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一类具有输入量化和未知扰动的非线性系统的自适应有限时间动态面控制

摘要

本文研究了具有量化输入信号和未知扰动的非线性系统的有限时间自适应输出反馈动态面控制问题.在控制设计过程中,利用模糊逻辑系统对系统中的非线性项进行逼近.然后引入一种滞回量化器来避免量化信号中的抖振,并且构造模糊观测器来估计系统中不可测的状态.为了提出一种有限时间控制策略,首先给出了半全局实际有限时间稳定的判据.在此基础上,将动态面控制技术与反步法相结合,设计了自适应模糊控制器.该控制器不仅能保证跟踪误差在有限时间内收敛到原点的一个小邻域,而且可以保证闭环系统中所有信号的有界性.最后通过一个仿真实例验证了该控制方法的有效性和可行性.

关键词

量化输入信号;模糊逻辑系统;动态面控制;反步法;有限时间跟踪控制

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0 引言

在过去的几十年里,自适应控制方法作为求解参数不确定的非线性系统控制问题的主要方法之一得到了广泛的应用[1-6].此外,为了克服复杂未知的非线性函数对非线性系统的影响,利用模糊逻辑系统(FLSs)[7]或神经网络(NNs)[8],提出了许多针对不确定非线性系统的模糊或神经网络控制方法[9-11].近年来,将自适应控制方法与模糊逻辑系统或神经网络相结合,取得了许多有意义的研究成果.比如,文献[12-14]针对不确定的严格反馈非线性系统,构造了基于 FLSs 或 NNs 的自适应控制方法.在文献[15-16]中,基于 FLSs 或 NNs,提出了非线性纯反馈系统的自适应控制方法.文献[17-18]研究了非线性非严格反馈系统的自适应智能控制问题.在已有方法的基础上,本文采用模糊自适应控制方法,提出了一种有限时间自适应模糊控制器.

目前,量化控制已经成为控制工程中的一个重要课题.它已经广泛应用于数字控制系统、混合系统和网络控制系统中,如文献[19-21]研究了非线性系统的量化控制问题.这些系统的一个共同特点是需要通过组件之间的无线媒体传输信息.由于无线通信网络的物理局限性,所以引入了量化技术来降低通信速率.设计量化控制系统的控制方案,其基础问题是保证系统在低带宽下能够正常运行.因此量化对于许多实际控制系统是必要的,也是有益的.本文采用滞后量化器来消除文献[22]中提出的对数量化器所引起的抖振现象.

值得注意的是,在传统的反步技术中,由于某些非线性函数在每一步的重复微分会导致"复杂性爆炸".因此,为了避免这一问题,提出了动态表面控制(DSC)技术.该方法将一阶滤波器引入到反步法的每一步中,将原微分运算转化为代数运算,使得在实际中难以控制的模型易于实现.近几十年来,动态表面控制技术在不确定非线性系统的自适应控制中得到了广泛的应用.例如,文献[23-25]针对严格反馈或纯反馈非线性系统,研究了基于 FLSs 或 NNs 的自适应动态表面控制策略.文献[26-28]研究了基于动态面技术的非严格反馈非线性系统的自适应控制方法.虽然上述文献所设计的控制器可以保证闭环系统的有界性,但不能保证系统在有限时间内的稳定性.因此,本文将研究闭环系统的有限时间稳定性.

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上述研究问题主要与无限时间跟踪控制有关.然而,在实际工程中,控制目标往往需要在有限的时间内收敛.有限时间控制可以使闭环系统具有更快的响应速度、更高的跟踪精度和更好的抗干扰能力.因此,近年来有限时间控制的分析与综合越来越受到人们的重视.例如,文献[29]针对一类具有输入饱和的非线性严格反馈系统,提出了一种模糊自适应有限时间控制设计方法;文献[30]研究了非线性纯反馈系统的自适应有限时间跟踪控制方法.随后,文献[31-32]设计了状态观测器,消除了文献[29-30]中要求状态完全可测量的限制,提出了有限时间自适应控制策略.然而,如何有效地解决具有输入量化和未知扰动的非线性系统的有限时间自适应控制问题仍是一个棘手的问题.

本文的主要贡献如下:

1)针对一类具有量化输入和未知扰动的非线性系统,提出了一种新的自适应控制方案.与文献[5]和文献[16]相比,本文不仅考虑了系统的量化输入和未知扰动,而且提出了一种有限时间自适应模糊控制策略.

2)本文提出了一种输出反馈控制方案,设计了模糊自适应观测器来估计系统中的不可测状态.并且,本文采用滞回量化器对输入信号进行量化,避免了量化信号中的抖振.

3)本文采用动态面控制技术,克服了反步设计中"复杂性爆炸"的缺点,降低了控制算法的计算复杂度.

1 问题形成和准备

1.1 预备知识

定义 1 称非线性系统 $\dot{\zeta} = f(\zeta)$ 在平衡点 $\zeta = 0$ 是半全局实际有限时间稳定(SGPFS),若对任意 $\zeta(t_0) = \zeta_0$,存在一个常数 $\varepsilon > 0$ 和稳定时间 $T(\varepsilon, \zeta_0) < \infty$,使得对于任意 $t \geq t_0 + T$,都有

$$\| \zeta(t) \| < \varepsilon.$$

引理 1[29] 对于任意 $z_i \in \mathbf{R}, i = 1, 2, \cdots, n$ 和一个实数 $0 < p \leq 1$,以下不等式成立:

$$\left(\sum_{i=1}^{n} | z_i | \right)^p \leq \sum_{i=1}^{n} | z_i |^p \leq n^{1-p} \left(\sum_{i=1}^{n} | z_i | \right)^p.$$

引理 2[29] 对于实变量 z, ζ 以及任意正常数 μ, υ, ι,以下不等式成立:

$$| z |^{\mu} | \zeta |^{\upsilon} \leq \frac{\mu}{\mu + \upsilon} \iota | z |^{\mu+\upsilon} + \frac{\upsilon}{\mu + \upsilon} \iota^{\frac{-\mu}{\upsilon}} | \zeta |^{\mu+\upsilon}.$$

引理 3[29] 考虑微分方程 $\dot{\hat{\zeta}}(t) = -\vartheta \hat{\zeta}(t) + \kappa \upsilon(t)$,其中 $\vartheta > 0$ 和 $\kappa > 0$ 是常数以及 $\upsilon(t)$ 是一个函数.如果对于任意 $t \geq t_0$,都有 $\upsilon(t) > 0$,则在初值条件 $\hat{\zeta}(t_0) \geq 0$ 下,对于任意 $t \geq t_0$,都有 $\zeta(t) \geq 0$.

引理 4[29] 考虑系统 $\dot{\zeta} = f(\zeta)$,如果存在一个光滑的正定函数 $V(\zeta)$ 以及标量 $c > 0, 0 < \beta < 1$ 和 $\rho > 0$,使得

$$\dot{V}(\zeta) \leq -c V^{\beta}(\zeta) + \rho,$$

则非线性系统 $\dot{\zeta} = f(\zeta)$ 为 SGPFS.定义到达时间为

$$T_{\text{reach}} = \frac{1}{(1-\beta)\eta c} \left[V^{1-\beta}(\zeta(0)) - \left(\frac{\rho}{(1-\eta)c} \right)^{\frac{1-\beta}{\beta}} \right],$$

其中 $V(\zeta(0))$ 为 $V(\zeta)$ 的初值.

1.2 系统描述及假设

考虑如下具有量化输入和外界扰动的非线性系统:

$$\begin{cases} \dot{x}_1 = x_2 + f_1(x_1) + d_1(\boldsymbol{x}), \\ \dot{x}_i = x_{i+1} + f_i(\bar{\boldsymbol{x}}_i) + d_i(\boldsymbol{x}), i = 2, \cdots, n-1, \\ \dot{x}_n = Q(u) + f_n(\boldsymbol{x}) + d_n(\boldsymbol{x}), \\ y = x_1, \end{cases} \tag{1}$$

其中,$\boldsymbol{x} = [x_1, x_2, \cdots, x_n]^{\mathrm{T}} \in \mathbf{R}^n$ 为系统的状态向量,$\bar{\boldsymbol{x}}_i = [x_1, x_2, \cdots, x_i]^{\mathrm{T}}, i = 1, 2, \cdots, n; u \in \mathbf{R}$ 为系统输入;$Q(u) \in \mathbf{R}$ 为量化输入;$y \in \mathbf{R}$ 为系统的输出;$f_i(\bar{\boldsymbol{x}}_i), i = 1, 2, \cdots, n$ 为未知的光滑非线性函数;$d_i(\boldsymbol{x})$ 是未知的外界扰动.假设系统的状态 $x_2(t), \cdots, x_n(t)$ 是不可测的,只有输出 y 是可测的.

