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基于量化脉冲控制策略的领导跟随多智能体系统的固定时间一致性

摘要

本文研究了基于量化脉冲控制的非线性领导跟随多智能体系统的一致性问题.基于矩阵理论、李雅普诺夫函数和利普希茨不等式,给出了若干假设和充分条件.通过构造比较系统,利用微分方程理论,给出了固定时间下的一致性准则,且计算了达到一致需要的时间.最后,选择合适的参数,通过数值仿真验证了理论分析的有效性.

关键词

多智能体系统;领导跟随;对数量化器;脉冲效应;固定时间一致性

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0 引言

随着人工智能的发展,多智能体系统[1]作为人工智能的一个重要分支,受到了广泛的关注并成为研究的热点.在各学科专家、学者的共同努力下,多智能体系统在一致性[2-3]、同步[4-5]、群集[6-7]、协调[8-9]、优化控制[10]等领域取得了丰硕的成果.此外,关于一致性问题有两个常见的研究方向,一个是领导跟随模型[11],另一个是无领导监督模型[12].

领导跟随一致性[13-16],意味着多智能体系统内,跟随者的动态行为要与领导者达到统一.需要指出的是,一致性类型有很多,如渐近一致性[17]、指数一致性[18]、有限时间一致性[19]、固定时间一致性[20-21]等.指数一致性作为一种特殊的渐近一致性,已经被很多学者研究.然而,渐近一致性和指数一致性通常是在时间趋于无穷时实现的,这在现实生活中很难实现.近年来,有限时间一致性的概念被提出,解决了渐近一致性和指数一致性的缺陷,且达到一致的时间是可计算的.但它仍有缺陷,因为该时间依赖于系统的初始状态.作为一种特殊的有限时间一致,固定时间一致是避免这种缺陷的好方法[22-24].但现有关于领导跟随多智能体系统的文献中,研究固定时间的成果相对较少.因此,研究领导跟随多智能体系统的固定时间一致性具有重要的意义.不失一般性,本文建立的是最基本的非线性领导跟随多智能体系统模型.

当多智能体之间的连接很弱时,没有控制器就很难实现系统的同步.迄今为止,已经发表了很多控制方案,如自适应控制[25]、事件触发控制[26-27]、协同控制[28]、脉冲控制[29]、状态反馈控制[30]、采样数据控制[31]、间歇控制[32]等.脉冲控制是一种典型的控制方法,广泛应用于生物、医学、物理、航天等领域.一方面,系统不可避免地会受到干扰和中断,脉冲效应可以很好地模拟状态的突变;另一方面,它可以有效缓解信息传递的压力.

由于通信信道的比特率、能量和带宽是有限的,信息量化[33]在控制方案中起着重要作用.一般有两种量化器:均匀量化器[34-36]和对数量化器[37-39].本文考虑对数量化器.

考虑到上述几个因素,本文通过量化脉冲控制的方法分析了非

线性领导跟随多智能体系统的一致性问题.本文的新颖之处在于:1)建立领导跟随多智能体系统模型,并设计了合适的控制协议;2)将通信数据量化,且在脉冲时刻进行系统内的信息交换,大大降低了通信带宽和能耗;3)所建立的模型是最基本的非线性多智能体系统.

本文的其余内容如下所示:第1节是预备知识,并介绍一些必要的假设、定义和引理;第2节建立了非线性领导跟随多智能体系统模型,并基于李雅普诺夫函数给出了定理;第3节通过选择合适的参数进行仿真,验证了上述理论分析的正确性;第4节是结论.

注1 本文中, $\mathbf{R}$ 表示实数集, $\boldsymbol{I}_N$ 表示 $N\times N$ 单位矩阵, $\boldsymbol{X}^{\mathrm{T}}$ 表示矩阵 $\boldsymbol{X}$ 的转置矩阵, $\mathrm{diag}(x_1,x_2,\cdots,x_N)$ 表示 $N\times N$ 对角阵, $\max\{x_i\}$ 表示 $x_1,x_2,\cdots,x_N$ 的最大值.令 $\boldsymbol{M}$ 为一个实对称矩阵,则 $\boldsymbol{M}>0(\boldsymbol{M}<0)$ 表示正(负)定矩阵.

1 预备知识

1.1 图论

$\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ 是一个无向图.在图 $\mathcal{G}$ 中, $\mathcal{V}=\{v_1,v_2,\cdots,v_N\}$ 是节点集, $\mathcal{E}$ 是边集,其中边表示为 $e_{ij}=(v_i,v_j)$. $\mathcal{A}=[a_{ij}]_{N\times N}$ 是邻接矩阵,其元素 a_{ij} 的值依赖于对应的两个节点是否连通.当 $e_{ij}\in\mathcal{E}$ 时 $a_{ij}=1$,否则 $a_{ij}=0$.第 i 个节点 v_i 的邻域表示为 $\mathcal{N}_i=\{v\in\mathcal{V}:(v_i,v_j)\in\mathcal{E}\}$.度矩阵 $\mathcal{D}$ 是对角阵,其元素 $d_i=\sum_{j=1}^{N}a_{ij}$.

