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高阶非线性系统自适应模糊有限时间状态约束控制

摘要

针对一类具有状态约束的非严格反馈高阶非线性系统,研究一种自适应模糊有限时间跟踪控制问题.首先,利用模糊逻辑系统逼近不确定性非线性函数,在此基础上,采用障碍 Lyapunov 函数,解决状态约束问题,通过障碍加幂积分方法和反步递推技术,提出了一种有限时间控制设计方法.在有限时间 Lyapunov 稳定意义下,严格证明闭环系统半全局实际有限时间稳定且系统的状态不超出给定的约束边界,并实现了有限时间跟踪控制目标.最后,仿真研究进一步验证了所提出控制方法的有效性.

关键词

有限时间稳定;模糊控制;加幂积分;障碍 Lyapunov 函数;高阶非线性系统

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0 引言

在过去十几年中,学者们对高阶非线性系统控制问题的关注度逐年提高,并取得了一些有价值的理论成果[1-4].相对于文献[5-6]中严格反馈非线性系统,高阶非线性系统在线性化过程中存在不可控情况,同时虚拟和实际控制输入中存在指数幂次 p_i,这增加了控制器的设计难度,因此传统的反步递推方法不再适用.为了解决高阶非线性系统的控制问题,Lin 等[1]在反步递推方法的基础上首次提出了加幂积分控制技术.随后,许多学者利用加幂积分方法对高阶非线性系统进行了广泛研究.文献[2-3]针对单输入单输出高阶非线性系统,研究了状态反馈和输出反馈控制设计问题,在 Lyapunov 稳定意义下,保证了被控系统渐近稳定.

值得注意的是,渐近稳定不能给被控系统提供更高精度的控制方案.在飞行器姿态、感应电机等实际控制中,一般希望控制系统能在有限时间内满足期望的控制性能.此外,有限时间控制具有许多潜在的好处,如较强的收敛速度和较强的鲁棒性等.因此,近几年有限时间控制问题的研究取得了很大的进展.文献[7-8]通过加幂积分技术和齐次占优技术,研究了高阶非线性系统的全局有限时间控制问题;文献[9]针对一类具有参数不确定性的非线性系统在给定的瞬态指标基础上,提出了一种自适应有限时间控制方法;文献[10]基于扰动观测器讨论了一类不确定非线性系统的终端滑模控制;文献[11-12]分别针对严格反馈和非严格反馈非线性系统,基于自适应模糊和神经网络控制方法,在有限时间 Lyapunov 稳定理论的框架下,保证了被控系统实际有限时间稳定.虽然上述工作[7-12]对非线性系统的有限时间控制设计问题进行了相应的研究,但仍存在一些问题值得进一步研究.

除了稳定性问题,当考虑到性能规格以及安全等因素,非线性系统会受到状态或输出的约束,在系统运行期间,如果状态或输出违反约束条件,可能会使系统性能下降或损坏.文献[13]结合时变非对称障碍 Lyapunov 函数,对一类具有全状态约束的严格反馈非线性系统,提出了一种自适应跟踪控制方案;文献[14]对一类非三角结构系统,在考虑执行器故障和误差约束的条件下,提出了一种自适应模糊控制方案.通过结合 tan 型障碍 Lyapunov 函数,文献[15-16]针对具有部分状态约束和全状态约束的高阶非线性系统,分别构造了两种不同

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的控制器.但是上述工作[13-16]均未考虑有限时间控制.在实际工程中,如机器人机械手、电机系统等,不仅存在状态约束现象,而且需要考虑系统的收敛时间问题.近些年来,许多学者致力于非线性系统的状态约束和有限时间控制问题的研究,并取得了许多具有标志性的成果[17-20].文献[17-18]分别针对具有死区非线性和切换行为的非线性系统,研究了有限时间全状态约束控制设计问题;文献[20]将文献[17-18]所提出的控制设计算法应用到了实际直流电机系统.在文献[17-20]所考虑的非线性系统中,虚拟控制和实际控制的幂次均为1.

受到以上研究成果的启发,本文针对一类非严格反馈高阶非线性系统,提出一种自适应模糊有限时间跟踪控制方法.结合反步递推法、加幂积分技术和障碍 Lyapunov 函数,设计了一种自适应有限时间控制器.所设计的控制器能够同时确保输出有限时间内跟踪给定的参考信号,且闭环系统的状态不超出给定的约束边界.与现有的文献相比,本文的贡献概括为以下三方面:

1) 结合模糊逻辑系统辨识未知非线性函数,放宽了文献[1-4,7-8]中系统非线性函数项需要已知或满足类似于不等式 $|g_i(x)| \leqslant \lambda_i(\bar{x}_i)(|x_1|^{\frac{r_i+\tau}{r_1}} + |x_2|^{\frac{r_i+\tau}{r_2}} + \cdots + |x_i|^{\frac{r_i+\tau}{r_i}})$ 的线性增长条件,其中,$\lambda_i(\bar{x}_i)$ 是一个已知的光滑函数.在本文中,非线性函数 $g_i(x)$ 是完全未知的,不需要满足上述假设.

2) 相较于文献[17-20]系统幂次为1的约束问题,本文解决了控制输入幂次是正奇数比的非线性系统的状态约束控制问题.

3) 本文所设计的有限时间控制器不仅可以保证系统状态不超出给定约束边界,而且能确保闭环系统的所有信号在有限时间内收敛到包含原点的一个小邻域内.而文献[15-16]在系统满足状态约束条件下,没有考虑闭环系统收敛时间.

1 模型描述和预备知识

1.1 模型描述

考虑如下一类非线性系统:

$$\begin{cases} \dot{x}_1 = h_1(\boldsymbol{x}) x_2^{p_1} + g_1(\boldsymbol{x}), \\ \dot{x}_2 = h_2(\boldsymbol{x}) x_3^{p_2} + g_2(\boldsymbol{x}), \\ \quad\vdots \\ \dot{x}_n = h_n(\boldsymbol{x}) u^{p_n} + g_n(\boldsymbol{x}), \\ y = x_1, \end{cases} \tag{1}$$

其中:$\boldsymbol{x} = [x_1, x_2, \cdots, x_n]^{\mathrm{T}} \in \mathbf{R}^n$ 是系统的状态向量;$u \in \mathbf{R}$ 和 $y \in \mathbf{R}$ 分别是系统的控制输入和输出;$h_i(\boldsymbol{x})$ 和 $g_i(\boldsymbol{x})$ 是光滑未知非线性函数;指数 $p_i(i = 1,2,\cdots,n)$ 是两个正奇数的比值.$\boldsymbol{x}(0) = [0,\cdots,0]^{\mathrm{T}}$ 是系统的平衡点.对于任意给定的正常数 k_{ci},系统所有的状态收敛到给定的紧集内,即 $\Omega_{x_i} = \{x_i \in \mathbf{R} \mid |x_i| < k_{ci}(i = 1,2,\cdots,n)\}$.