本文采用迟滞量化器来减小抖振,量化器 $Q(u)$ 的定义如下所示:

$$Q(u) = \begin{cases} u_i \mathrm{sgn}(u), & \dfrac{u_i}{1+\delta} < | u | \leq u_i, \dot{u} < 0, \\ & \text{或} \ u_i < | u | \leq \dfrac{u_i}{1-\delta}, \dot{u} > 0, \\ u_i(1+\delta)\mathrm{sgn}(u), & u_i < | u | \leq \dfrac{u_i}{1-\delta}, \dot{u} < 0, \\ & \text{或} \ \dfrac{u_i}{1-\delta} < | u | \leq \dfrac{u_i(1+\delta)}{1-\delta}, \dot{u} > 0, \\ 0, & 0 \leq | u | < \dfrac{u_{\min}}{1+\delta}, \dot{u} < 0, \\ & \text{或} \ \dfrac{u_{\min}}{1+\delta} \leq | u | \leq u_{\min}, \dot{u} > 0, \\ Q(u(t^-)), & \dot{u} = 0, \end{cases} \tag{2}$$

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齐晓静,等.一类具有输入量化和未知扰动的非线性系统的自适应有限时间动态面控制.
QI Xiaojing, et al.Adaptive finite-time dynamic surface control for nonlinear systems with input quantization and unknown disturbances.

其中,$u_i = \rho^{1-i} u_{\min} (i = 1,2,\cdots)$,$\delta = (1-\rho)/(1+\rho)$,参数 $u_{\min} > 0$ 以及 $0 < \rho < 1$.

$Q(u) \in U = \{0, \pm u_m, \pm u_m(1+\delta), m = 1,2, \cdots\}$,$u_{\min} > 0$ 决定 $Q(u)$ 的死区范围,参数 ρ 表示量化密度的度量.

本文的控制目标是设计一个自适应模糊控制器,保证闭环系统中的所有信号的有界性,并且保证系统的输出 $y(t)$ 能够在有限时间内很好地跟踪给定的参考信号 $y_r(t)$.

为了便于控制器设计,给出以下假设:

假设 1 存在常数 D_i,它是外部扰动输入 $d_i(x)$ 上界,即满足 $|d_i(x)| \leqslant D_i$.

假设 2 给定参考信号 $y_r(t)$,$\dot{y}_r(t)$ 及 $\ddot{y}_r(t)$ 都是有界的.

注意 $Q(u)$ 是一个量化的值,这使得控制器的设计变得困难.为了便于控制器的设计,我们首先给出了 $Q(u)$ 与 u 之间的关系定理.

定理 1 存在函数 $A(u)$ 和 $B(t)$,使得量化后的值 $Q(u)$ 满足以下等式:

$$Q(u) = A(u)u(t) + B(t), \tag{3}$$

其中 $A(u)$ 和 $B(t)$ 满足如下不等式:

$$1 - \delta \leqslant A(u) \leqslant 1 + \delta, |B(t)| \leqslant u_{\min}.$$

我们可将系统(1)重新描述为以下形式:

$$\dot{x} = Ax + ky + \sum_{i=1}^{n} B_i[f_i(\bar{x}_i) + d_i] + BQ(u), \tag{4}$$

其中

$$A = \begin{bmatrix} -k_1 & & \\ -k_2 & & I_{n-1} \\ \vdots & & \\ -k_n & 0 & \cdots & 0 \end{bmatrix}, \quad k = \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}, \quad B_i = [\underbrace{0\cdots1\cdots0}_{i}]^{\mathrm{T}}.$$

选择向量 k,使得 A 为一个严格的 Hurwitz 矩阵.因此对于任意给定的矩阵 $Q = Q^{\mathrm{T}} > 0$,存在一个正定矩阵 P,使得

$$A^{\mathrm{T}}P + PA = -2Q. \tag{5}$$

1.3 模糊逻辑系统

利用模糊逻辑系统对系统中的非线性项进行逼近,描述如下:

设模糊推理 IF-Then 规则模糊语句为

R^l:如果 x_1 是 F_1^l 且 x_2 是 F_2^l 且 $\cdots$ 且 x_n 是 F_n^l,则 y 是 G^l,$l = 1,2,\cdots,N$.

其中,F_i^l 和 G^l 是对应于模糊隶属函数 $\mu_{F_i^l}(x_i)$ 和 $\mu_{G^l}(y)$ 的模糊集,N 为模糊规则数.

由单点模糊化、乘积推理和中心加权解模糊化构成的模糊逻辑系统为

$$y(x) = \frac{\sum_{l=1}^{N} \bar{y}_l \prod_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{N} \left[\prod_{i=1}^{n} \mu_{F_i^l}(x_i)\right]}, \tag{6}$$

其中 $\bar{y}_l = \max_{y \in \mathbf{R}} \mu_{G^l}(y)$.定义模糊基函数:

$$\varphi_l = \frac{\prod_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{N} \left[\prod_{i=1}^{n} \mu_{F_i^l}(x_i)\right]}. \tag{7}$$

令 $\boldsymbol{\theta} = [\bar{y}_1, \bar{y}_2, \cdots, \bar{y}_N]^{\mathrm{T}} = [\theta_1, \theta_2, \cdots, \theta_N]^{\mathrm{T}}$,$\boldsymbol{\varphi}^{\mathrm{T}}(x) = [\varphi_1(x), \cdots, \varphi_N(x)]$,则模糊逻辑系统可进一步表示为

$$y(x) = \boldsymbol{\theta}^{\mathrm{T}} \boldsymbol{\varphi}(x). \tag{8}$$

引理 5[11] 设 $f(x)$ 是紧集 Ω 上定义的连续函数,对于任意常数 $\varepsilon > 0$,都存在一个模糊逻辑系统(8),使得

$$\sup_{x \in \Omega} |f(x) - \boldsymbol{\theta}^{\mathrm{T}} \boldsymbol{\varphi}(x)| \leqslant \varepsilon. \tag{9}$$

2 模糊状态观测器的设计

在这一部分中,假设系统(1)的状态 $x_2(t), \cdots, x_n(t)$ 是不可测的,只有输出 y 是可测的.所以需要构造状态观测器来估计这些状态,然后建立模糊状态观测器.

由于(1)中的 $f(x)$ 为非线性项,所以根据引理5 可得 $\hat{f}_i(\bar{x}_i \mid \boldsymbol{\theta}_i) = \boldsymbol{\theta}_i^{\mathrm{T}} \boldsymbol{\varphi}_i(\bar{x}_i) \hat{f}_i(\hat{\bar{x}}_i \mid \boldsymbol{\theta}_i) = \boldsymbol{\theta}_i^{\mathrm{T}} \boldsymbol{\varphi}_i(\hat{\bar{x}}_i)$,其中 $\hat{\bar{x}}_i = [\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_i]^{\mathrm{T}}$ 为 $\bar{x}_i = [x_1, x_2, \cdots, x_i]^{\mathrm{T}}$ 的估计.将模糊最小逼近误差 ε_i 和 δ_i 定义为:

$\varepsilon_i = f_i(\bar{x}_i) - \hat{f}_i(\hat{\bar{x}}_i \mid \boldsymbol{\theta}_i^*)$,$\delta_i = f_i(\bar{x}_i) - \hat{f}_i(\hat{\bar{x}}_i \mid \boldsymbol{\theta}_i)$,其中 $\boldsymbol{\theta}_i^*$ 为最优参数向量,令 $\delta_i' = \delta_i + d_i$,$\varepsilon_i' = \varepsilon_i + d_i$,并且存在常数 $\bar{\varepsilon}_i$ 和 $\bar{\delta}_i$,$i = 1,2,\cdots,n$,使得 $|\varepsilon_i'| \leqslant \bar{\varepsilon}_i$,$|\delta_i'| \leqslant \bar{\delta}_i$.