通过计算 $\mathcal{L}=\mathcal{D}-\mathcal{A}$ 可以得到拉普拉斯矩阵 $\mathcal{L}=(l_{ij})_{N\times N}$,其元素满足:

$$l_{ij}=\begin{cases}\sum_{j=1,j\neq i}^{N}a_{ij}, & i=j,\\ -a_{ij}, & i\neq j.\end{cases}\tag{1}$$

1.2 对数量化器

在量化过程中,给定量化参数,将整个取值范围划分为若干个量化电平.量化级数与量化误差成反比,与量化质量成正比.给定量化器的密度 $\iota\in(0,1)$ 和准确度 $\zeta=\dfrac{1-\iota}{1+\iota}\in(0,1)$.对于 $\forall x\in\mathbf{R}$,定义数量化器 $Q(\cdot):\mathbf{R}\to\tilde{\omega}$ 为

$$Q(m)=\begin{cases}\tilde{\omega}_i, & \dfrac{1}{1+\zeta}\tilde{\omega}_i<m<\dfrac{1}{1-\zeta}\tilde{\omega}_i,\\ 0, & m=0,\\ -Q(-m), & m<0.\end{cases}$$

这是一组量化电平的映射,量化电平为 $\tilde{\omega}_i=\{\pm\tilde{\omega}_i:\tilde{\omega}_i=\iota^i\tilde{\omega}_0,i=0,\pm1,\pm2,\cdots\}$,其中 $\tilde{\omega}_0>0$.

量化器关于原点对称,且满足 $Q(m)=(1+\Xi_i)m$, $\exists\,\Xi_i\in[-\zeta,\zeta]$.而且,如果 $\boldsymbol{m}\in\mathbf{R}^n$,有 $Q(\boldsymbol{m})=(Q(\boldsymbol{m})_1,Q(\boldsymbol{m})_2,\cdots,Q(\boldsymbol{m})_N)$.

1.3 假设和相关引理

在这部分,将介绍后续证明需要用到的假设、定义和引理.

假设1 给定利普希茨常数 $l_f>0$ 满足

$|f(t,s_j(t))-f(t,s_i(t))|\leqslant l_f|s_j(t)-s_i(t)|$.

假设2 给定利普希茨常数 $l_{\xi_1},l_{\xi_2},\cdots,l_{\xi_N}>0$ 满足 $|\xi_i(s_j(t))-\xi_i(s_i(t))|\leqslant l_{\xi_i}|s_j(t)-s_i(t)|$,其中 $i=1,2,\cdots,N$.

引理1[40] 当满足 $\lim\limits_{t\to\infty}|s_i(t)-s_0(t)|=0$, $i=1,2,\cdots,N$ 时,可以认为在控制协议下跟随者与领导者达到一致.当总存在 $T>t_0$ 满足 $\lim\limits_{t\to T}|s_i(t)-s_0(t)|=0$, $i=1,2,\cdots,N$ 时,可以认为在控制协议下跟随者与领导者达到固定时间一致.

引理2[41] 令 $\theta_1,\theta_2,\cdots,\theta_N\geqslant0$, $n>1$, $0<m\leqslant1$,则满足: $\sum_{i=1}^{N}\theta_i^m\geqslant\left(\sum_{i=1}^{N}\theta_i\right)^m$ 和 $\sum_{i=1}^{N}\theta_i^n\geqslant N^{1-n}\left(\sum_{i=1}^{N}\theta_i\right)^n$.

引理3[42] 假设在 (t,T) 区间上有脉冲序列 $\tau=t_1,t_2,\cdots$,令 T 表示平均脉冲间隔, $K_\xi(t,t_0)$ 表示给定区间内的脉冲次数,存在 $K_0\in\mathbf{N}^+$ 和 $T\in\mathbf{R}^+$ 满足

$$\frac{t-t_0}{T}-K_0\leqslant K_\xi(t,t_0)\leqslant\frac{t-t_0}{T}+K_0.$$

引理4[43] 众所周知,对于无向连通图 $\mathcal{G}$ 满足 $\boldsymbol{m}^{\mathrm{T}}\mathcal{L}\boldsymbol{m}\geqslant\sum_{i=1}^{N}\lambda_i m_i^2$,其中 λ_i 是拉普拉斯矩阵 $\mathcal{L}$ 的特征根,且满足 $0=\lambda_1\leqslant\lambda_2\leqslant\cdots\leqslant\lambda_n$.另外,图的代数连通度满足 $\lambda_2(\mathcal{L})=\min\limits_{\|m\|\neq0,1_N^{\mathrm{T}}m=0}\dfrac{\boldsymbol{m}^{\mathrm{T}}\mathcal{L}\boldsymbol{m}}{\|\boldsymbol{m}\|}$,因此得出当 $\mathbf{1}_N^{\mathrm{T}}\boldsymbol{m}=0$ 时有 $\boldsymbol{m}^{\mathrm{T}}\mathcal{L}\boldsymbol{m}\geqslant\lambda_2(\mathcal{L})\boldsymbol{m}^{\mathrm{T}}$.

