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基于多切换模式的多混沌系统有限时组合同步

摘要

本文研究了多切换模式下混沌系统的有限时同步控制问题.针对多个不同阶实变量混沌系统,研究了其多切换同步行为,给出了有限时多切换组合同步的定义,进而,在给出误差系统的基础上,设计了一类实现快速同步的有限时控制方案,并给出了误差系统有限时稳定的充分条件.最后,仿真结果表明所设计控制方案具有快速收敛性,较好地验证了有效性.

关键词

混沌系统;多切换模式;组合同步;有限时控制

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0 引言

混沌系统的同步控制作为保密通信的关键技术,自20世纪90年代以来受到了越来越多的关注并得到了迅猛的发展[1-5],对其研究不仅让人们更好地了解和认识混沌机理及其复杂性,对改善通信系统的保密性能和扩大应用范围也有着积极的指导意义.近年来,随着混沌同步研究的不断深入,如何给出具有保密性和安全性更好的同步控制方案成为研究热点.事实上,在实际的物理、自动控制以及信息通信等系统中,仅仅考虑两个混沌系统有很大的局限性,尤其是在多方保密通信的加密信号传输过程中,这将会大大降低系统的通信效率,甚至会影响混沌系统的同步.因此,建立多个混沌系统的同步机制越来越显现出它的重要性.

近年来,如何实现多个混沌系统同步已成为很多学者讨论和研究的焦点.文献[6]给出了 N 个异结构环链耦合混沌系统完全同步控制器的设计方法;文献[7-8]对 N 个分数阶耦合混沌系统的完全同步问题进行了研究;文献[9]研究了多个实变量混沌系统的组合同步问题;文献[10]考虑多个混沌系统之间的同步关系,给出了一种新的混沌同步模式——传输同步的定义;文献[11-13]基于网络传输同步模式设计了若干多混沌系统同步控制方案.为了较好地探索更复杂的混沌同步模式和提高传输信号的安全性,文献[14-15]给出了混沌同步过程中的多切换模式的定义.进而,文献[16-18]对多个混沌系统的多切换同步进行了深入的讨论,并给出了多切换同步控制器的设计方法.为了实现混沌误差系统的快速同步,有限时稳定已经成为研究热点并取得了一系列结果[16-18].文献[19]给出了一类更有效的实现快速稳定的非线性系统有限时稳定的设计方法.基于此,如何给出更有效地实现快速同步的控制方案成为亟待解决的问题.

为此,本文针对多个实变量混沌系统,研究多切换模式下多混沌系统的有限时同步控制问题,研究信号传输过程中的多切换行为,给出一类新的较好实现快速同步的有限时控制方案,给出误差系统有限时稳定的充分条件,所设计控制方案通过数值仿真进行有效性验证.

1 系统模型与问题描述

考虑如下具有不同阶数的混沌系统模型:

$$\begin{cases}\dot{x}_{11}(t)=\boldsymbol{A}_{11}\boldsymbol{x}_1(t)+\boldsymbol{f}_{11}(x_{11}(t),\cdots,x_{1n_1}(t)),\\ \dot{x}_{12}(t)=\boldsymbol{A}_{12}\boldsymbol{x}_1(t)+\boldsymbol{f}_{12}(x_{11}(t),\cdots,x_{1n_1}(t)),\\ \vdots\\ \dot{x}_{1n_1}(t)=\boldsymbol{A}_{1n_1}\boldsymbol{x}_1(t)+\boldsymbol{f}_{1n_1}(x_{11}(t),\cdots,x_{1n_1}(t)),\end{cases}\tag{1}$$

$$\begin{cases}\dot{x}_{i1}(t)=\boldsymbol{A}_{i1}\boldsymbol{x}_i(t)+\boldsymbol{f}_{i1}(x_{i1}(t),\cdots,x_{in_i}(t))+u_{i-1,1},\\ \dot{x}_{i2}(t)=\boldsymbol{A}_{i2}\boldsymbol{x}_i(t)+\boldsymbol{f}_{i2}(x_{i1}(t),\cdots,x_{in_i}(t))+u_{i-1,2},\\ \vdots\\ \dot{x}_{in_i}(t)=\boldsymbol{A}_{in_i}\boldsymbol{x}_i(t)+\boldsymbol{f}_{in_i}(x_{i1}(t),\cdots,x_{in_i}(t))+u_{i-1,n_i},\end{cases}\tag{2}$$