模糊状态观测器设计如下:

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + ky + \sum_{i=1}^{n} B_i \hat{f}_i(\hat{\bar{x}}_i \mid \boldsymbol{\theta}_i) + BQ(u), \\ \hat{y} = C\hat{x}, \end{cases} \tag{10}$$

其中 $C = [1\cdots0\cdots0]$.令 $\tilde{x} = x - \hat{x} = [\tilde{x}_1, \tilde{x}_2, \cdots, \tilde{x}_n]^{\mathrm{T}}$ 为观测器误差.由(4)和(10)得:

$$\dot{\tilde{x}} = A\tilde{x} + \sum_{i=1}^{n} B_i [f_i(\bar{x}_i) - \hat{f}(\hat{\bar{x}}_i \mid \theta_i) + d_i] = A\tilde{x} + \delta, \tag{11}$$

其中 $\delta = [\delta_1', \delta_2', \cdots, \delta_n']$,并且令 $\bar{\delta} = \parallel \delta \parallel^2 = \sum_{i=1}^{n} \delta_i'^2$.

为了保证状态估计误差的收敛性,选择 Lyapunov 函数:

$$V_0 = \frac{1}{2}\tilde{x}^T P \tilde{x}. \tag{12}$$

由(5)和(12)可得:

$$\dot{V}_0 = \frac{1}{2}\tilde{x}^T(A^T P + PA)\tilde{x} + \tilde{x}^T P\delta = -\tilde{x}^T Q\tilde{x} + \tilde{x}^T P\delta. \tag{13}$$

根据 Young 不等式可得:

$$\tilde{x}^T P\delta \leqslant \frac{1}{2}(\parallel \tilde{x} \parallel^2 + \parallel P\delta \parallel^2). \tag{14}$$

将(14)代入(13)得:

$$\dot{V}_0 = -\tilde{x}^T Q\tilde{x} + \tilde{x}^T P\delta \leqslant -(\lambda_{\min}(Q) - \frac{1}{2})\parallel \tilde{x} \parallel^2 + \frac{1}{2}\parallel P\delta \parallel^2. \tag{15}$$

注 1 由(15)可以明显看出,设计的模糊状态观测器(10)不能保证观测器误差的收敛性,所以在下一节中我们在有限时间 Lyapunov 稳定性意义下,设计了控制器来保证闭环系统的稳定性.

3 控制器的设计与稳定性分析

在本节中,我们将利用反步法,提出一种基于观测器的有限时间模糊自适应输出反馈控制策略,并在有限时间李雅普诺夫稳定性理论下对系统进行稳定性分析.

3.1 有限时间自适应控制器的设计

为了便于后续控制器的设计,采用以下坐标变换:

$$\begin{aligned} e_1 &= y - y_r, \\ e_i &= \hat{x}_i - \alpha_i^c, i = 2,3,\cdots,n, \\ w_i &= \alpha_i^c - \alpha_{i-1}, \end{aligned} \tag{16}$$

其中,e_1 为跟踪误差,e_i 为误差面,α_{i-1} 为虚拟控制函数,α_i^c 是新引入的变量,它可以通过 α_{i-1} 上的一阶滤波器得到,w_i 是一阶滤波器的输出误差.

第 1 步.定义跟踪误差 $e_1 = y - y_r$,且计算跟踪误差的导数为

$$\dot{e}_1 = e_2 + w_2 + \alpha_1 + \tilde{x}_2 + \theta_1^T \varphi_1(\hat{x}_1) + \tilde{\theta}_1^T \varphi_1(\hat{x}_1) + \varepsilon_1' - \dot{y}_r, \tag{17}$$

其中 $\tilde{\theta}_1 = \theta_1^* - \theta_1$.

选择如下 Lyapunov 函数:

$$V_1 = V_0 + \frac{1}{2}e_1^2 + \frac{1}{2r_1}\tilde{\theta}_1^T \tilde{\theta}_1, \tag{18}$$

其中 $r_1 > 0$ 是一个设计参数.由(17)和(18)可得 V_1 对时间的导数为

$$\dot{V}_1 \leqslant -(\lambda_{\min}(Q) - \frac{1}{2})\parallel \tilde{x} \parallel^2 + \frac{1}{2}\parallel P\delta \parallel^2 + e_1[e_2 + w_2 + \alpha_1 + \tilde{x}_2 + \theta_1^T \varphi_1(\hat{x}_1) + \tilde{\theta}_1^T \varphi_1(\hat{x}_1) + \varepsilon_1' - \dot{y}_r] - \frac{1}{r_1}\tilde{\theta}_1^T \dot{\theta}_1. \tag{19}$$

根据 Young 不等式可得:

$$e_1(\tilde{x}_2 + \varepsilon_1') \leqslant e_1^2 + \frac{1}{2}\parallel x \parallel^2 + \frac{1}{2}\bar{\varepsilon}_1^2. \tag{20}$$

将式(20)代入式(19)可得:

$$\dot{V}_1 \leqslant -[\lambda_{\min}(Q) - 1]\parallel \tilde{x} \parallel + e_1[e_1 + e_2 + w_2 + \alpha_1 + \theta_1^T \varphi_1(\hat{x}_1) - \dot{y}_r] + \frac{1}{2}(\parallel P\delta \parallel^2 + \bar{\varepsilon}_1^2) + \frac{1}{r_1}\tilde{\theta}_1^T[r_1\varphi_1(\hat{x}_1)e_1 - \dot{\theta}_1]. \tag{21}$$

选择虚拟控制器 α_1 和自适应律 θ_1 为

$$\alpha_1 = -c_1 e_1^{2\beta-1} - e_1 - \theta_1^T \varphi_1(\hat{x}_1) + \dot{y}_r, \tag{22}$$

$$\dot{\theta}_1 = r_1\varphi_1(\hat{x}_1)e_1 - \sigma_1\theta_1, \tag{23}$$

其中 $c_1 > 0$ 和 $\sigma_1 > 0$ 是设计参数,并且 α_1 是 $\hat{x}_1, \theta_1, y$ 和 y_r 的函数.

将(22)和(23)代入(21)可得:

$$\dot{V}_1 \leqslant -[\lambda_{\min}(Q) - 1]\parallel \tilde{x} \parallel^2 - c_1 e_1^{2\beta} + e_1(e_2 + w_2) + \frac{\sigma_1}{r_1}\tilde{\theta}_1^T \theta_1 + \omega_1, \tag{24}$$

其中 $\omega_1 = \frac{1}{2}(\parallel P \parallel^2 \bar{\delta} + \bar{\varepsilon}_1^2)$.

根据文献[32],引入一个新的状态变量 α_2^c,然后令 α_1 通过具有时间常数 λ_2 的一阶滤波器得到 α_2^c,如下所示:

$$\lambda_2\dot{\alpha}_2^c + \alpha_2^c = \alpha_1, \alpha_2^c(0) = \alpha_1(0), \tag{25}$$

于是,由(25)可得:

$$\dot{\alpha}_2^c = \frac{\alpha_1 - \alpha_2^c}{\lambda_2} = -\frac{w_2}{\lambda_2}. \tag{26}$$

根据(16)和(26)可得:

$$\dot{w}_2 = \dot{\alpha}_2^c - \dot{\alpha}_1 = -\frac{w_2}{\lambda_2} + B_2(e_1, e_2, w_2, y_r, \dot{y}_r, \ddot{y}_r, \theta_1), \tag{27}$$

其中

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$$B_2(\cdot) = (2\beta - 1)c_1 \cdot e_1^{2\beta-2}\dot{e}_1 + \dot{e}_1 - \ddot{y}_r +$$

$$\dot{\boldsymbol{\theta}}_1^{\mathrm{T}}\varphi_1(\hat{x}_1) + \frac{\boldsymbol{\theta}_1^{\mathrm{T}}\mathrm{d}\varphi_1}{\mathrm{d}\hat{x}_1}\dot{\hat{x}}_1.$$

第 2 步.定义 $e_2 = \hat{x}_2 - \alpha_2^c$,并计算 e_2 的导数为

$$\dot{e}_2 = \dot{\hat{x}}_2 - \dot{\alpha}_2^c = \hat{x}_3 + k_2\tilde{x}_1 + \hat{f}_2(\hat{\bar{\boldsymbol{x}}}_2 \mid \boldsymbol{\theta}_2) - \dot{\alpha}_2^c =$$
$$e_3 + w_3 + \alpha_2 + k_2\tilde{x}_1 + \boldsymbol{\theta}_2^{\mathrm{T}}\varphi_2(\hat{\bar{\boldsymbol{x}}}_2) +$$
$$\tilde{\boldsymbol{\theta}}_2^{\mathrm{T}}\varphi_2(\hat{\bar{\boldsymbol{x}}}_2) + \varepsilon_2 - \delta_2 - \dot{\alpha}_2^c. \tag{28}$$

选择如下 Lyapunov 函数:

$$V_2 = V_1 + \frac{1}{2}e_2^2 + \frac{1}{2}w_2^2 + \frac{1}{2r_2}\tilde{\boldsymbol{\theta}}_2^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_2, \tag{29}$$

其中 $\tilde{\boldsymbol{\theta}}_2 = \boldsymbol{\theta}_2^* - \boldsymbol{\theta}_2, r_2 > 0$ 为设计参数.