假设 1[15] 存在常数 $h_i > 0$ 和 $\bar{h}_i > 0, i = 1,2,\cdots,n$,非线性函数 $h_i(x)$ 满足如下不等式:

$$0 < h_i \leqslant h_i(\boldsymbol{x}) \leqslant \bar{h}_i. \tag{2}$$

1.2 预备知识

引理 1[6,14] 在紧集 Ω 上定义连续函数 $h(\boldsymbol{x})$,对于任意给定正常数 $\eta > 0$,存在模糊逻辑系统 $\boldsymbol{\Theta}^{*\mathrm{T}}\boldsymbol{\varphi}(\boldsymbol{x})$,满足:

$$\sup_{x \in \Omega} |h(\boldsymbol{x}) - \boldsymbol{\Theta}^{*\mathrm{T}}\boldsymbol{\varphi}(\boldsymbol{x})| \leqslant \eta, \tag{3}$$

其中,$\boldsymbol{\varphi}(\boldsymbol{x}) = [\varphi_1(\boldsymbol{x}),\cdots,\varphi_N(\boldsymbol{x})]^{\mathrm{T}}$ 是模糊基函数,η 是最小模糊逼近误差,$\boldsymbol{\Theta}^* = [\boldsymbol{\Theta}_1^*,\cdots,\boldsymbol{\Theta}_N^*]^{\mathrm{T}}$ 是真值权重向量,且 N 是模糊规则数.

引理 2[17-19] 考虑如下非线性系统:

$$\dot{\boldsymbol{x}} = g(\boldsymbol{x}), g(0) = 0, \boldsymbol{x} \in \mathbf{R}^n \tag{4}$$

存在连续可微正定函数 $V(\boldsymbol{x})$ 和常数 $c > 0, \rho > 0, \alpha \in (0,1)$,满足 $\dot{V}(\boldsymbol{x}) \leqslant -cV^{\alpha}(\boldsymbol{x}) + \rho$,则系统(4)是半全局实际有限时间稳定,即:对于所有 $\boldsymbol{x}(t_0) = x_0$,存在一个常数 $\varepsilon > 0$ 和驻留时间 $T(\varepsilon, x_0) < \infty$,使得对于所有 $t \geqslant t_0 + T$ 有 $\|\boldsymbol{x}\| < \varepsilon$.

控制目标:对于系统(1),设计有限时间自适应模糊控制器,使得:

1) 闭环系统的所有信号在有限时间内收敛到包含原点的一个小邻域内;

2) 系统输出在有限时间内跟踪给定信号 $y_r(|y_r| \leqslant y_0, y_0$ 是已知正常数);

3) 系统所有状态收敛到一个给定的约束边界内.

2 模糊有限时间控制器设计和稳定性分析

在本节中,基于自适应反步递推方法和加幂积分技术,设计模糊自适应有限时间控制器,同时依据引理2,结合所设计的控制器证明系统(1)是半全局实际有限时间稳定的.

2.1 模糊有限时间控制器设计

定义坐标转换:

$$\begin{cases}\beta_1 = x_1 - y_r, \\ \beta_i = x_i^{r_i} - (x_i^*)^{r_i}, i = 2, \cdots, n,\end{cases} \tag{5}$$

其中,β_i 是虚拟误差,x_i^* 是虚拟控制器,实际控制器 u 将会在最后一步设计给出.定义 $1 + p_i/r_{i+1} = 1/r_i + \tau$,设计参数 $r_{i,1} = 1, r_i > 1\ (i = 2, \cdots, n)$,则 $\tau \in (0,1)$.

步骤 1.根据式(1)和(5),β_1 的导数为

$$\dot{\beta}_1 = h_1(\boldsymbol{x}) x_2^{p_1} + g_1(\boldsymbol{x}) - \dot{y}_r. \tag{6}$$

定义如下障碍 Lyapunov 函数:

$$V_1 = \frac{1}{2}\log \frac{k_{b1}^2}{k_{b1}^2 - \beta_1^2} + \frac{1}{2\eta_1}\tilde{\theta}_1^2, \tag{7}$$

其中,η_1 是正设计参数,$\tilde{\theta}_1$ 是 $\theta_1^* = \|\boldsymbol{\Theta}_1^*\|^{1+\tau}$ 的估计,$\tilde{\theta}_1 = \theta_1^* - \hat{\theta}_1$ 是参数估计误差.在紧集 $\Omega_{\beta_1} = \{\beta_1 \mid |\beta_1| < k_{b1}\}$ 中,V_1 是连续的.根据式(6)和(7),可得:

$$\dot{V}_1 = \frac{\beta_1 \dot{\beta}_1}{k_{b1}^2 - \beta_1^2} + \frac{1}{\eta_1}\tilde{\theta}_1 \dot{\tilde{\theta}}_1 = \frac{\beta_1}{k_{b1}^2 - \beta_1^2}(h_1(\boldsymbol{x}) x_2^{p_1} + g_1(\boldsymbol{x}) - \dot{y}_r) + \frac{1}{\eta_1}\tilde{\theta}_1 \dot{\tilde{\theta}}_1. \tag{8}$$

因为 $g_1(\boldsymbol{x})$ 是未知光滑非线性函数,根据引理1,可以使用模糊逻辑系统 $\hat{\bar{g}}(z_1 \mid \hat{\boldsymbol{\Theta}}_1) = \hat{\boldsymbol{\Theta}}_1^{\mathrm{T}}\boldsymbol{\varphi}(z_1)\ (z_1 = [\boldsymbol{x}, y_r, \dot{y}_r]^{\mathrm{T}})$ 逼近未知函数 $\bar{g}_1(z_1) = \dfrac{g_1(\boldsymbol{x}) - \dot{y}_r}{k_{b1}^2 - \beta_1^2}$,并假设:

$$\bar{g}_1 = \boldsymbol{\Theta}_1^{*\mathrm{T}}\boldsymbol{\varphi}_1(z_1) + \varepsilon_1(z_1), \tag{9}$$

