, 0.2]^T$ , 跟随者初始位姿为  $[2, 2.5, 0.5]^T$ ,  $d = 0.25$  m, 给定领航者的参考速度  $v_r = 2$  m/s, 参考角速度  $\omega_r = 0.3$  rad/s, 跟随者参考点  $o_f$  与领航者驱动轮轴中心点之间期望的参考距离  $L_{rf} = 1$  m, 领航者与跟随者之间期望的参考相对相角  $\varphi_{rf} = 150^\circ$ . 控制器参数选取为  $\alpha_{f11} = 4, \alpha_{f12} = 12, \alpha_{f21} = 4, \alpha_{f22} = 12, p_{f11} = 3, p_{f12} = 15, q_{f11} = 15, q_{f12} = 5, p_{f21} = 3, p_{f22} = 15, q_{f21} = 15, q_{f22} = 5$ .

系统轨迹如图 3 所示,跟随者可以在较短的时间内到达期望位置并保持稳定的编队结构.跟随者的线速度、角速度分别如图 4 和图 5 所示,可知,跟随者的线速度和角速度也都在很短的时间内跟踪到领航者的速度.

## 4 总结

本文研究了轮式移动机器人的编队控制问题.基于领航跟随法,将编队控制问题转化为多个跟随者的轨迹跟踪控制问题.在此基础上,基于双幂次滑模趋近律,设计了跟随者的线速度和角速度控制器,保证了跟踪能够快速收敛到零,从而保证了编队队形的稳定.最后,通过仿真验证了本文所提方法的有效性.