根据(28)和(29)可得 V_2 的导数为

$$\dot{V}_2 \leqslant - [\lambda_{\min}(\boldsymbol{Q}) - 1]\|\tilde{\boldsymbol{x}}\|^2 - c_1e_1^{2\beta} +$$
$$e_1(e_2 + w_2) + \frac{\sigma_1}{r_1}\tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\boldsymbol{\theta}_1 + \omega_1 + e_2[e_3 + w_3 +$$
$$\alpha_2 + k_2\tilde{x}_1 + \boldsymbol{\theta}_2^{\mathrm{T}}\varphi_2(\hat{\bar{\boldsymbol{x}}}_2) + \varepsilon_2 - \delta_2 - \dot{\alpha}_2^c] +$$
$$w_2\dot{w}_2 + \frac{1}{r_2}\tilde{\boldsymbol{\theta}}_2^{\mathrm{T}}[r_2\varphi_2(\hat{\bar{\boldsymbol{x}}}_2)e_2 - \dot{\boldsymbol{\theta}}_2]. \tag{30}$$

根据 Young 不等式可得:

$$e_2(\varepsilon_2 - \delta_2) \leqslant e_2^2 + \frac{1}{2}(\bar{\varepsilon}_2 + \bar{\delta}_2). \tag{31}$$

将(31)代入(30)式可得:

$$\dot{V}_2 \leqslant - [\lambda_{\min}(\boldsymbol{Q}) - 1]\|\tilde{\boldsymbol{x}}\|^2 - c_1e_1^{2\beta} +$$
$$e_1(e_2 + w_2) + \frac{\sigma_1}{r_1}\tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\boldsymbol{\theta}_1 + \omega_1 + e_2[e_2 + e_3 +$$
$$w_3 + \alpha_2 + k_2\tilde{x}_1 + \boldsymbol{\theta}_2^{\mathrm{T}}\varphi_2(\hat{\bar{\boldsymbol{x}}}_2) - \dot{\alpha}_2^c] + w_2\dot{w}_2 +$$
$$\frac{1}{r_2}\tilde{\boldsymbol{\theta}}_2^{\mathrm{T}}[r_2\varphi_2(\hat{\bar{\boldsymbol{x}}}_2)e_2 - \dot{\boldsymbol{\theta}}_2] + \frac{1}{2}(\bar{\varepsilon}_2 + \bar{\delta}_2). \tag{32}$$

选择虚拟控制器 α_2 和自适应律 $\boldsymbol{\theta}_2$ 为

$$\alpha_2 = - c_2e_2^{2\beta-1} - e_2 - k_2\tilde{x}_1 - \boldsymbol{\theta}_2^{\mathrm{T}}\varphi_2(\hat{\bar{\boldsymbol{x}}}_2) + \dot{\alpha}_2^c, \tag{33}$$
$$\dot{\boldsymbol{\theta}}_2 = r_2\varphi_2(\hat{\bar{\boldsymbol{x}}}_2)e_2 - \sigma_2\boldsymbol{\theta}_2, \tag{34}$$

其中 $c_2 > 0$ 和 $\sigma_2 > 0$ 为设计参数.

将(33)和(34)代入(32)可得:

$$\dot{V}_2 \leqslant - [\lambda_{\min}(\boldsymbol{Q}) - 1]\|\tilde{\boldsymbol{x}}\|^2 - \sum_{j=1}^2 c_je_j^{2\beta} +$$
$$\sum_{j=1}^2 e_j(e_{j+1} + w_{j+1}) + \sum_{j=1}^2 \frac{\sigma_j}{r_j}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\boldsymbol{\theta}_j +$$
$$w_2\left(- \frac{w_2}{\lambda_2} + B_2\right) + \omega_2, \tag{35}$$

其中 $\omega_2 = \omega_1 + \frac{1}{2}(\bar{\varepsilon}_2 + \bar{\delta}_2)$.

根据文献[32],引入一个新的状态变量 α_3^c,然后令 α_2 通过具有时间常数 λ_3 的一阶滤波器得到 α_3^c,如下所示:

$$\lambda_3\dot{\alpha}_3^c + \alpha_3^c = \alpha_2, \alpha_3^c(0) = \alpha_2(0), \tag{36}$$

于是,由(36)可得:

$$\dot{\alpha}_3^c = \frac{\alpha_2 - \alpha_3^c}{\lambda_3} = - \frac{w_3}{\lambda_3}. \tag{37}$$

根据(16)和(37)可得:

$$\dot{w}_3 = \dot{\alpha}_3^c - \dot{\alpha}_2 = - \frac{w_3}{\lambda_3} +$$
$$B_3(e_1, e_2, e_3, w_2, w_3, y_r, \dot{y}_r, \ddot{y}_r, \boldsymbol{\theta}_1, \boldsymbol{\theta}_2), \tag{38}$$

其中

$$B_3(\cdot) = (2\beta - 1)c_2 \cdot e_2^{2\beta-2}\dot{e}_2 + \dot{e}_2 + k_2\dot{\tilde{x}}_1 +$$
$$\dot{\boldsymbol{\theta}}_2^{\mathrm{T}}\varphi_2(\hat{\bar{\boldsymbol{x}}}_2) + \boldsymbol{\theta}_2^{\mathrm{T}}\frac{\partial(\varphi_2)}{\partial \hat{\bar{\boldsymbol{x}}}_2}[\dot{\hat{x}}_1, \dot{\hat{x}}_2]^{\mathrm{T}} + \frac{\dot{w}_2}{\lambda_2}.$$

第 i 步($3 \leqslant i \leqslant n - 1$).定义 $e_i = \hat{x}_i - \alpha_i^c$,并计算 e_i 的导数为

$$\dot{e}_i = \dot{\hat{x}}_i - \dot{\alpha}_i^c =$$
$$\hat{x}_{i+1} + k_i\tilde{x}_1 + \hat{f}_i(\hat{\bar{\boldsymbol{x}}}_i \mid \boldsymbol{\theta}_2) - \dot{\alpha}_i^c =$$
$$e_{i+1} + w_{i+1} + \alpha_i + k_i\tilde{x}_1 + \boldsymbol{\theta}_i^{\mathrm{T}}\varphi_i(\hat{\bar{\boldsymbol{x}}}_i) +$$
$$\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\varphi_i(\hat{\bar{\boldsymbol{x}}}_i) + \varepsilon_i - \delta_i - \dot{\alpha}_i^c. \tag{39}$$

选择如下 Lyapunov 函数:

$$V_i = V_{i-1} + \frac{1}{2}e_i^2 + \frac{1}{2}w_i^2 + \frac{1}{2r_i}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_i, \tag{40}$$

其中 $\tilde{\boldsymbol{\theta}}_i = \boldsymbol{\theta}_i^* - \boldsymbol{\theta}_i, r_i > 0$ 为设计参数.

根据(39)和(40),可得 V_i 的导数为

$$\dot{V}_i \leqslant - [\lambda_{\min}(\boldsymbol{Q}) - 1]\|\tilde{\boldsymbol{x}}\|^2 - \sum_{j=1}^{i-1} c_je_j^{2\beta} +$$
$$\sum_{j=1}^i e_j(e_{j+1} + w_{j+1}) + \sum_{j=1}^{i-1} \frac{\sigma_j}{r_j}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\boldsymbol{\theta}_j +$$
$$\sum_{j=2}^{i-1} w_j\left(- \frac{w_j}{\lambda_j} + B_j\right) + \omega_{i-1} + e_i[\alpha_i +$$
$$k_i\tilde{x}_1 + \boldsymbol{\theta}_i^{\mathrm{T}}\varphi_i(\hat{\bar{\boldsymbol{x}}}_i) + \varepsilon_i - \delta_i - \dot{\alpha}_i^c] + w_i\dot{w}_i +$$
$$\frac{1}{r_i}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}[r_i\varphi_i(\hat{\bar{\boldsymbol{x}}}_i)e_i - \dot{\boldsymbol{\theta}}_i]. \tag{41}$$

根据 Young 不等式可得:

$$e_i(\varepsilon_i - \delta_i) \leqslant e_i^2 + \frac{1}{2}(\bar{\varepsilon}_i + \bar{\delta}_i). \tag{42}$$