其中,$|\varepsilon(z_1)| \leqslant \varepsilon_1^*$,且 ε_1^* 是正常数.将式(9)代入式(8)中,可得:

$$\dot{V}_1 = \frac{\beta_1}{k_{b1}^2 - \beta_1^2}h_1(\boldsymbol{x}) x_2^{p_1} + \beta_1(\boldsymbol{\Theta}_1^{*\mathrm{T}}\boldsymbol{\varphi}_1(z_1) + \varepsilon_1(z_1)) + \frac{1}{\eta_1}\tilde{\theta}_1 \dot{\tilde{\theta}}_1. \tag{10}$$

根据 Young 不等式[14]:

$$\boldsymbol{a}^{\mathrm{T}}\boldsymbol{b} \leqslant \frac{m^\mu}{\mu}\|\boldsymbol{a}\|^\mu + \frac{1}{\lambda m^\lambda}\|\boldsymbol{b}\|^\lambda, \tag{11}$$

其中,$\boldsymbol{a}, \boldsymbol{b} \in \mathbf{R}^n, m > 0, \mu > 1, \lambda > 1$,并且 $(\mu - 1)(\lambda - 1) = 1$.由式(11),可得:

$$\beta_1 \boldsymbol{\Theta}_1^{*\mathrm{T}}\boldsymbol{\varphi}_1(z_1) + \beta_1 \varepsilon_1(z_1) \leqslant \beta_1^{1+\tau} \upsilon_1^{1+\tau}\|\boldsymbol{\varphi}_1\|^{1+\tau}\theta_1^* + \upsilon_1^{-(1+\tau)/\tau} + \beta_1^{1+\tau}\sigma_1^{1+\tau} + \sigma_1^{-(1+\tau)/\tau}\varepsilon_1^{*(1+\tau)/\tau}, \tag{12}$$

其中,$\upsilon_1 > 0, \sigma_1 > 0$ 是设计参数.将式(12)代入式(10),可得:

$$\dot{V}_1 \leqslant \frac{\beta_1}{k_{b1}^2 - \beta_1^2}h_1(\boldsymbol{x})(x_2^*)^{p_1} + \frac{\beta_1}{k_{b1}^2 - \beta_1^2}[(\beta_1^\tau \upsilon_1^{1+\tau}\|\boldsymbol{\varphi}_1\|^{1+\tau}\hat{\theta}_1 + \beta_1^\tau \sigma_1^{1+\tau})(k_{c1}^2 - \beta_1^2)] - \frac{1}{\eta_1}\tilde{\theta}_1(\eta_1 \beta_1^{1+\tau}\upsilon_1^{1+\tau}\|\boldsymbol{\varphi}_1\|^{1+\tau} - \dot{\hat{\theta}}_1) + \frac{\beta_1}{k_{b1}^2 - \beta_1^2}h_1(\boldsymbol{x})(x_2^{p_1} - (x_2^*)^{p_1}) + a_1, \tag{13}$$

其中 $a_1 = \upsilon_1^{-(1+\tau)/\tau} + \sigma_1^{-(1+\tau)/\tau}\varepsilon_1^{*(1+\tau)/\tau}$.

设计虚拟控制函数 x_2^* 和参数 $\hat{\theta}_1$ 的自适应律为

$$x_2^* = -\mu_1 \beta_1^{1/r_2}, \tag{14}$$

$$\dot{\hat{\theta}}_1 = \eta_1 \upsilon_1^{1+\tau}\beta_1^{1+\tau}\|\boldsymbol{\varphi}_1\|^{1+\tau} - \gamma_1 \hat{\theta}_1, \tag{15}$$

其中,$\mu_1 = [((n + \upsilon_1^{1+\tau}\|\boldsymbol{\varphi}_1\|^{1+\tau}\hat{\theta}_1 + \sigma_1^{1+\tau})(k_{b1}^2 - \beta_1^2))/h_1]^{1/p_1}$,$\gamma_1 > 0$ 是设计参数.

根据式(13),(14)和(15),$\dot{V}_1$ 可改写为

$$\dot{V}_1 \leqslant -n\frac{\beta_1^{1+\tau}}{k_{b1}^2 - \beta_1^2} + \frac{\beta_1}{k_{b1}^2 - \beta_1^2}h_1(\boldsymbol{x})(x_2^{p_1} - (x_2^*)^{p_1}) + \frac{\gamma_1}{\eta_1}\hat{\theta}_1\tilde{\theta}_1 + a_1. \tag{16}$$

式(16)的第二项结合不等式 $|a|^m |b|^n \leqslant \dfrac{m}{(m+n)}\gamma |a|^{m+n} + \dfrac{n}{(m+n)}\gamma^{\frac{-m}{n}}|b|^{m+n}$($m, n$ 和 γ 是正常数)[17],可得:

$$\frac{\beta_1}{k_{b1}^2 - \beta_1^2}h_1(\boldsymbol{x})(x_2^{p_1} - (x_2^*)^{p_1}) = h_1(\boldsymbol{x})\frac{\beta_1}{k_{b1}^2 - \beta_1^2}\left((x_2^{r_2})^{\frac{p_1}{r_2}} - ((x_{i,2}^*)^{r_2})^{\frac{p_1}{r_2}}\right) \leqslant \frac{\beta_1^{1+\tau}}{k_{b1}^2 - \beta_1^2} + e_2 \beta_2^{1+\tau}, \tag{17}$$

其中,$e_2 > 0$ 是一个设计参数.

根据式(17),式(16)可重写为

$$\dot{V}_1 \leqslant -(n-1)\frac{\beta_1^{1+\tau}}{k_{b1}^2 - \beta_1^2} + e_2 \beta_2^{1+\tau} + \frac{\gamma_1}{\eta_1}\hat{\theta}_1\tilde{\theta}_1 + a_1. \tag{18}$$

步骤 $i\,(2 \leqslant i \leqslant n-1)$.在这一步中,引入障碍加幂积分技术.定义如下标量函数

$$K_i = \int_{x_i^*}^{x_i}\frac{\zeta_i(w)^{2-1/r_i}}{k_{bi}^2 - \zeta_i^2(w)}\mathrm{d}w, \tag{19}$$

其中,$\zeta_i(w) = w^{r_i} - (x_i^*)^{r_i}$.