2 主要结果

本节建立了领导跟随多智能体系统模型.此外,通过构造李雅普诺夫函数得到了同步标准和一些充分条件,证明了领导跟随多智能体系统在量化脉冲控制下的固定时间一致性,并给出了同步时间.

考虑含 N 个多智能体的非线性多智能体系统,

拓扑结构用无向图 $\mathcal{G}$ 表示,标记为 $1,2,\cdots,N$ 的多智能体可以被动态描述为

$$\dot{s}_i(t)=f(t,s_i(t))+u_i(t),\tag{1}$$

其中,$x_i\in\mathbf{R}$ 描述第 i 个多智能体的状态,$f:\mathbf{R}\times\mathbf{R}\to\mathbf{R}$ 是一个非线性函数,且 $u_i\in\mathbf{R}$ 是第 i 个多智能体的控制协议.

标记为 0 的多智能体称为领导者,是根据实际情况设定的理想目标轨迹,$s_0(t)$ 可以被动态描述为

$$\dot{s}_0(t)=f(t,s_0(t)).\tag{2}$$

第 i 个多智能体的控制协议被设计为

$$\begin{aligned}u_i(t)=&-\alpha\Big(\sum_{j\in N_i}a_{ij}(s_j(t)-s_i(t))\Big)^{\frac{x}{y}}-\\&\beta\Big(\sum_{j\in N_i}a_{ij}(s_j(t)-s_i(t))\Big)^{\frac{p}{q}}+\\&\sum_{k=1}^{+\infty}g_iQ(s_i-s_0)\delta(t-t_k),\end{aligned}\tag{3}$$

其中,控制参数 $\alpha,\beta>0,x,y,p$ 和 q 都是正奇数,且满足 $x<y$ 和 $q>p$.

在控制协议(3)下,系统(1)被改写为

$$\begin{cases}\dot{s}_i(t)=f(t,s_i(t))+\alpha\Big(\sum\limits_{j\in N_i}a_{ij}(s_j(t)-s_i(t))\Big)^{\frac{x}{y}}+\\\quad\beta\Big(\sum\limits_{j\in N_i}a_{ij}(s_j(t)-s_i(t))\Big)^{\frac{p}{q}},\quad t\neq t_k,\\\Delta s_i(t)=\sum\limits_{j\in N_i}g_iQ(s_i(t_k^-)-s_0(t_k^-)),\quad t=t_k.\end{cases}\tag{4}$$

令 $c_i(t)=s_i(t)-s_0(t)$,可以得到误差系统.

$$\begin{cases}\dot{c}_i(t)=f(t,s_i(t))+\alpha\Big(\sum\limits_{j\in N_i}a_{ij}(s_j(t)-s_i(t))\Big)^{\frac{x}{y}}+\\\quad\beta\Big(\sum\limits_{j\in N_i}a_{ij}(s_j(t)-s_i(t))\Big)^{\frac{p}{q}},\quad t\neq t_k,\\\Delta c_i(t)=g_iQ(c_i(t_k^-)),\quad t=t_k.\end{cases}\tag{5}$$

定理 1 假定假设 1 和 2 满足且 $\gamma_k\in(0,1)$.当 $(\boldsymbol{I}_N+\boldsymbol{G\Xi}+\boldsymbol{G})^{\mathrm{T}}(\boldsymbol{I}_N+\boldsymbol{G\Xi}+\boldsymbol{G})-\gamma_k\boldsymbol{I}_N<0$ 时,非线性多智能体系统(4)将在固定时间内达到领导跟随一致性,一致时间为

$$T=\frac{T_a}{(1-\omega)\ln\gamma}\ln\left(1-\frac{\gamma^{N_0(1-\omega)\ln\gamma}}{T_a(\eta_3-\eta_1)}\right)+\frac{T_a}{(1-\upsilon)\ln\gamma}\ln\left(\frac{T_a(\eta_2-\eta_1)}{T_a(\eta_2-\eta_1)\gamma^{2N_0(1-\upsilon)}-\ln\gamma}\right)+2T_aN_0,$$

其中,$\eta_1=2l_f,\eta_2=2^{\frac{x}{y}}\alpha(\lambda_2(\mathcal{L}))^{\frac{x+y}{2y}},\eta_3=2^{\frac{p}{q}}\beta N^{\frac{q-p}{2q}}(\lambda_2(\mathcal{L}))^{\frac{p+q}{2q}}$,$\mathcal{L}$ 为图 $\mathcal{G}$ 的拉普拉斯矩阵.