将式(42)代入式(41)可得:

$$\dot{V}_i \leqslant - [\lambda_{\min}(\boldsymbol{Q}) - 1]\|\tilde{\boldsymbol{x}}\|^2 - \sum_{j=1}^{i-1} c_je_j^{2\beta} +$$
$$\sum_{j=1}^i e_j(e_{j+1} + w_{j+1}) + \sum_{j=1}^{i-1} \frac{\sigma_j}{r_j}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\boldsymbol{\theta}_j +$$

$$\sum_{j=2}^{i} w_j \left(-\frac{w_j}{\lambda_j} + B_j \right) + e_i [e_i + \alpha_i + k_i \tilde{x}_1 +$$

$$\boldsymbol{\theta}_i^{\mathrm{T}} \boldsymbol{\varphi}_i(\hat{\bar{\boldsymbol{x}}}_i) - \dot{\alpha}_i^c] + \frac{1}{r_i} \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}} [r_i \boldsymbol{\varphi}_i(\hat{\bar{\boldsymbol{x}}}_i) e_i - \dot{\boldsymbol{\theta}}_i] +$$

$$\omega_{i-1} + \frac{1}{2} (\bar{\varepsilon}_i + \bar{\delta}_i). \tag{43}$$

选择虚拟控制器 α_i 和自适应律 $\boldsymbol{\theta}_i$ 为

$$\alpha_i = -c_i e_i^{2\beta-1} - e_i - k_i \tilde{x}_1 - \boldsymbol{\theta}_i^{\mathrm{T}} \boldsymbol{\varphi}_i(\hat{\bar{\boldsymbol{x}}}_i) + \alpha_i^c, \tag{44}$$

$$\dot{\boldsymbol{\theta}}_i = r_i \boldsymbol{\varphi}_i(\hat{\bar{\boldsymbol{x}}}_i) e_i - \sigma_i \boldsymbol{\theta}_i, \tag{45}$$

其中 $c_i > 0$ 和 $\sigma_i > 0$ 为设计参数.

将(44)和(45)代入(43)可得:

$$\dot{V}_i \leqslant - [\lambda_{\min}(\boldsymbol{Q}) - 1] \parallel \tilde{\boldsymbol{x}} \parallel^2 - \sum_{j=1}^{i} c_j e_j^{2\beta} +$$

$$\sum_{j=1}^{i} e_j (e_{j+1} + w_{j+1}) + \sum_{j=1}^{i} \frac{\sigma_j}{r_j} \tilde{\boldsymbol{\theta}}_j^{\mathrm{T}} \boldsymbol{\theta}_j +$$

$$\sum_{j=2}^{i} w_j \left(-\frac{w_j}{\lambda_j} + B_j \right) + \omega_i, \tag{46}$$

其中 $\omega_i = \omega_{i-1} + \frac{1}{2} (\bar{\varepsilon}_i + \bar{\delta}_i)$.

根据文献[32],引入一个新的状态变量 α_{i+1}^c,然后令 α_i 通过具有时间常数 λ_{i+1} 的一阶滤波器得到 α_{i+1}^c,如下所示:

$$\lambda_{i+1} \dot{\alpha}_{i+1}^c + \alpha_{i+1}^c = \alpha_i, \alpha_{i+1}^c(0) = \alpha_i(0), \tag{47}$$

于是,由(47)可得:

$$\dot{\alpha}_{i+1}^c = \frac{\alpha_i - \alpha_{i+1}^c}{\lambda_{i+1}} = -\frac{w_{i+1}}{\lambda_{i+1}}. \tag{48}$$

根据(16)和(48)可得:

$$\dot{w}_{i+1} = \dot{\alpha}_{i+1}^c - \dot{\alpha}_i = -\frac{w_{i+1}}{\lambda_{i+1}} +$$

$$B_{i+1}(e_1, \cdots, e_{i+1}, w_2, \cdots, w_{i+1}, y_r, \dot{y}_r, \ddot{y}_r, \boldsymbol{\theta}_1, \cdots, \boldsymbol{\theta}_i), \tag{49}$$

其中

$$B_{i+1}(\cdot) = (2\beta - 1) c_i e_i^{2\beta-2} \dot{e}_i + k_i \dot{\tilde{\boldsymbol{x}}}_1 + \dot{e}_i +$$

$$\dot{\boldsymbol{\theta}}_i^{\mathrm{T}} \boldsymbol{\varphi}_i(\hat{\bar{\boldsymbol{x}}}_i) + \boldsymbol{\theta}_i^{\mathrm{T}} \frac{\partial(\boldsymbol{\varphi}_i)}{\partial \hat{\bar{\boldsymbol{x}}}_i} [\dot{\hat{x}}_1, \cdots, \dot{\hat{x}}_i]^{\mathrm{T}} + \frac{\dot{w}_i}{\lambda_i}.$$

第 n 步.定义 $e_n = \hat{x}_n - \alpha_n^c$,并计算 e_n 的导数为

$$\dot{e}_n = \dot{\hat{x}}_n - \dot{\alpha}_n^c =$$

$$k_n \tilde{x}_1 + \hat{f}_n(\hat{\boldsymbol{x}} \mid \boldsymbol{\theta}_n) + Q(u) - \dot{\alpha}_n^c =$$

$$k_n \tilde{x}_1 + \boldsymbol{\theta}_n^{\mathrm{T}} \boldsymbol{\varphi}_n(\hat{\boldsymbol{x}}) + A(u) u(t) + B(t) +$$

$$\tilde{\boldsymbol{\theta}}_n^{\mathrm{T}} \boldsymbol{\varphi}_n(\hat{\boldsymbol{x}}) + \varepsilon_n - \delta_n - \dot{\alpha}_n^c, \tag{50}$$

选择如下 Lyapunov 函数:

$$V_n = V_{n-1} + \frac{1}{2} e_n^2 + \frac{1}{2} w_n^2 + \frac{1}{2r_n} \tilde{\boldsymbol{\theta}}_n^{\mathrm{T}} \tilde{\boldsymbol{\theta}}_n, \tag{51}$$

其中 $\tilde{\boldsymbol{\theta}}_n = \boldsymbol{\theta}_n^* - \boldsymbol{\theta}_n, r_n > 0$ 为设计参数.

根据(46)和(51)可得 V_n 的导数为

$$\dot{V}_n \leqslant - [\lambda_{\min}(\boldsymbol{Q}) - 1] \parallel \tilde{\boldsymbol{x}} \parallel^2 - \sum_{j=1}^{n-1} c_j e_j^{2\beta} +$$

$$\sum_{j=1}^{n-1} e_j (e_{j+1} + w_{j+1}) + \sum_{j=1}^{n-1} \frac{\sigma_j}{r_j} \tilde{\boldsymbol{\theta}}_j^{\mathrm{T}} \boldsymbol{\theta}_j +$$

$$\sum_{j=2}^{n-1} w_j \left(-\frac{w_j}{\lambda_j} + B_j \right) + \omega_{n-1} + e_n [k_n \tilde{x}_1 + \boldsymbol{\theta}_n^{\mathrm{T}} \boldsymbol{\varphi}_n(\hat{\bar{\boldsymbol{x}}}_n) +$$

$$A(u) u(t) + B(t) + \varepsilon_n - \delta_n - \dot{\alpha}_n^c] +$$

$$w_n \left(-\frac{w_n}{\lambda_n} + B_n \right) + \frac{1}{r_n} \tilde{\boldsymbol{\theta}}_n^{\mathrm{T}} [r_n \boldsymbol{\varphi}_n(\hat{\bar{\boldsymbol{x}}}_n) e_n - \dot{\boldsymbol{\theta}}_n]. \tag{52}$$