选取如下 Lyapunov 函数:

$$V_i = V_{i-1} + K_i + \frac{1}{2\eta_i}\tilde{\theta}_i^2, \tag{20}$$

其中, $\hat{\theta}_i$ 是 $\theta_i^* = \|\boldsymbol{\Theta}_i^*\|^{1+\tau}$ 的估计, $\tilde{\theta}_i = \theta_i^* - \hat{\theta}_i$ 是参数估计误差.在紧集 $\Omega_{\beta_i} = \{\beta_i \mid |\beta_i| < k_{bi}\}$ 中, V_i 是关于时间 t 的连续正定函数.式(18) 对时间 t 求导,可得:

$$\dot{V}_i = \dot{V}_{i-1} + \frac{\partial K_i}{\partial x_i}\dot{x}_i + \frac{\partial K_i}{\partial x_{i-1}}\dot{x}_{i-1} + \frac{\partial K_i}{\partial \hat{\theta}_{i-1}}\dot{\hat{\theta}}_{i-1} + \frac{1}{\eta_i}\tilde{\theta}_i\dot{\tilde{\theta}}_i. \tag{21}$$

根据式(19),可得:

$$\frac{\partial K_i}{\partial x_i \dot{x}_i} = \frac{(x_i^{r_i} - (x_i^*)^{r_i})^{2-1/r_i}}{k_{bi}^2 - \beta_i^2}\dot{x}_i = \frac{\beta_i^{2-1/r_i}}{k_{bi}^2 - \beta_i^2}(h_i(\boldsymbol{x})x_{i+1}^{p_i} + g_i(x)), \tag{22}$$

$$\frac{\partial K_i}{\partial x_{i-1}\dot{x}_{i-1}} = -\frac{1}{r_i}\frac{\partial (x_i^*)^{r_i}}{\partial x_{i-1}\dot{x}_{i-1}} \times \int_{x_i^*}^{x_i}\frac{\zeta_i(w)^{1-1/r_{i,2}}}{k_{bi}^2 - \zeta_i^2(w)}\mathrm{d}w. \tag{23}$$

根据式(23)和不等式 $|a^\eta - b^\eta| \leqslant 2^{1-\eta}|a - b|^\eta$ [18] $(a, b \in \mathbf{R}$ 且 $0 < \eta < 1)$,可得:

$$\left|\frac{\partial K_i}{\partial x_{i-1}\dot{x}_{i-1}}\right| = \frac{1}{r_i}\frac{\partial (x_i^*)^{r_i}}{\partial x_{i-1}\dot{x}_{i-1}} \times |x_i - x_i^*| \left|\frac{(x_i^{r_i} - (x_i^*)^{r_i})^{1-1/r_i}}{k_{bi}^2 - \beta_i^2(w)}\right| \leqslant \frac{1}{r_i}|(x_i^{r_i})^{1/r_i} - ((x_i^*)^{r_i})^{1/r_i}| \times \left|\frac{\partial (x_i^*)^{r_i}}{\partial x_{i-1}\dot{x}_{i-1}}\right| \times \left|\frac{[(x_i^{r_i} - (x_i^*)^{r_i})^{1-1/r_i}]}{[k_{bi}^2 - \beta_i^2(w)]}\right| \leqslant b_i\left|\frac{\beta_i}{k_{bi}^2 - \beta_i^2(w)}\right|\left|\frac{\partial (x_i^*)^{r_i}}{\partial x_{i-1}\dot{x}_{i-1}}\right|, \tag{24}$$

其中, $b_i \geqslant (1/r_i) \times 2^{1-1/r_i}$.

同理可得:

$$\left|\frac{\partial K_i}{\partial \hat{\theta}_{i-1}\dot{\hat{\theta}}_{i-1}}\right| \leqslant b_i\left|\frac{\beta_i}{k_{bi}^2 - \beta_i^2(w)}\right|\left|\frac{\partial (x_i^*)^{r_i}}{\partial \hat{\theta}_{i-1}\dot{\hat{\theta}}_{i-1}}\right|. \tag{25}$$

将式(22)—(25)代入式(21),可得:

$$\dot{V}_i \leqslant \sum_{j=1}^{i-1} -(n-i+2)\frac{\beta_j^{1+\tau}}{k_{bj}^2 - \beta_j^2} + \sum_{j=1}^{i-1}\left(\frac{\gamma_j}{\eta_j}\hat{\theta}_j\tilde{\theta}_j + a_j\right) + \frac{\beta_i^{1+\tau}}{k_{bi}^2 - \beta_i^2} + e_i\beta_i^{1+\tau} + \frac{\beta_i^{2-1/r_i}}{k_{bi}^2 - \beta_i^2}h_i(\boldsymbol{x})(x_{i+1}^*)^{p_i} + \frac{\beta_i^{2-1/r_i}}{k_{bi}^2 - \beta_i^2}g_i(\boldsymbol{x}) + \sum_{j=1}^{i-1}b_i\left|\frac{\beta_i}{k_{bi}^2 - \beta_i^2(w)}\right|\left(\left|\frac{\partial (x_i^*)^{r_i}}{\partial x_j}\dot{x}_j\right| + \left|\frac{\partial (x_i^*)^{r_i}}{\partial \hat{\theta}_j\dot{\hat{\theta}}_j}\right|\right) + \frac{\beta_i^{2-1/r_i}}{k_{bi}^2 - \beta_i^2}h_i(\boldsymbol{x})(x_{i+1}^{p_i} - (x_{i+1}^*)^{p_i}) + \frac{1}{\eta_i}\tilde{\theta}_i\dot{\tilde{\theta}}_i. \tag{26}$$

令

$$\bar{g}_i = e_i\beta_i^\tau + \frac{\beta_i^{1-1/r_i}}{k_{bi}^2 - \beta_i^2}g_i(\boldsymbol{x}) + \sum_{j=1}^{i-1}b_i\left|\frac{1}{k_{bi}^2 - \beta_i^2(w)}\right| \times \left(\left|\frac{\partial (x_i^*)^{r_i}}{\partial x_j\dot{x}_j}\right| + \left|\frac{\partial (x_i^*)^{r_i}}{\partial \hat{\theta}_j\dot{\hat{\theta}}_j}\right|\right). \tag{27}$$

由于 $g_i(\boldsymbol{x})$ 是未知光滑非线性函数,根据引理1,利用模糊逻辑系统 $\hat{\bar{g}}_i(z_i \mid \hat{\boldsymbol{\Theta}}_i) = \hat{\boldsymbol{\Theta}}_i^{\mathrm{T}}\boldsymbol{\varphi}(z_i)$, $z_i = [\boldsymbol{x}, \hat{\theta}_i]^{\mathrm{T}}$ 逼近未知函数 $\bar{g}_i(z_i)$,并假设:

$$\bar{g}_i = \boldsymbol{\Theta}_i^{*\mathrm{T}}\boldsymbol{\varphi}_i(z_i) + \varepsilon_i(z_i), \tag{28}$$