证明 选择李雅普诺夫函数[44] $V(\boldsymbol{c}(t))=\frac{1}{2}\sum_{i=1}^{n}(c_i(t))^2$.对于 $t\neq t_k$,可以从式(5)推导出

$$\begin{aligned}\dot{V}(\boldsymbol{c}(t))=&\sum_{i=1}^{n}\sum_{j=1}^{n}c_i(t)\dot{c}_i(t)=\\&\sum_{i=1}^{n}c_i(t)\Big[f(t,s_i(t))-f(t,s_0(t))+\\&\alpha\Big(\sum_{j\in N_i}a_{ij}(s_j(t)-s_i(t))\Big)^{\frac{x}{y}}+\\&\beta\Big(\sum_{j\in N_i}a_{ij}(s_j(t)-s_i(t))\Big)^{\frac{p}{q}}\Big]=\\&\sum_{i=1}^{n}c_i(t)(f(t,s_i(t))-f(t,s_0(t)))+\\&\sum_{i=1}^{n}c_i(t)\Big[\alpha\Big(\sum_{j\in N_i}a_{ij}(s_j(t)-s_i(t))\Big)^{\frac{x}{y}}+\\&\beta\Big(\sum_{j\in N_i}a_{ij}(s_j(t)-s_i(t))\Big)^{\frac{p}{q}}\Big]=V_1(t)+V_2(t).\end{aligned}\tag{6}$$

根据假设 1,有

$$\begin{aligned}V_1(\boldsymbol{c}(t))=&\sum_{i=1}^{n}c_i(t)(f(t,s_i(t))-f(t,s_0(t)))=\\&\sum_{i=1}^{n}|c_i(t)||f(t,s_i(t))-f(t,s_0(t))|\leqslant\\&\sum_{i=1}^{n}|c_i(t)|l_f|s_i(t)-s_0(t)|=\\&l_f\sum_{i=1}^{n}c_i^2(t)\leqslant 2l_fV(\boldsymbol{c}(t))=\boldsymbol{\eta}_1V(\boldsymbol{c}(t)).\end{aligned}\tag{7}$$

其中,$\boldsymbol{\eta}_1=2l_f>0$.

由于图 $\mathcal{G}$ 是无向图和奇函数的性质,有

$$\begin{aligned}&\sum_{i=1}^{N}c_i(t)\sum_{j\in N_i}a_{ij}(s_j(t)-s_i(t))^{\frac{x}{y}}=\\&\sum_{i=1}^{N}c_i(t)\sum_{j=1}^{N}a_{ij}(c_j(t)-c_i(t))^{\frac{x}{y}}=\\&\frac{1}{2}\sum_{i=1}^{N}c_i(t)\sum_{j=1}^{N}a_{ij}(c_j(t)-c_i(t))^{\frac{x}{y}}+\\&\frac{1}{2}\sum_{j=1}^{N}c_i(t)\sum_{i=1}^{N}a_{ji}(c_i(t)-c_j(t))^{\frac{x}{y}}=\\&\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}(c_i(t)-c_j(t))(c_j(t)-c_i(t))^{\frac{x}{y}}=\\&-\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}(c_j(t)-c_i(t))^{\frac{x+y}{y}}.\end{aligned}\tag{8}$$

根据引理 4,有

$$\begin{aligned}V_2(\boldsymbol{c}(t))=&\sum_{i=1}^{n}c_i(t)\Big[\alpha\Big(\sum_{j\in N_i}a_{ij}(s_j(t)-s_i(t))\Big)^{\frac{x}{y}}+\\&\beta\Big(\sum_{j\in N_i}a_{ij}(s_j(t)-s_i(t))\Big)^{\frac{p}{q}}\Big]=\end{aligned}$$

$$-\frac{1}{2}\alpha\Big[\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}^{\frac{2y}{x+y}}(c_j(t)-c_i(t))^2\Big]^{\frac{x+y}{2y}}-$$
$$\frac{1}{2}\beta N^{\frac{q-p}{2q}}\Big[\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}^{\frac{2q}{p+q}}(c_j(t)-c_i(t))^2\Big]^{\frac{p+q}{2q}}. \quad (9)$$

由于 $\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}^{\frac{2y}{x+y}}(c_j(t)-c_i(t))^2=2\boldsymbol{c}^{\mathrm{T}}(t)\boldsymbol{\mathcal{L}}\boldsymbol{c}(t)\geqslant 4\lambda_2(\mathcal{L})V(\boldsymbol{c}(t))$ 和 $\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}^{\frac{2q}{p+q}}(c_j(t)-c_i(t))^2=2\boldsymbol{c}^{\mathrm{T}}(t)\mathcal{L}\boldsymbol{c}(t)\geqslant 4\lambda_2(\boldsymbol{\mathcal{L}})V(\boldsymbol{c}(t))$,可以得到

$$V_2(\boldsymbol{c}(t))\leqslant -2^{\frac{x}{y}}\alpha(\lambda_2(\boldsymbol{\mathcal{L}}))^{\frac{x+y}{2y}}V^{\frac{x+y}{2y}}(\boldsymbol{c}(t))-$$
$$2^{\frac{p}{q}}\beta N^{\frac{q-p}{2q}}(\lambda_2(\boldsymbol{\mathcal{L}}))^{\frac{p+q}{2q}}V^{\frac{p+q}{2q}}(\boldsymbol{c}(t)). \quad (10)$$