根据 Young 不等式可得:

$$e_n (\varepsilon_n - \delta_n) \leqslant e_n^2 + \frac{1}{2} (\bar{\varepsilon}_n + \bar{\delta}_n), \tag{53}$$

$$e_n B(t) \leqslant \frac{1}{2} e_n^2 + \frac{1}{2} u_{\min}^2. \tag{54}$$

将(53)和(54)代入(52)式可得:

$$\dot{V}_n \leqslant - [\lambda_{\min}(\boldsymbol{Q}) - 1] \parallel \tilde{\boldsymbol{x}} \parallel^2 - \sum_{j=1}^{n-1} c_j e_j^{2\beta} +$$

$$\sum_{j=1}^{n-1} e_j (e_{j+1} + w_{j+1}) + \sum_{j=1}^{n-1} \frac{\sigma_j}{r_j} \tilde{\boldsymbol{\theta}}_j^{\mathrm{T}} \boldsymbol{\theta}_j +$$

$$\sum_{j=2}^{n} w_j \left(-\frac{w_j}{\lambda_j} + B_j \right) + \omega_{n-1} + e_n \Big[k_n \tilde{x}_1 + \frac{3}{2} e_n +$$

$$\boldsymbol{\theta}_n^{\mathrm{T}} \boldsymbol{\varphi}_n(\hat{\bar{\boldsymbol{x}}}_n) + A(u) u(t) - \dot{\alpha}_n^c \Big] +$$

$$\frac{1}{r_n} \tilde{\boldsymbol{\theta}}_n^{\mathrm{T}} [r_n \boldsymbol{\varphi}_n(\hat{\bar{\boldsymbol{x}}}_n) e_n - \dot{\boldsymbol{\theta}}_n] + \frac{1}{2} (\bar{\varepsilon}_n + \bar{\delta}_n) + \frac{1}{2} u_{\min}. \tag{55}$$

选择虚拟控制器 α_n 和自适应律 $\boldsymbol{\theta}_n$ 为

$$u = \frac{1}{1-\delta} \Big[-c_n e_n^{2\beta-1} - \frac{3}{2} e_n - k_n \tilde{x}_1 - \boldsymbol{\theta}_n^{\mathrm{T}} \boldsymbol{\varphi}_n(\hat{\bar{\boldsymbol{x}}}_n) + \alpha_n^c \Big], \tag{56}$$

$$\dot{\boldsymbol{\theta}}_n = r_n \boldsymbol{\varphi}_n(\hat{\bar{\boldsymbol{x}}}_n) e_n - \sigma_n \boldsymbol{\theta}_n, \tag{57}$$

其中 $c_n > 0$ 和 $\sigma_n > 0$ 为设计参数.

根据(56)和定理1可得:

$$A(u) u(t) \leqslant - c_n e_n^{2\beta-1} - \frac{3}{2} e_n - k_n \tilde{x}_1 - \boldsymbol{\theta}_n^{\mathrm{T}} \boldsymbol{\varphi}_n(\hat{\bar{\boldsymbol{x}}}_i) + \alpha_n^c. \tag{58}$$

将(57)和(58)代入(55)可得:

$$\dot{V}_n \leqslant - [\lambda_{\min}(\boldsymbol{Q}) - 1] \parallel \tilde{\boldsymbol{x}} \parallel^2 - \sum_{j=1}^{n} c_j e_j^{2\beta} +$$

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$$\sum_{j=1}^{n-1} e_j(e_{j+1} + w_{j+1}) + \sum_{j=1}^{n} \frac{\sigma_j}{r_j}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\boldsymbol{\theta}_j +$$

$$\sum_{j=2}^{n} w_j\left(-\frac{w_j}{\lambda_j} + B_j\right) + \omega_n, \quad (59)$$

其中 $\omega_n = \omega_{n-1} + \frac{1}{2}(\bar{\varepsilon}_n + \bar{\delta}_n) + \frac{1}{2}u_{\min}.$

3.2 闭环系统的稳定性分析

定理 2 针对具有量化输入及外部扰动的非线性系统(1),在假设 1、假设 2、引理 1—5 及定理 1下,本文所设计的自适应控制器(56),模糊状态观测器(10),参数自适应律(23)、(34)、(45)、(57)以及虚拟控制函数(22)、(33)和(44),能够保证闭环系统的所有信号都是有界的,并且能够保证观测器误差和跟踪误差在有限时间内收敛到原点的一个小邻域内.

证明 选择 Lyapunov 函数 $V = V_n$,故由(59)得:

$$\dot{V} \leqslant -[\lambda_{\min}(\boldsymbol{Q}) - 1] \| \tilde{\boldsymbol{x}} \|^2 - \sum_{j=1}^{n} c_j e_j^{2\beta} +$$

$$\sum_{j=1}^{n-1} e_j(e_{j+1} + w_{j+1}) + \sum_{j=1}^{n} \frac{\sigma_j}{r_j}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\boldsymbol{\theta}_j +$$

$$\sum_{j=2}^{n} w_j\left(-\frac{w_j}{\lambda_j} + B_j\right) + \omega_n. \quad (60)$$

根据 Young 不等式以下不等式成立:

$$\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\boldsymbol{\theta}_j \leqslant -\frac{1}{2}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_j + \frac{1}{2} \| \boldsymbol{\theta}_j^* \|^2,$$

$$e_j(e_{j+1} + w_{j+1}) \leqslant e_j^2 + \frac{1}{2}e_{j+1}^2 + \frac{1}{2}w_{j+1}^2, \quad (61)$$

$$| w_j B_j | \leqslant \frac{w_j^2 \bar{B}_j^2}{2} + \frac{1}{2}.$$

将式(61)代入式(60)可得:

$$\dot{V} \leqslant -[\lambda_{\min}(\boldsymbol{Q}) - 1] \| \tilde{\boldsymbol{x}} \|^2 - \left(\sum_{j=1}^{n} c_j e_j^{2\beta} - \right.$$

$$\left.\sum_{j=1}^{n-1} e_j^2 - \frac{1}{2}\sum_{j=2}^{n} e_j^2\right) - \frac{1}{2}\sum_{j=1}^{n} \frac{\sigma_j}{r_j}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_j -$$

$$\sum_{j=2}^{n}\left(\frac{1}{\lambda_j} - \frac{\bar{B}_j^2}{2} - \frac{1}{2}\right)w_j^2 + \omega, \quad (62)$$

其中 $\omega = \omega_n + \frac{1}{2}\sum_{j=1}^{n}\frac{\sigma_j}{r_j} \| \boldsymbol{\theta}_j^* \|^2 + \frac{n-1}{2}.$

我们可以把(62)式重新描述为

$$\dot{V} \leqslant -\frac{\lambda_{\min}(\boldsymbol{Q}) - 1}{\lambda_{\min}(\boldsymbol{Q})}(\lambda_{\min}(\boldsymbol{Q}) \| \tilde{\boldsymbol{x}} \|^2)^\beta +$$

$$\frac{\lambda_{\min}(\boldsymbol{Q}) - 1}{\lambda_{\min}(\boldsymbol{Q})}(\lambda_{\min}(\boldsymbol{Q}) \| \tilde{\boldsymbol{x}} \|^2)^\beta - [\lambda_{\min}(\boldsymbol{Q}) -$$

$$1] \| \tilde{\boldsymbol{x}} \|^2 - \left[\sum_{j=1}^{n} c e_j^{2\beta} - \left(\sum_{j=1}^{n-1} e_j^2\right)^\beta + \left(\sum_{j=1}^{n-1} e_j^2\right)^\beta - \right.$$

$$\sum_{j=1}^{n-1} e_j^2 - \left(\sum_{j=2}^{n} \frac{1}{2}e_j^2\right)^\beta + \left(\sum_{j=2}^{n} \frac{1}{2}e_j^2\right)^\beta - \sum_{j=2}^{n} \frac{1}{2}e_j^2\right] -$$

$$c\left(\sum_{j=1}^{n} \frac{1}{2r_j}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_j\right)^\beta + c\left(\sum_{j=1}^{n} \frac{1}{2r_j}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_j\right)^\beta - c\sum_{j=1}^{n} \frac{1}{2r_j}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_j -$$

$$c\left(\sum_{j=2}^{n} w_j^2\right)^\beta + c\left(\sum_{j=2}^{n} w_j^2\right)^\beta - c\sum_{j=2}^{n} w_j^2 + \omega, \quad (63)$$

其中 $c = \min\{c_i, \sigma_i, 1/\lambda_j, -\bar{B}_j^2/2 - 1/2, i = 1, 2, \cdots, n\}.$