其中, $|\varepsilon(z_i)| \leqslant \varepsilon_i^*$,且 ε_i^* 是正常数.将式(28) 代入式(26) 中,可得:

$$\dot{V}_i \leqslant \sum_{j=1}^{i-1} -(n-i+1)\frac{\beta_j^{1+\tau}}{k_{bj}^2 - \beta_j^2} + \sum_{j=1}^{i-1}\left(\frac{\gamma_j}{\eta_j}\hat{\theta}_j\tilde{\theta}_j + a_j\right) + \frac{\beta_i^{2-1/r_i}}{k_{bi}^2 - \beta_i^2}h_i(\boldsymbol{x})(x_{i+1}^*)^{p_i} + \beta_i(\boldsymbol{\Theta}_i^{*\mathrm{T}}\boldsymbol{\varphi}_i(z_i) + \varepsilon_i(z_i)) + \frac{1}{\eta_i}\tilde{\theta}_i\dot{\tilde{\theta}}_i + \frac{\beta_i^{2-1/r_i}}{k_{bi}^2 - \beta_i^2}h_i(\boldsymbol{x})(x_{i+1}^{p_i} - (x_{i+1}^*)^{p_i}). \tag{29}$$

根据 Young 不等式,可得:

$$\beta_i\boldsymbol{\Theta}_i^{*\mathrm{T}}\boldsymbol{\varphi}_i(z_i) + \beta_i\varepsilon_i(z_i) \leqslant \beta_i^{1+\tau}\boldsymbol{\upsilon}_i^{1+\tau}\|\boldsymbol{\varphi}_i\|^{1+\tau}\theta_i^* + \boldsymbol{\upsilon}_i^{-(1+\tau)/\tau} + \beta_i^{1+\tau}\boldsymbol{\sigma}_i^{1+\tau} + \boldsymbol{\sigma}_i^{-(1+\tau)/\tau}\varepsilon_i^{*(1+\tau)/\tau}, \tag{30}$$

其中, $\boldsymbol{\upsilon}_i > 0, \boldsymbol{\sigma}_i > 0$ 是设计参数.

将式(30) 代入式(29),可得:

$$\dot{V}_i \leqslant \sum_{j=1}^{i-1} -(n-i+1)\frac{\beta_j^{1+\tau}}{k_{bj}^2 - \beta_j^2} + \sum_{j=1}^{i-1}\left(\frac{\gamma_j}{\eta_j}\hat{\theta}_j\tilde{\theta}_j + a_j\right) + \frac{\beta_i^{2-1/r_i}}{k_{bi}^2 - \beta_i^2}h_i(\boldsymbol{x})(x_{i+1}^*)^{p_i} + \frac{\beta_i}{k_{bi}^2 - \beta_i^2}[(\beta_i^\tau\boldsymbol{\upsilon}_i^{1+\tau}\|\boldsymbol{\varphi}_i\|^{1+\tau}\hat{\theta}_i + \beta_i^\tau\boldsymbol{\sigma}_i^{1+\tau})(k_{bi}^2 - \beta_i^2)] - \frac{1}{\eta_i}\tilde{\theta}_i(\eta_i\beta_i^{1+\tau}\boldsymbol{\upsilon}_i^{1+\tau}\|\boldsymbol{\varphi}_i\|^{1+\tau} - \dot{\hat{\theta}}_i) + \frac{\beta_i^{2-1/r_i}}{k_{bi}^2 - \beta_i^2}h_i(\boldsymbol{x})(x_{i+1}^{p_i} - (x_{i+1}^*)^{p_i}) + a_i, \tag{31}$$

其中 $a_i = \boldsymbol{\upsilon}_i^{-(1+\tau)/\tau} + \boldsymbol{\sigma}_i^{-(1+\tau)/\tau}\varepsilon_i^{*(1+\tau)/\tau}$.

设计虚拟控制函数 x_{i+1}^* 和参数 $\hat{\theta}_i$ 的自适应律为

$$x_{i+1}^* = -\mu_i\beta_i^{1/r_{i+1}}, \tag{32}$$

$$\dot{\hat{\theta}}_i = \eta_i\upsilon_i^{1+\tau}\beta_i^{1+\tau}\|\boldsymbol{\varphi}_i\|^{1+\tau} - \gamma_i\hat{\theta}_i, \tag{33}$$

其中, $\mu_i = [(((n-i+1) + \upsilon_i^{1+\tau}\|\boldsymbol{\varphi}_i\|^{1+\tau}\hat{\theta}_i + \boldsymbol{\sigma}_i^{1+\tau})(k_{bi}^2 - \beta_i^2))/h_i]^{1/p_i}$, $\gamma_i > 0$ 是设计参数.

根据式(31),(32)和(33),$\dot{V}_i$ 可改写为

$$\dot{V}_i \leqslant -(n-i+1)\sum_{j=1}^{i}\frac{\beta_j^{1+\tau}}{k_{bj}^2-\beta_j^2}+\sum_{j=1}^{i}\left(\frac{\gamma_j}{\eta_j}\hat{\theta}_j\tilde{\theta}_j+a_j\right)+\frac{\beta_i^{2-1/r_i}}{k_{bi}^2-\beta_i^2}h_i(\boldsymbol{x})(x_{i+1}^{p_i}-(x_{i+1}^*)^{p_i}). \quad (34)$$

步骤 n.选取如下 Lyapunov 函数:

$$V_n=V_{n-1}+K_n+\frac{1}{2\eta_n}\tilde{\theta}_n^2, \quad (35)$$

其中,$\hat{\theta}_n$ 是 $\theta_n^*=\|\boldsymbol{\Theta}_n^*\|^{1+\tau}$ 的估计,$\tilde{\theta}_n=\theta_n^*-\hat{\theta}_n$ 是参数估计误差.$K_n=\int_{x_n^*}^{x_n}\frac{\zeta_n(w)^{2-1/r_n}}{k_{bn}^2-\zeta_n^2(w)}\mathrm{d}w$,并且 $\zeta_n(w)=w^{r_n}-(x_n^*)^{r_n}$.在紧集 $\Omega_{\beta_n}=\{\beta_n \mid |\beta_n|<k_{bn}\}$ 中,V_n 是正定且连续的.式(35)对时间 t 求导,可得:

$$\dot{V}_n=\dot{V}_{n-1}+\frac{\partial k_n}{\partial x_n}\dot{x}_n+\frac{\partial k_n}{\partial x_{n-1}}\dot{x}_{n-1}+\frac{\partial k_n}{\partial\hat{\theta}_{n-1}}\dot{\hat{\theta}}_{n-1}+\frac{1}{\eta_n}\tilde{\theta}_n\dot{\tilde{\theta}}_n\leqslant \sum_{j=1}^{n-1}-\frac{\beta_j^{1+\tau}}{k_{bj}^2-\beta_j^2}+\sum_{j=1}^{n-1}\left(\frac{\gamma_j}{\eta_j}\hat{\theta}_j\tilde{\theta}_j+a_j\right)+\frac{\beta_n^{2-1/r_n}}{k_{bn}^2-\beta_n^2}h_n(\boldsymbol{x})u^{p_n}+\frac{\beta_n}{k_{bn}^2-\beta_n^2}[(\beta_n^{\tau}\upsilon_n^{1+\tau}\|\boldsymbol{\varphi}_n\|^{1+\tau}\hat{\theta}_n+\beta_n^{\tau}\sigma_n^{1+\tau})(k_{bn}^2-\beta_n^2)]-\frac{1}{\eta_n}\tilde{\theta}_n(\eta_n\beta_n^{1+\tau}\upsilon_n^{1+\tau}\|\boldsymbol{\varphi}_n\|^{1+\tau}-\dot{\hat{\theta}}_n)+a_n. \quad (36)$$

设计控制器 u 和参数 $\hat{\theta}_n$ 的自适应律为

$$u=-\mu_n\beta_n^{1/r_{n+1}}, \quad (37)$$

$$\dot{\hat{\theta}}_n=\eta_n\upsilon_n^{1+\tau}\beta_n^{1+\tau}\|\boldsymbol{\varphi}_n\|^{1+\tau}-\gamma_n\hat{\theta}_n, \quad (38)$$

其中,$\mu_n=[((1+\upsilon_n^{1+\tau}\|\boldsymbol{\varphi}_n\|^{1+\tau}\hat{\theta}_n+\sigma_n^{1+\tau})(k_{bn}^2-\beta_n^2))/\underline{h}_n]^{1/p_n}$,$\gamma_n>0$ 是设计参数.

根据式(31),(32)和(33),可得 V_n 的导数为

$$\dot{V}_n\leqslant-\sum_{j=1}^{n}\frac{\beta_j^{1+\tau}}{k_{bj}^2-\beta_j^2}+\sum_{j=1}^{n}\left(\frac{\gamma_j}{\eta_j}\hat{\theta}_j\tilde{\theta}_j+a_j\right). \quad (39)$$

根据 Young 不等式和 $\tilde{\theta}_j=\theta_j^*-\hat{\theta}_j$,下列不等式成立:

$$\frac{\gamma_j}{\eta_j}\hat{\theta}_j\tilde{\theta}_j\leqslant-\frac{\gamma_j}{2\eta_j}\tilde{\theta}_i^2+\frac{\gamma_j}{2\eta_j}(\theta_j^*)^2. \quad (40)$$

由式(40),$\dot{V}_n$ 可改写为

$$\dot{V}_n\leqslant-\sum_{j=1}^{n}\frac{\beta_j^{1+\tau}}{k_{bj}^2-\beta_j^2}+\sum_{j=1}^{n}\left(-\frac{\gamma_j}{2\eta_j}\tilde{\theta}_j^2+\frac{\gamma_j}{2\eta_j}(\theta_j^*)^2+a_j\right). \quad (41)$$

2.2 稳定性分析

下面的定理给出了所设计的有限时间控制器所具有的性质.

定理 1 针对非线性系统(1),在假设 1 的条件下,控制器(37),虚拟控制函数(14)、(32)和参数自适应律(15)、(33)和(38),能保证:

1)闭环系统的所有信号半全局实际有限时间稳定,且输出信号 y 在有限时间内跟踪给定信号 y_r;

2)系统状态 $x_i(t)$ 不超出给定的约束边界 k_{ci}.

证明 选取如下 Lyapunov 函数:

$$V=\frac{1}{2}\log\frac{k_{b1}^2}{k_{b1}^2-\beta_1^2}+\sum_{i=2}^{n}\int_{x_i^*}^{x_i}\frac{\zeta_i(w)^{2-1/r_i}}{k_{bi}^2-\zeta_i^2(w)}\mathrm{d}w+\sum_{i=1}^{n}\frac{1}{2\eta_i}\tilde{\theta}_i^2. \quad (42)$$

注意到,对于任意给定正常数 k_{b1},β_1 满足 $|\beta_1|<k_{b1}$,以下不等式成立:

$$\log\frac{k_{b1}^2}{k_{b1}^2-\beta_1^2}<\frac{\beta_1^2}{k_{b1}^2-\beta_1^2}. \quad (43)$$

根据式(42)和式(43),可得:

$$V\leqslant\frac{1}{2}\frac{\beta_1^2}{k_{b1}^2-\beta_1^2}+\sum_{i=2}^{n}(x_i-x_i^*)\frac{[x_i^{r_i}-(x_i^*)^{r_i}]^{2-1/r_i}}{k_{bi}^2-\beta_i^2}+\sum_{i=1}^{n}\frac{1}{2\eta_i}\tilde{\theta}_i^2\leqslant\frac{1}{2}\frac{\beta_1^2}{k_{b1}^2-\beta_1^2}+\sum_{i=2}^{n}((x_i^{r_i})^{1/r_i}-((x_i^*)^{r_i})^{1/r_i})\times[x_i^{r_i}-(x_i^*)^{r_i}]^{2-1/r_i}/[k_{bi}^2-\beta_i^2]+\sum_{i=1}^{n}\frac{1}{2\eta_i}\tilde{\theta}_i^2\leqslant2\sum_{i=1}^{n}\frac{\beta_i^2}{k_{bi}^2-\beta_i^2}+\sum_{i=1}^{n}\frac{1}{2\eta_i}\tilde{\theta}_i^2. \quad (44)$$

对于 $\tau\in(0,1)$ 一定存在一个常数 $\ell=(1+\tau)/2$,使得 $\ell\in(0,1)$.结合式(44),进一步可得到:

$$V^{\ell}\leqslant\left(2\sum_{i=1}^{n}\frac{\beta_i^2}{k_{bi}^2-\beta_i^2}+\sum_{i=1}^{n}\frac{1}{2\eta_i}\tilde{\theta}_i^2\right)^{\ell}\leqslant 2^{\ell}\sum_{i=1}^{n}\frac{\beta_i^{2\ell}}{(k_{bi}^2-\beta_i^2)^{\ell}}+\sum_{i=1}^{n}\left(\frac{1}{2\eta_i}\tilde{\theta}_i^2\right)^{\ell}\leqslant 2\sum_{i=1}^{n}\frac{\beta_i^{1+\tau}}{k_{bi}^2-\beta_i^2}+\sum_{i=1}^{n}\left(\frac{1}{2\eta_i}\tilde{\theta}_i^2\right)^{\ell}. \quad (45)$$