综上所述,李雅普诺夫函数可以被改写为

$$\dot{V}(\boldsymbol{c}(t))\leqslant \eta_1 V(\boldsymbol{c}(t))-\eta_2 V^{\frac{p+q}{2q}}(\boldsymbol{c}(t))-$$
$$\eta_3 V^{\frac{p+q}{2q}}(\boldsymbol{c}(t)), \quad (11)$$

其中,$\eta_2\leqslant 2^{\frac{x}{y}}\alpha(\lambda_2(\boldsymbol{\mathcal{L}}))^{\frac{x+y}{2y}}$,$\eta_3= 2^{\frac{p}{q}}\beta N^{\frac{q-p}{2q}}(\lambda_2(\boldsymbol{\mathcal{L}}))^{\frac{p+q}{2q}}$.

当 $t=t_k$ 时,

$$V(\boldsymbol{c}(t_k^+))=\frac{1}{2}\boldsymbol{c}^{\mathrm{T}}(t_k^+)\boldsymbol{c}(t_k^+)=$$
$$\frac{1}{2}\boldsymbol{c}^{\mathrm{T}}(t_k^-)(\boldsymbol{I}_N+\boldsymbol{G\Xi}+\boldsymbol{G})^{\mathrm{T}}(\boldsymbol{I}_N+\boldsymbol{G\Xi}+\boldsymbol{G})\boldsymbol{c}(t_k^-)=$$
$$\frac{1}{2}\boldsymbol{c}^{\mathrm{T}}(t_k^-)\,[(\boldsymbol{I}_N+\boldsymbol{G\Xi}+\boldsymbol{G})^{\mathrm{T}}(\boldsymbol{I}_N+\boldsymbol{G\Xi}+\boldsymbol{G})-$$
$$\gamma_k\boldsymbol{I}_N\,]+\frac{1}{2}\gamma_k\boldsymbol{c}^{\mathrm{T}}(t_k^-)\boldsymbol{c}(t_k^-). \quad (12)$$

由于 $(\boldsymbol{I}_N+\boldsymbol{G\Xi}+\boldsymbol{G})^{\mathrm{T}}(\boldsymbol{I}_N+\boldsymbol{G\Xi}+\boldsymbol{G})-\gamma_k\boldsymbol{I}_N$ 是负定的,因此

$$V(\boldsymbol{c}(t_k^+))\leqslant\frac{1}{2}\gamma_k\boldsymbol{c}^{\mathrm{T}}(t_k^-)\boldsymbol{c}(t_k^-)\leqslant\gamma V(\boldsymbol{c}(t_k^-)). \quad (13)$$

方便起见,用 $\gamma=\max\{\gamma_1,\gamma_2,\cdots,\gamma_k\}$ 来代替 γ_k.

根据式(6)—(13),可以推导出

$$\begin{cases}\dot{V}(\boldsymbol{c}(t))\leqslant\eta_1V(\boldsymbol{c}(t))-\eta_2V^{\frac{p+q}{2q}}(\boldsymbol{c}(t))-\\ \quad\eta_3V^{\frac{p+q}{2q}}(\boldsymbol{c}(t)), & t\neq t_k,\\ V(\boldsymbol{c}(t_k^+))\leqslant\gamma V(\boldsymbol{c}(t_k^-)), & t=t_k.\end{cases} \quad (14)$$

显而易见,当 $0<V(\boldsymbol{c}(t))<1$ 时,$\eta_3V^{\frac{p+q}{2q}}(\boldsymbol{c}(t))$ 趋近于0,当 $V(\boldsymbol{c}(t))\geqslant 1$ 时,$\eta_2V^{\frac{p+q}{2q}}(\boldsymbol{c}(t))$ 趋近于0,所以忽略不计.因此将式(14)改写成以下形式.

$$\begin{cases}\dot{V}(\boldsymbol{c}(t))\leqslant\begin{cases}0, & V(t)=0,t\neq t_k,\\ -(\eta_2-\eta_1)V^{\nu}(t), & 0<V(t)<1,t\neq t_k,\\ -(\eta_3-\eta_1)V^{\omega}(t), & V(t)\geqslant 1,t\neq t_k,\end{cases}\\ V(\boldsymbol{c}(t_k^+))\leqslant\gamma V(\boldsymbol{c}(t_k^-)), \quad t=t_k.\end{cases} \quad (15)$$

其中 $\upsilon=\dfrac{x+y}{2y}$,$\omega=\dfrac{p+q}{2q}$.