引理 2 中的参数取特殊值,取 $z = 1, \zeta_1 = \lambda_{\min}(\boldsymbol{Q}) \| \tilde{\boldsymbol{x}} \|^2, \zeta_2 = \sum_{j=1}^{n-1} e_j^2, \zeta_3 = \sum_{j=2}^{n} \frac{1}{2}e_j^2, \zeta_4 = \sum_{j=1}^{n} \frac{1}{2r_j}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_j, \zeta_5 = \sum_{j=2}^{n} w_j^2, \mu = 1 - \beta, \upsilon = \beta$ 以及 $\iota = \beta^{\frac{\beta}{1-\beta}}$,可得以下不等式:

$$(\lambda_{\min}(\boldsymbol{Q}) \| \tilde{\boldsymbol{x}} \|^2)^\beta \leqslant (1 - \beta)\iota + \lambda_{\min}(\boldsymbol{Q}) \| \tilde{\boldsymbol{x}} \|^2,$$

$$\left(\sum_{j=1}^{n-1} e_j^2\right)^\beta \leqslant (1 - \beta)\iota + \sum_{j=1}^{n-1} e_j^2,$$

$$\left(\sum_{j=2}^{n} \frac{1}{2}e_j^2\right)^\beta \leqslant (1 - \beta)\iota + \sum_{j=2}^{n} \frac{1}{2}e_j^2,$$

$$\left(\sum_{j=1}^{n} \frac{1}{2r_j}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_j\right)^\beta \leqslant (1 - \beta)\iota + \sum_{j=1}^{n} \frac{1}{2r_j}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_j,$$

$$\left(\sum_{j=2}^{n} w_j^2\right)^\beta \leqslant (1 - \beta)\iota + \sum_{j=2}^{n} w_j^2. \quad (64)$$

将(64)代入(63)可得:

$$\dot{V} \leqslant -\frac{\lambda_{\min}(\boldsymbol{Q}) - 1}{\lambda_{\min}(\boldsymbol{Q})}(\lambda_{\min}(\boldsymbol{Q}) \| \tilde{\boldsymbol{x}} \|^2)^\beta -$$

$$\left[\sum_{j=1}^{n} c e_j^{2\beta} - \left(\sum_{j=1}^{n-1} e_j^2\right)^\beta - \left(\sum_{j=2}^{n} \frac{1}{2}e_j^2\right)^\beta\right] -$$

$$c\left(\sum_{j=1}^{n} \frac{1}{2r_j}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_j\right)^\beta - c\left(\sum_{j=2}^{n} w_j^2\right)^\beta + \omega +$$

$$(2c + 3)(1 - \beta)\iota. \quad (65)$$

根据引理 1 可得:

$$\left(\sum_{j=1}^{n-1} e_j^2\right)^\beta \leqslant \sum_{j=1}^{n-1} e_j^{2\beta}, \left(\sum_{j=2}^{n} \frac{1}{2}e_j^2\right)^\beta \leqslant \sum_{j=2}^{n} \frac{1}{2^\beta}e_j^{2\beta}. \quad (66)$$

将式(66)代入式(65)可得:

$$\dot{V} \leqslant -\frac{\lambda_{\min}(\boldsymbol{Q}) - 1}{\lambda_{\min}(\boldsymbol{Q})}(\lambda_{\min}(\boldsymbol{Q}) \| \tilde{\boldsymbol{x}} \|^2)^\beta -$$

$$\sum_{j=2}^{n-1}\left(c - 1 - \frac{1}{2^\beta}\right)e_j^{2\beta} - (c - 1)e_1^{2\beta} -$$

$$\left(c - \frac{1}{2^\beta}\right)e_n^{2\beta} - c\left(\sum_{j=1}^{n} \frac{1}{2r_j}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_j\right)^\beta - c\left(\sum_{j=2}^{n} w_j^2\right)^\beta +$$

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$$\omega + (2c + 3)(1 - \beta)\iota. \tag{67}$$

又根据引理 1 可得:

$$-\sum_{j=2}^{n-1}\left(c - 1 - \frac{1}{2^{\beta}}\right)e_j^{2\beta} \leqslant -2^{\beta}\left(c - 1 - \frac{1}{2^{\beta}}\right)\left(\sum_{j=2}^{n-1}\frac{e_j^2}{2}\right)^{\beta}. \tag{68}$$

将式(68)代入式(67)可得:

$$\dot{V} \leqslant -\frac{\lambda_{\min}(Q) - 1}{\lambda_{\min}(Q)}(\lambda_{\min}(Q)\|\tilde{x}\|^2)^{\beta} - $$
$$2^{\beta}\left(c - 1 - \frac{1}{2^{\beta}}\right)\left(\sum_{j=2}^{n-1}\frac{e_j^2}{2}\right)^{\beta} - 2^{\beta}(c - 1)\left(\frac{e_1^2}{2}\right)^{\beta} - $$
$$2^{\beta}\left(c - \frac{1}{2^{\beta}}\right)\left(\frac{e_n^2}{2}\right)^{\beta} - c\left(\sum_{j=1}^{n}\frac{1}{2r_j}\tilde{\theta}_j^T\tilde{\theta}_j\right)^{\beta} - $$
$$c\left(\sum_{j=2}^{n}w_j^2\right)^{\beta} + \omega + (2c + 3)(1 - \beta)\iota, \tag{69}$$

所以有

$$\dot{V} \leqslant -\bar{c}V^{\beta} + \rho, \tag{70}$$

其中

$$\bar{c} = \min\left\{2^{\beta}c, c, \frac{\lambda_{\min}(Q) - n}{\lambda_{\min}(Q)}\right\}, \tag{71}$$

$$\rho = \omega + (2c + 3)(1 - \beta)\iota. \tag{72}$$

令

$$T^* = \frac{1}{(1 - \beta)\eta\bar{c}}\left[V^{1-\beta}(e(0), \tilde{\theta}(0)) - \left(\frac{\rho}{(1 - \eta)\bar{c}}\right)^{(1-\beta)/\beta}\right], \tag{73}$$

其中

$$e(0) = [e_1(0), e_2(0), \cdots, e_n(0)]^T,$$
$$\tilde{\theta}(0) = [\tilde{\theta}_1(0), \tilde{\theta}_2(0), \cdots, \tilde{\theta}_n(0)]^T.$$

因此根据引理 4 可知,对于 $\forall t \geqslant T^*$,有 $V^{\beta}(e, \tilde{\theta}) \leqslant \frac{\rho}{(1 - \eta)\bar{c}}$,这意味着闭环系统中的所有信号都是 SGPFS.

此外,对于 $\forall t \geqslant T^*$,以下不等式成立:

$$|y - y_r| \leqslant 2\left(\frac{\rho}{(1 - \eta)\bar{c}}\right)^{\frac{1}{2\beta}}, \tag{74}$$

也就是说,观测器和跟踪误差收敛到原点的一个小邻域,并在有限时间 T^* 后仍保持在该邻域内.

4 仿真实例

以下仿真实例可以验证上述方法的有效性.

考虑如下具有量化输入和未知扰动的非线性系统:

$$\begin{cases} \dot{x}_1 = x_2 + 0.1x_1^3 + 0.2x_1 + d_1, \\ \dot{x}_2 = Q(u) + 0.3x_1x_2^2 + 0.1\cos x_1 + d_2, \\ y = x_1, \end{cases} \tag{75}$$

其中,x_1, x_2 为系统状态,u 和 y 是系统的输入和输出,外部扰动 $d_1 = 0.02\cos(2t)$,$d_2 = 0.01\sin(t + 0.25)$.$Q(u)$ 为(2)所定义的量化输入,以及 $\delta = 0.2$,$u_{\min} = 0.1$.给定初始值 $x_1(0) = 0.1$,$x_2(0) = 0.2$,给定参考信号 $y_r = \sin(0.5t) + \cos t$,控制目标是设计一个有限时间控制器,使系统的输出能够良好地跟随参考信号.

模糊状态观测器设计为

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + k_1(x_1 - \hat{x}_1) + \theta_1^T\varphi_1(\hat{x}_1), \\ \dot{\hat{x}}_2 = Q(u) + k_2(x_1 - \hat{x}_1) + \theta_2^T\varphi_2(\hat{x}_1, \hat{x}_2), \end{cases} \tag{76}$$

其中初始值为 $\hat{x}_1(0) = 0$,$\hat{x}_2(0) = 0$.

选择模糊隶属函数

$$\mu_{F_1^i}(\hat{x}_1) = \exp\left[\frac{-(\hat{x}_1 + 2 - 0.5i)^2}{2}\right],$$

$$\mu_{F_2^i}(\hat{x}_1, \hat{x}_2) = $$
$$\exp\left[\frac{-(\hat{x}_1 + 2 - 0.5i)^2}{2} + \frac{-(\hat{x}_2 + 2 - 0.5i)^2}{2}\right],$$

其中,$i = 1, 2, \cdots, 7$.