根据式(41)和(45),可得:

$$\dot{V}\leqslant-1/2V^{\ell}+\frac{1}{2}\sum_{i=1}^{n}\left(\frac{1}{2\eta_i}\tilde{\theta}_i^2\right)^{\ell}+\sum_{j=1}^{n}\left(-\frac{\gamma_j}{2\eta_j}\tilde{\theta}_j^2+\frac{\gamma_j}{2\eta_j}(\theta_j^*)^2+a_j\right). \quad (46)$$

类似于式(40),以下不等式成立:

$$\frac{1}{2}\left(\frac{1}{2\eta_i}\tilde{\theta}_i^2\right)^{\ell}=\left[\left(\frac{1}{2}\right)^{\frac{1}{\ell}}\left(\frac{1}{2\eta_i}\tilde{\theta}_i^2\right)\right]^{\ell}\leqslant$$

$$\left(\frac{1}{2}\right)^{\frac{1}{\ell}}\left(\frac{1}{2\eta_i}\tilde{\theta}_i^2\right)+l. \tag{47}$$

选择参数 γ_i 使 $\gamma_j/2\eta_j > \left(\frac{1}{2}\right)^{\frac{1}{\ell}}$,则式(46)可改写为

$$\dot{V}\leqslant-\frac{1}{2}V^{\ell}+\pi, \tag{48}$$

其中,$\pi=\sum_{i=1}^{n}\left(\frac{\gamma_j}{2\eta_j}(\theta_j^*)^2+a_i\right)+nl.$

令到达时间为 $T=\frac{2}{(1-\ell)\xi}[V^{1-\ell}(\boldsymbol{\beta}(0),\tilde{\boldsymbol{\theta}}(0))-(2\pi/(1-\xi))^{\frac{1-\ell}{\ell}}]$,常数 $0<\xi<1$ 且初始值 $\boldsymbol{\beta}(0)=[\beta_0(0),\cdots,\beta_n(0)]^{\mathrm{T}}$,$\tilde{\boldsymbol{\theta}}(0)=[\tilde{\theta}_1(0),\cdots,\tilde{\theta}_n(0)]^{\mathrm{T}}$.因此,对于 $\forall t\geqslant T$,$V^{\ell}(\boldsymbol{\beta},\tilde{\boldsymbol{\theta}})\leqslant\frac{2\pi}{(1-\xi)}$ 满足引理2,则闭环系统是半全局实际有限时间稳定的.另外,对于 $\forall t\geqslant T$,以下不等式成立:

$$|y-y_r|\leqslant k_{b1}\sqrt{1-\mathrm{e}^{-2\left[\frac{2\pi}{(1-\xi)}\right]^{1/\ell}}}. \tag{49}$$

根据以上分析以及式(15),(33)和(38),得到:

$$|\tilde{\theta}_i|\leqslant 2(2\pi\eta_i^{\ell}/(1-\xi))^{1/2\ell}. \tag{50}$$

根据 $\hat{\theta}_i=\theta_i^*-\tilde{\theta}_i$,则 $\hat{\theta}_i$ 是有界的.

根据 $\beta_1=x_1-y_r$ 和 $|y_r|\leqslant y_0$,得到 $|x_1|\leqslant|y_r|+|\beta_1|<y_0+k_{b1}$.定义 $k_{b1}=k_{c1}-y_0$,则有 $|x_1|<k_{c1}$.根据式(48)可以得到 K_i 是有界的,则 $|\beta_i|<k_{bi}(i=2,\cdots,n)$.又因为 $\hat{\theta}_i$ 有界的,所以 x_i^* 是有界的,则可得到 $(x_i^*)^{r_i}$ 是有界的.类似于 $|x_1|<k_{c1}$,结合式(5),可得到 x_i 有界,即 $|x_i|<k_{ci}$.因此,系统所有状态 $x_1,\cdots,x_n$ 都有界并且不超过给定的边界 k_{ci}.

定理1证明完毕.

3 仿真实例

考虑如下非线性系统:

$$\begin{cases}\dot{x}_1=h_1(x_1,x_2)x_2^{p_1}+g_1(x_1,x_2),\\ \dot{x}_2=h_2(x_1,x_2)u^{p_2}+g_2(x_1,x_2),\\ y=x_1,\end{cases} \tag{51}$$

其中,非线性函数 $h_1(\boldsymbol{x})=3+0.5x_1x_2\sin(x_1x_2)$,$h_2(\boldsymbol{x})=8+x_1x_2\cos(x_1x_2)$,$g_1(\boldsymbol{x})=-0.2\mathrm{e}^{-0.5x_1^2x_2^2}$ 和 $g_2(\boldsymbol{x})=0.1x_2\cos(1/(1+x_1^2))$,$\boldsymbol{x}=[x_1,x_2]^{\mathrm{T}}$,参考信号为 $y_r=\sin(t)$.

选择隶属函数为

$\mu_{F_i^1}(x_i)=\exp[-(x_i-2)^2/2]$,
$\mu_{F_i^2}(x_i)=\exp[-(x_i-1)^2/2]$,
$\mu_{F_i^3}(x_i)=\exp[-(x_i-0)^2/2]$,
$\mu_{F_i^4}(x_i)=\exp[-(x_i+1)^2/2]$,
$\mu_{F_i^5}(x_i)=\exp[-(x_i+2)^2/2]$,$i=1,2.$

选择初始条件为:$x_1(0)=-0.23$,$x_2(0)=-0.2$,$\hat{\theta}_1(0)=0.5$,$\hat{\theta}_2(0)=0.2$.系统状态变量 x_1 和 x_2 约束为 $|x_1|<k_{c1}=1.3$ 和 $|x_2|<k_{c2}=1$.选择控制器 u,虚拟控制器 x_2^*,自适应律 $\hat{\theta}_1$ 和 $\hat{\theta}_2$ 中的设计参数为 $\eta_1=1.2$,$\eta_2=1.3$,$\gamma_1=1.3$,$\gamma_2=1.5$,$\sigma_1=1.3$,$\sigma_2=1.5$,$v_1=1$,$v_2=1$,$k_{b1}=1.1$,$k_{b2}=0.8$.