因此构建如下比较系统[45]:

$$\begin{cases}\nu(t)=\begin{cases}0, & \nu(t)=0,t\neq t_k,\\ -(\eta_2-\eta_1)\nu^{\upsilon}(t), & 0<\nu(t)<1,t\neq t_k,\\ -(\eta_3-\eta_1)\nu^{\omega}(t), & \nu(t)\geqslant 1,t\neq t_k,\end{cases}\\ \nu(t_k)=\gamma\nu(t_k^-), \quad t=t_k,\\ \nu(0)=V(0)=\nu_0.\end{cases} \quad (16)$$

观察可知,在误差系统(14)中存在平衡点 $\boldsymbol{c}(t)=0$.当 $\boldsymbol{c}(t)=0$ 时,$\dot{V}(\boldsymbol{c}(t))=0$,$\nu(\boldsymbol{c}(t))=0$.通过比较式(16)和(15)发现,当 $t\geqslant 0$ 时 $0<V(t)<\nu(t)$.因此,将通过求 $\nu(t)$ 来求 $V(t)$.根据参数的不同取值范围,微分方程(16)的解将分两部分考虑.

考虑 $0<\gamma<1$,为了方便计算,当 $\nu(t)\geqslant 1$ 时,假设 $r(t)=\nu^{1-\omega}(t)$,式(15)可改写为

$$\begin{cases}\dot{r}(t)=-(\eta_3-\eta_1)(\omega-1), & 0<r(t)\leqslant 1,t\neq t_k,\\ r(t_k^+)=\gamma_1 r(t_k), & t=t_k,\\ r(0)=r_0=\nu_0^{1-\omega}.\end{cases} \quad (17)$$

其中,$\gamma_1=\gamma^{1-\omega}\geqslant 1$.如果画出 $r(t)$ 关于 $\nu(t)$ 的曲线,可以看出连续部分大致是下降的,且当 $\nu(t)\to 1$ 时,$r(t)\to 1$,当 $\nu(t)\to\infty$ 时,$r(t)\to 0$.

式(16)的解可以通过数学归纳法[46]求出.

$$r(t)=\gamma_1^{N_\zeta(0,t)}h(0)+(\eta_3-\eta_1)(\omega-1)\int_0^t\gamma_1^{N_\zeta(\sigma,t)}\mathrm{d}\sigma, \quad (18)$$

这表明 $r(t)$ 是单调递增的,且 $r(0)<1$,$\underset{t\to\infty}{r(t)}=+\infty$.因此不难发现必然存在一个时间点 T_1 满足 $\underset{t\to T_1}{r(t)}=1$.

接下来求解 T_1,

$$\gamma_1^{N_\zeta(0,t)}h(0)+(\eta_3-\eta_1)(\omega-1)\int_0^t\gamma_1^{N_\zeta(\sigma,t)}\mathrm{d}\sigma=1. \quad (19)$$

由于 $N_\zeta(0,t)>0$ 未知,因此只能估算出 T_1 的最大值.由于 $\gamma_1^{N_\zeta(0,t)}h(0)>0$,有

$$(\eta_3-\eta_1)(\omega-1)\int_0^t\gamma_1^{N_\zeta(\sigma,t)}\mathrm{d}\sigma\leqslant 1. \quad (20)$$

根据 $N_\zeta(t,T)$ 的定义,有 $\int_0^t\gamma_1^{N_\zeta(\sigma,t)}\mathrm{d}\sigma\geqslant\int_0^t\gamma_1^{\frac{t-\sigma}{T_a}-N_0}\mathrm{d}\sigma$.令 $\tilde{\sigma}=\dfrac{t-\sigma}{T_a}-N_0$,可以得到 $-T_a(\eta_3-\eta_1)(\omega-1)\int_{\frac{t}{T_a}-N_0}^{t}\gamma_1^{\tilde{\sigma}}\mathrm{d}\tilde{\sigma}\leqslant 1$.解这个微分不等式可以得到

$$t\leqslant\frac{T_a}{\ln\gamma_1}\ln\left(1-\frac{\gamma_1^{N_0}\ln\gamma_1}{T_a(\eta_3-\eta_1)(1-\omega)}\right). \quad (21)$$

令 $\gamma_1=\gamma^{1-\omega}$,可以求得 $\underset{t\to T_1}{r(t)}=1$ 中

$$T_1=\frac{T_a}{(1-\omega)\ln\gamma}\ln\left(1-\frac{(1-\omega)\gamma^{N_0(1-\omega)}\ln\gamma}{T_a(\eta_3-\eta_1)}\right).\qquad(22)$$

接下来考虑另一种情况 $0<\nu(t)<1$,设定 $r(t)=\nu^{1-\upsilon}$,可以得到

$$\begin{cases}\dot r(t)=-(\eta_2-\eta_1)(1-\upsilon),\quad 0<r(t)\leqslant 1,\\ \qquad t\neq t_k,t\geqslant T,\\ r(t_k^+)=\gamma_2 r(t_k),\quad t=t_k,t\geqslant T,\\ r(T_1)=1.\end{cases}\qquad(23)$$

其中 $0<\gamma_2<1$ 满足 $\gamma_2=\gamma^{1-\upsilon}$.如果画出 $r(t)$ 关于 $\nu(t)$ 的曲线,可以看出连续部分大致是上升的,且当 $\nu(t)\to 1$ 时,$r(t)\to 1$,当 $\nu(t)\to 0$ 时,$r(t)\to 0$.