将模糊基函数定义为

$$\varphi_1(\hat{x}_1) = [\varphi_{11}(\hat{x}_1), \cdots, \varphi_{17}(\hat{x}_1)]^T,$$
$$\varphi_2(\hat{x}_1, \hat{x}_2) = [\varphi_{21}(\hat{x}_1, \hat{x}_2), \cdots, \varphi_{27}(\hat{x}_1, \hat{x}_2)]^T,$$

其中

$$\varphi_{1j}(\hat{x}_1) = \frac{\mu_{F_1^j}(\hat{x}_1)}{\sum_{j=1}^{7}\mu_{F_1^j}(\hat{x}_1)}, j = 1, \cdots, 7,$$

$$\varphi_{2j}(\hat{x}_1, \hat{x}_2) = \frac{\mu_{F_2^j}(\hat{x}_1, \hat{x}_2)}{\sum_{j=1}^{7}\mu_{F_2^j}(\hat{x}_1, \hat{x}_2)}, j = 1, \cdots, 7.$$

令参数 θ_1 和 θ_2 为

$$\theta_1 = [\theta_{11}, \theta_{12}, \theta_{13}, \theta_{14}, \theta_{15}, \theta_{16}, \theta_{17}]^T,$$
$$\theta_2 = [\theta_{21}, \theta_{22}, \theta_{23}, \theta_{24}, \theta_{25}, \theta_{26}, \theta_{27}]^T,$$

因此,模糊逻辑系统可以表示为

$$\theta_1^T\varphi_1(\hat{x}_1) = \sum_{i=1}^{7}\theta_{1i}\varphi_{1i}(\hat{x}_1),$$

$$\theta_2^T\varphi_2(\hat{x}_1, \hat{x}_2) = \sum_{i=1}^{7}\theta_{2i}\varphi_{2i}(\hat{x}_1, \hat{x}_2).$$

设计以下具有观测器的自适应模糊控制器

$$u = \frac{1}{1 - \delta}\left[-c_2e_2^{2\beta-1} - \frac{3}{2}e_2 - k_2\tilde{x}_1 - \theta_2^T\varphi_2(\hat{\bar{x}}_2) + \alpha_2^c\right],$$

定义 $\dot{\theta}_1$ 和 $\dot{\theta}_2$ 为

$$\dot{\theta}_1 = r_1e_1\varphi_1(\hat{x}_1) - \sigma_1\theta_1, \dot{\theta}_2 = r_2e_2\varphi_2(\hat{x}_1, \hat{x}_2) - \sigma_2\theta_2,$$

其中,$e_1 = y - y_r$,$e_2 = \hat{x}_2 - \alpha_2^c$,$w_2 = \alpha_2^c - \alpha_1$,虚拟控制

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器 $\alpha_1 = -c_1 e_1^{2\beta-1} - e_1 - \boldsymbol{\theta}_1^{\mathrm{T}} \boldsymbol{\varphi}_1(\hat{x}_1)$，自适应律的初值选择为 $\boldsymbol{\theta}_1(0) = [0,0.1,0,0.1,0,0.1,0]$，$\boldsymbol{\theta}_2(0) = [0.1,0,0.1,0,0,0,0.1]$.设计参数为 $k_1 = 6, k_2 = 8$，$r_1 = 0.01, r_2 = 0.01, \sigma_1 = 0.1, \sigma_2 = 0.1, c_1 = 48, c_2 = 32, \beta = 99/101$.

根据所设计的控制器 u，可由图1—6得出仿真结果.图1表示的是输出 y 的轨迹与跟踪信号 y_r 的轨迹，图2表示跟踪误差 e_1 的仿真结果.由图1和图2可以看出，在控制器 u 下实现了良好的跟踪性能.图3和图4表示系统状态 x_1, x_2 的轨迹及它们的估计 $\hat{x}_1$，$\hat{x}_2$ 的轨迹.图5表示 θ_1 与 θ_2 的轨迹，可看出 θ_1 与 θ_2 是有界的，图6表示了输入信号 $u(t)$ 以及量化输入信号 $Q(u(t))$.从以上仿真结果可以看出，本文所提出的自适应控制方法是有效的.

图3 状态变量 x_1 及其估计 $\hat{x}_1$ 的轨迹

Fig. 3 Trajectories of state variable x_1 and its estimation $\hat{x}_1$
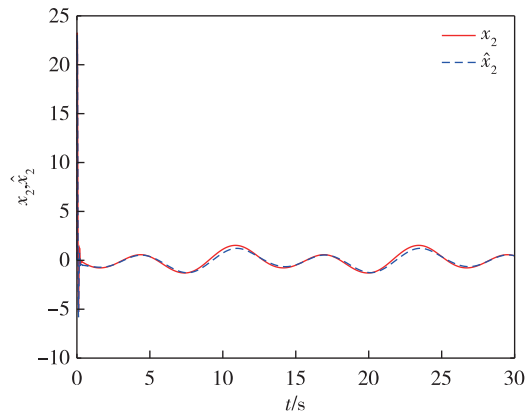
图4 状态变量 x_2 及其估计 $\hat{x}_2$ 的轨迹

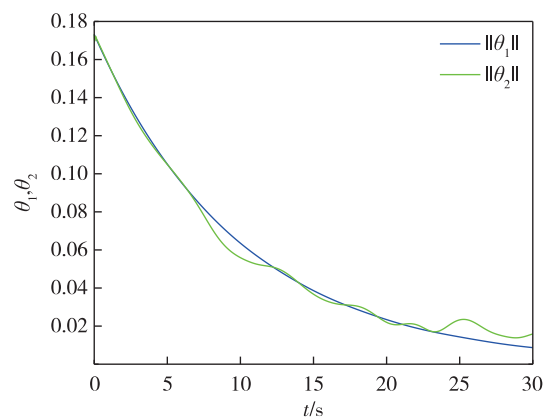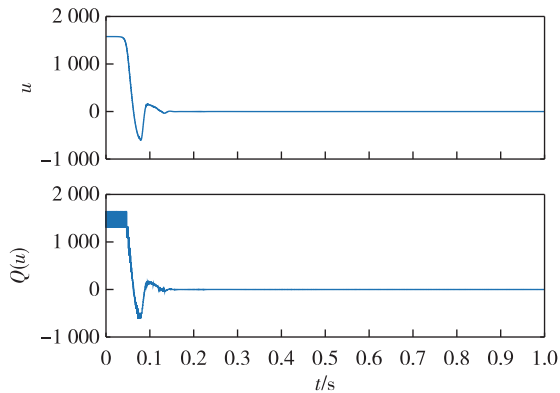Fig. 4 Trajectories of state variable x_2 and its estimation $\hat{x}_2$

图1 输出 y 与跟踪信号 y_r 的轨迹

Fig. 1 Trajectories of output y and tracking signal y_r

图2 跟踪误差 e_1 的轨迹

Fig. 2 Trajectory of tracking error e_1

图5 模糊自适应参数 θ_1 与 θ_2 的轨迹

Fig. 5 Trajectories of fuzzy adaptive parameters θ_1 and θ_2

5 结束语

本文针对一类具有量化输入和未知扰动的非线

性系统，提出了一种基于输出反馈的有限时间自适应模糊控制方案.采用模糊逻辑系统对系统中的非线性项进行逼近，利用一种滞回量化器来避免量化信号中的抖振.该控制方案可保证整个系统是

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图6　输入信号 u 及量化输入 $Q(u)$
Fig. 6　Trajectories of input u and quantized input $Q(u)$

SGPFS,保证闭环系统中的所有信号都是有界的,并且观测器误差及跟踪误差能够在有限时间内收敛到原点的一个小邻域,可以获得良好的跟踪性能.仿真结果验证了该控制方法的有效性和可行性.

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Adaptive finite-time dynamic surface control for nonlinear systems with input quantization and unknown disturbances

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Abstract   In this paper,the problem of finite-time adaptive output feedback dynamic surface control is studied for a class of nonlinear systems with quantized input signals and unknown disturbances.In the control design process, the nonlinear terms in the system are approximated by the fuzzy logic systems.A hysteretic quantizer is introduced to avoid chattering in the quantized signals,and the fuzzy state observer is constructed to estimate the unmeasurable states of the system.In order to propose the finite-time control strategy,firstly,a semi-global practical finite-time stability criterion is given.On this basis,an adaptive fuzzy controller is designed by combining the dynamic surface control technology with backstepping method.The controller can not only ensure that the observer and tracking error converge to a small neighborhood of the origin in a finite time,but also keep all the signals in the closed-loop system bounded.Finally,a simulation example is given to verify the effectiveness and feasibility of the control method.

Key words   quantized input signals;fuzzy logic systems;dynamic surface control;backstepping;finite-time tracking control