仿真结果如图1—6所示.

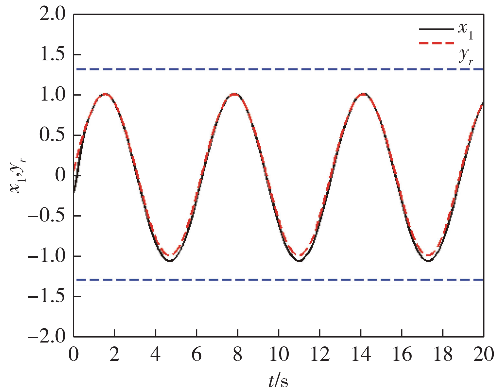

图1 状态 x_1 与跟踪信号 y_r 的轨迹

Fig. 1 Trajectories of state x_1 and tracking signal y_r

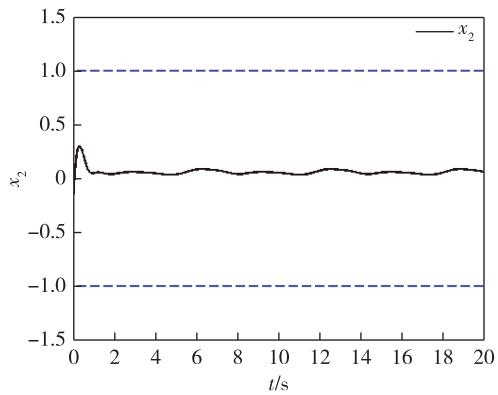

图2 状态 x_2 的轨迹

Fig. 2 Trajectory of state x_2

根据仿真结果图1—6可知,系统的状态没有超出给定的约束边界,系统输出在有限时间内很好地跟踪给定的参考信号,同时闭环系统的所有信号在有限时间收敛到包含原点的一个小邻域内.

4 结论

本文针对一类具有全状态约束的非严格反馈高

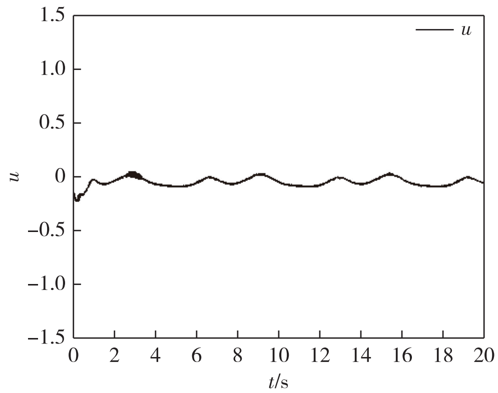

图3 控制器 u 的轨迹

Fig. 3 Trajectory of controller u

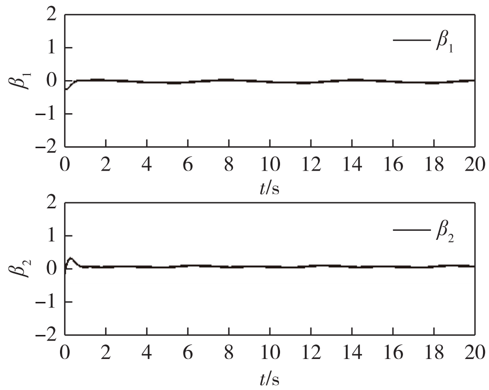

图4 跟踪误差 β_1 和 β_2 的轨迹

Fig. 4 Trajectories of tracking errors β_1 and β_2

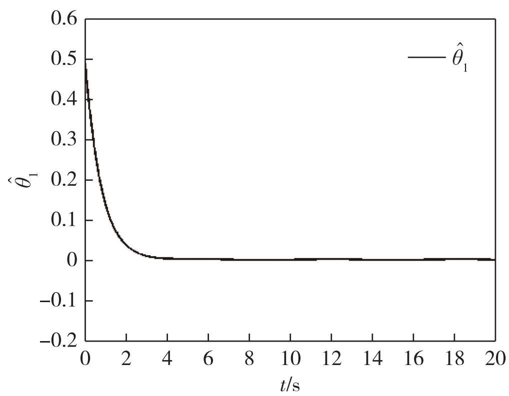

图5 自适应参数 $\hat{\theta}_1$ 的轨迹

Fig. 5 Trajectory of adaptive parameter $\hat{\theta}_1$

阶非线性系统,在自适应模糊理论框架下,研究了一种有限时间自适应控制设计问题.首先,利用模糊逻辑系统对未知非线性函数进行辨识;其次,采用加幂积分技术,构造了一种新的障碍 Lyapunov 函数,结合反步递推技术,解决了高阶非线性系统的有限时间状态约束问题;最后,通过有限时间 Lyapunov 稳

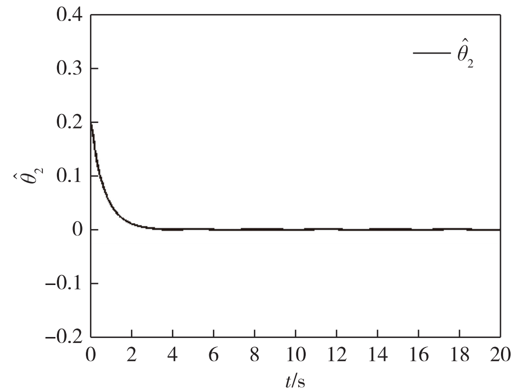

图6 自适应参数 $\hat{\theta}_2$ 的轨迹

Fig. 6 Trajectory of adaptive parameter $\hat{\theta}_2$

定性理论,严格证明了跟踪误差的有限时间收敛性和闭环系统的半全局实际有限时间稳定性.

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Adaptive fuzzy finite-time control for high-order nonlinear system with state constraints

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Abstract The adaptive fuzzy finite-time output tracking control is addressed for a class of non-strict feedback high-order nonlinear systems with state constraints.The fuzzy logic systems are utilized to identify the unknown nonlinearities of the controlled systems,based on which the problem of state constraints is tackled by adopting barrier Lyapunov function.An adaptive finite-time control approach is developed combining backstepping control algorithm with adding a barrier power integrator.According to the finite-time Lyapunov stability theory,the proposed adaptive finite-time control scheme could guarantee the closed-loop system to be semi-globally practically finite-time stable and the states are ensured not to transgress their constrained sets. Moreover, the goal of finite-time tracking control is achieved.Finally,a numerical simulation is given to verify the effectiveness of the proposed adaptive finite-time control approach.

Key words finite-time stability;fuzzy control;adding a power integrator;barrier Lyapunov functions;high-order nonlinear system