通过数学归纳法,可推出

$$r(t)=\gamma_2^{N_\zeta(T_1,t)}r(T_1)-(\eta_2-\eta_1)(1-\upsilon)\int_{T_1}^{t}\gamma_2^{N_\zeta(\sigma,t)}\mathrm{d}\sigma,\qquad(24)$$

这表明 $r(t)$ 是单调递增的,且 $r(T_1)=1$.

根据 $N_\zeta(t,T)$ 的定义且 $0<\gamma_2<1$,可以有 $\gamma_2^{N_\zeta(T_1,t)}\leqslant\gamma_2^{\frac{t-\sigma}{T_a}-N_0}$,进而得出

$$\int_{T_1}^{t}\gamma_2^{N_\zeta(\sigma,t)}\mathrm{d}\sigma\geqslant\int_{T_1}^{t}\gamma_2^{\frac{t-\sigma}{T_a}+N_0}\mathrm{d}\sigma=-T_a\gamma_2^{N_0}\int_{T_1}^{t}\gamma_2^{\frac{t-\sigma}{T_a}}\mathrm{d}\frac{t-\sigma}{T_a}=-\frac{T_a\gamma_2^{N_0}}{\ln\gamma_2}\left(1-\gamma_2^{\frac{t-T_1}{T_a}}\right).$$

因此,式(24)可以重写为

$$r(t)\leqslant\gamma_2^{\frac{t-T_1}{T_a}N_0}h(T_1)-(\eta_2-\eta_1)(1-\upsilon)\int_{T_1}^{t}\gamma_2^{\frac{t-\sigma}{T_a}+N_0}\mathrm{d}\sigma=\left[1-\frac{T_a(\eta_2-\eta_1)(1-\upsilon)\gamma_2^{2N_0}}{\ln\gamma_2}\right]\gamma_2^{\frac{t-T_1}{T_a}-N_0}+\frac{T_a(\eta_2-\eta_1)(1-\upsilon)\gamma_2^{2N_0}}{\ln\gamma_2}=\tilde r(t).\qquad(25)$$

由于 $1-\dfrac{T_a(\eta_2-\eta_1)(1-\upsilon)\gamma_2^{2N_0}}{\ln\gamma_2}>0$,且 $\dfrac{T_a(\eta_2-\eta_1)(1-\upsilon)\gamma_2^{2N_0}}{\ln\gamma_2}<0$,必然存在一个时间点 t 满足:

$$t-T_1=2T_aN_0+\frac{T_a}{\ln\gamma_2}\ln\left(\frac{T_a(\eta_2-\eta_1)(1-\upsilon)}{T_a(\eta_2-\eta_1)(1-\upsilon)\gamma_2^{2N_0}-\ln\gamma_2}\right).$$

令 $\gamma_2=\gamma^{1-\upsilon}$,在 $r(t)\to 1$ 之后,可以认为存在一个时间 T_2,使得 $\underset{t\to T_2}{r(t)}\to 0$,其中

$$T_2=2T_aN_0+\frac{T_a}{(1-\upsilon)\ln\gamma}\ln\left(\frac{T_a(\eta_2-\eta_1)}{T_a(\eta_2-\eta_1)(1-\upsilon)\gamma^{2N_0(1-\upsilon)}-\ln\gamma}\right).\qquad(26)$$

根据上述分析可以得到同步时间为

$$T=T_1+T_2=\frac{T_a}{(1-\omega)\ln\gamma}\ln\left(1-\frac{(1-\omega)\gamma^{N_0(1-\omega)}\ln\gamma}{T_a(\eta_3-\eta_1)}\right)+2T_aN_0+\frac{T_a}{(1-\upsilon)\ln\gamma}\ln\left(\frac{T_a(\eta_2-\eta_1)}{T_a(\eta_2-\eta_1)(1-\upsilon)\gamma^{2N_0(1-\upsilon)}-\ln\gamma}\right).\qquad(27)$$

注 2 估算的 T 值是最大同步时间,且不确定是在连续时间还是脉冲跳变点.

3 数值仿真

本节选择一些合适的参数通过仿真验证以上理论分析的有效性.

考虑如下多智能体系统,网络拓扑如图 1 所示.