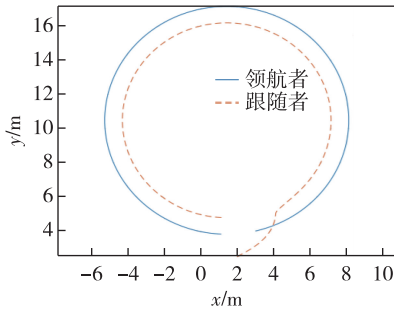


图3 领航者与跟随者编队轨迹

Fig. 3 Trajectories of the leader and follower

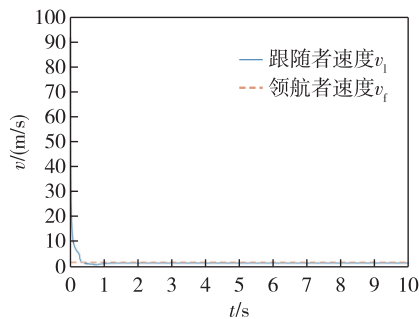


图4 跟随者跟踪领航者速度曲线

Fig. 4 Velocities of the leader and follower

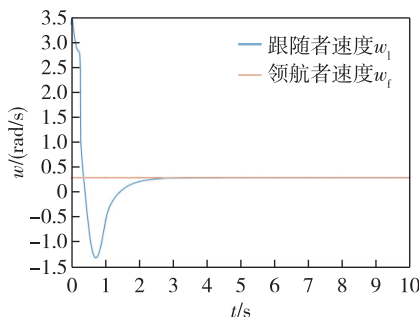


图5 跟随者跟踪领航者角速度曲线

Fig. 5 Angular velocities of the leader and follower

## 参考文献

### References

- [ 1 ] Tzafestas S G. Mobile robot control and navigation: a global overview[J]. Journal of Intelligent & Robotic Systems, 2018, 91(1): 35-58
- [ 2 ] Samson C, Morin P, Lenain R. Modeling and control of wheeled mobile robots[M] // Springer Handbook of Robotics. Cham: Springer International Publishing, 2016: 1235-1266
- [ 3 ] Burgard W, Moors M, Fox D, et al. Collaborative multi-robot exploration[C] // IEEE International Conference on Robotics and Automation, 2000: 476-481
- [ 4 ] Marco D B, Healey A J. Command, control, and navigation experimental results with the NPS ARIES AUV[J]. IEEE Journal of Oceanic Engineering, 2001, 26(4): 466-476
- [ 5 ] Ghosh A, Ghosh A, Konar A, et al. Multi-robot cooperative box-pushing problem using multi-objective particle swarm optimization technique[C] // World Congress on Information and Communication Technologies, 2012: 272-277
- [ 6 ] Oh K K, Park M C, Ahn H S. A survey of multi-agent formation control[J]. Automatica, 2015, 53: 424-440
- [ 7 ] Scharf D P, Hadaegh F Y, Ploen S R. A survey of spacecraft formation flying guidance and control. Part II: control[C] // Proceedings of the 2004 American Control Conference, 2004: 2976-2985
- [ 8 ] Desai J P, Ostrowski J, Kumar V. Controlling formations of multiple mobile robots[C] // IEEE International Conference on Robotics and Automation, 1998: 2864-2869
- [ 9 ] Das A K, Fierro R, Kumar V, et al. A vision-based formation control framework[J]. IEEE Transactions on Robotics and Automation, 2002, 18(5): 813-825
- [ 10 ] Consolini L, Morbidi F, Prattichizzo D, et al. Leader-follower formation control of nonholonomic mobile robots with input constraints[J]. Automatica, 2008, 44(5): 1343-1349
- [ 11 ] Loria A, Dastemir J, Alvarez Jarquin N. Leader-follower formation and tracking control of mobile robots along straight paths[J]. IEEE Transactions on Control Systems Technology, 2016, 24(2): 727-732
- [ 12 ] Wang H S, Guo D J, Liang X W, et al. Adaptive vision-based leader-follower formation control of mobile robots[J]. IEEE Transactions on Industrial Electronics, 2017, 64(4): 2893-2902
- [ 13 ] Sun Z J, Zhang G Q, Lu Y, et al. Leader-follower formation control of underactuated surface vehicles based on sliding mode control and parameter estimation[J]. ISA Transactions, 2018, 72: 15-24
- [ 14 ] 冯勇, 鲍晟, 余星火. 非奇异终端滑模控制系统的设计方法[J]. 控制与决策, 2002, 17(2): 194-198  
FENG Yong, BAO Sheng, YU Xinghuo. Design method of non-singular terminal sliding mode control systems[J]. Control and Decision, 2002, 17(2): 194-198
- [ 15 ] 李升波, 李克强, 王建强, 等. 非奇异快速的终端滑模控制方法[J]. 信息与控制, 2009, 38(1): 1-8  
LI Shengbo, LI Keqiang, WANG Jianqiang, et al. Nonsingular and fast terminal sliding mode control method[J]. Information and Control, 2009, 38(1): 1-8
- [ 16 ] 张合新, 范金锁, 孟飞, 等. 一种新型滑模控制双幂次趋近律[J]. 控制与决策, 2013, 28(2): 289-293  
ZHANG Hexin, FAN Jinsuo, MENG Fei, et al. A new double power reaching law for sliding mode control[J]. Control and Decision, 2013, 28(2): 289-293
- [ 17 ] 华玉龙, 孙伟, 迟宝山, 等. 非奇异快速终端滑模控制[J]. 系统工程与电子技术, 2017, 39(5): 1119-1125  
HUA Yulong, SUN Wei, CHI Baoshan, et al. Study on nonsingular fast terminal sliding mode control[J]. Systems Engineering and Electronics, 2017, 39(5): 1119-1125
- [ 18 ] Kanayama Y, Kimura Y, Miyazaki F, et al. A stable tracking control method for an autonomous mobile robot[C] // IEEE International Conference on Robotics and Automation, 1990: 384-389

## Formation control of the multi-robot system based on dualpower sliding mode

LI Tianlong<sup>1</sup> BAI Jianjun<sup>2</sup>

1 School of Modern Information Technology,Zhejiang Institute of Mechanical and Electrical Engineering, Hangzhou 310053

2 School of Automation (Artificial Intelligence), Hangzhou Dianzi University, Hangzhou 310018

**Abstract** The leader-follower formation control of the multi-robot system is studied in this paper. The expected trajectory of the follower is given according to the formation constraints, thus the formation issue is turned into trajectory tracking of a single follower. Then the linear velocity controller and angular velocity controller for the follower are designed based on dual power reaching law method, which can ensure that the tracking error quickly converge to zero, so as to keep the formation stability. Finally, a numerical example is given to verify the effectiveness of the proposed method.

**Key words** multi-robot formation; leader follower; dual power reaching law