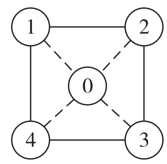

图 1 网络拓扑图

Fig. 1 Network topology map

$$\dot{\boldsymbol{s}}_0(t)=f(t,\boldsymbol{s}_0(t))$$

和

$$\begin{cases}\dot s_i(t)=f(t,s_i(t))+\alpha\Big(\sum\limits_{j\in N_i}a_{ij}(s_j(t)-s_i(t))\Big)^{\frac{x}{y}}+\\ \qquad\beta\Big(\sum\limits_{j\in N_i}a_{ij}(s_j(t)-s_i(t))\Big)^{\frac{p}{q}},\quad t\neq t_k,\\ \Delta s_i(t)=\sum\limits_{j\in N_i}g_iQ(s_i(t_k^-)-s_0(t_k^-)),t=t_k,\end{cases}$$

其中,$\boldsymbol{s}_0$ 是一个孤立的控制节点,满足初始值 $\boldsymbol{s}_0(0)=0.5$,其余 4 个多智能体满足初始值为 $\boldsymbol{s}_0(0)=[-0.1,\ -2.7,\ 1.8,\ 2.4]^{\mathrm{T}}$.$F(t,\boldsymbol{m}(t))=[f(t,m_1(t)),f(t,m_2(t)),f(t,m_3(t)),f(t,m_4(t))]$.$f(t,m_i(t))=\cos^2(m_i(t))-|\sin(m_i(t))|$,满足常数为 $\boldsymbol{l}_f=[1,1,1.8,1.2]^{\mathrm{T}}$ 的利普希茨条件.如图 2 所示.$\boldsymbol{s}(t)$ 的拉普拉斯矩阵为

$$\mathcal{L}=\begin{pmatrix}-2&1&0&1\\1&-2&1&0\\0&1&-2&1\\1&0&1&-2\end{pmatrix}.$$

令步长为 0.000 1,脉冲间隔为 0.002,脉冲增益 $\gamma=0.8$.取 $\alpha=4.3$,$\beta=2.5$,$x=1$,$y=3$,$p=5$,$q=3$.经过计算得,$\eta_1=4.4$,$\eta_2=9.798\ 8$,$\eta_3=16.459\ 1$,且 $(\boldsymbol{I}_N+\boldsymbol{G\Xi}+\boldsymbol{G})^{\mathrm{T}}(\boldsymbol{I}_N+\boldsymbol{G\Xi}+\boldsymbol{G})-\gamma_k\boldsymbol{I}_N<0$ 也满足.

无向图 $\mathcal{G}$ 的代数连通度为 2,经过计算得到 $T_1=0.071\ 8$ s,$T_2=0.102\ 0$ s,因此同步时间 $T=0.173\ 8$ s.

显而易见,定理 1 是有效的,图 2 展示了跟随者将在约 0.18 s 的位置与领导者达到一致.

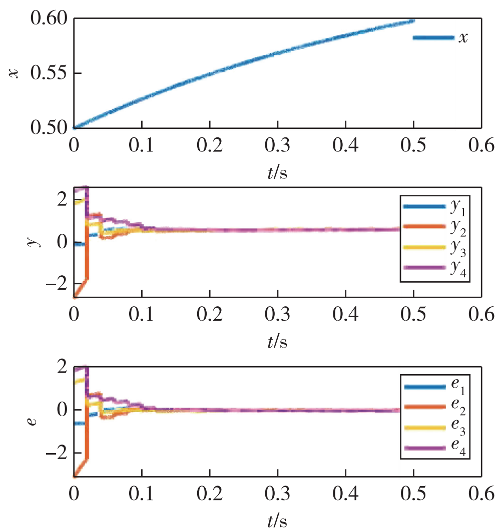

图 2 在脉冲控制下 $\boldsymbol{s}(t)$ 中的每一个节点都与 $\boldsymbol{s}_0(t)$ 达到一致

Fig. 2 Every node of $\boldsymbol{s}(t)$ achieves soon consensus with $\boldsymbol{s}_0(t)$ under impulsive control

4 结论

通过选择合适的脉冲控制协议,解决了非线性领导跟随多智能体系统的固定时间一致性问题.为减少通信损失,脉冲控制协议保证多智能体系统内的多智能体在脉冲时刻进行信息交互.利用李雅普诺夫函数、凸分析和利普希茨条件,得到了两个充分条件,使系统在固定时间内达到一致,该时间可计算且与初始状态无关.最后通过仿真验证了理论推导的可行性.

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Fixed-time consensus in leader-following multi-agent system based on quantized impulsive control

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Abstract This paper considers the consensus problem of nonlinear leader-following multi-agent system under quantized impulsive control.According to matrix theory,Lyapunov function and Lipshitz inequalities,some assumptions and sufficient conditions are given to make sure that the leader-following multi-agent system achieve fixed-time consensus.By constructing comparison system and using differential equations,the criteria for fixed-time consensus are set and the time interval to achieve consensus is calculated.Finally,the above theoretical analyses are verified by a simulation example.

Key words multi-agent system;leader-following;logarithmic quantizer;impulsive effect;fixed-time consensus