其中,$i=2,3;n_1\neq n_2\neq n_3.\boldsymbol{x}_1(t)=[x_{11},x_{12},\cdots,x_{1n_1}]^{\mathrm{T}}$,$\boldsymbol{x}_i(t)=[x_{i1},x_{i2},\cdots,x_{in_i}]^{\mathrm{T}}$为系统的状态向量.$\boldsymbol{f}_1(\boldsymbol{x}_1(t))=\boldsymbol{f}_{11}(x_{11}(t),\cdots,x_{1n_1}(t))$ 和 $\boldsymbol{f}_i(\boldsymbol{x}_i(t))=[\boldsymbol{f}_{i1},\boldsymbol{f}_{i2},\cdots,\boldsymbol{f}_{in_i}]^{\mathrm{T}}$ 是系统的非线性函数向量.$\boldsymbol{A}_1=[\boldsymbol{A}_{11},\boldsymbol{A}_{12},\cdots,\boldsymbol{A}_{1n_1}]^{\mathrm{T}}$ 和 $\boldsymbol{A}_i=[\boldsymbol{A}_{i1},\boldsymbol{A}_{i2},\cdots,\boldsymbol{A}_{in_i}]^{\mathrm{T}}$ 为系数矩阵.系统(2)的控制输入为 $\boldsymbol{u}_{i-1}=[u_{i-1,1},u_{i-1,2},\cdots,u_{i-1,n_i}]^{\mathrm{T}}$.

考虑 $s_{1jl}(j,l=1,\cdots,n_1),s_{ik_im_i}(k_i,m_i=1,\cdots,n_i)$ 为切换变量,且 $s_{1jl},s_{ik_im_i}\in\{0,1\}$,则在传输同步模式下,带有切换变量的组合同步误差变量为

$$\begin{cases}e_{11}=s_{211}x_{21}+s_{212}x_{22}+\cdots+s_{21n_2}x_{2n_2}-s_{111}x_{11}-\\ \qquad s_{112}x_{12}-\cdots-s_{11n_1}x_{1n_1},\\ e_{12}=s_{221}x_{21}+s_{222}x_{22}+\cdots+s_{22n_2}x_{2n_2}-s_{121}x_{11}-\\ \qquad s_{122}x_{12}-\cdots-s_{12n_1}x_{1n_1},\\ \vdots\\ e_{1n_1}=s_{2n_21}x_{21}+s_{2n_22}x_{22}+\cdots+s_{2n_2n_2}x_{2n_2}-s_{1n_11}x_{11}-\\ \qquad s_{1n_12}x_{12}-\cdots-s_{1n_1n_1}x_{1n_1},\end{cases}\tag{3}$$

$$\begin{cases}e_{21}=s_{311}x_{31}+s_{312}x_{32}+\cdots+s_{31n_3}x_{3n_3}-s_{221}x_{21}-\\ \qquad s_{222}x_{22}-\cdots-s_{22n_2}x_{2n_2},\\ e_{22}=s_{321}x_{31}+s_{322}x_{32}+\cdots+s_{32n_3}x_{3n_3}-s_{221}x_{21}-\\ \qquad s_{222}x_{22}-\cdots-s_{22n_2}x_{2n_2},\\ \vdots\\ e_{2n_2}=s_{3n_31}x_{31}+s_{3n_32}x_{32}+\cdots+s_{3n_3n_3}x_{3n_3}-s_{2n_21}x_{21}-\\ \qquad s_{2n_22}x_{22}-\cdots-s_{2n_2n_2}x_{2n_2}.\end{cases}\tag{4}$$

为了探索多切换组合同步行为,针对每一个误差变量,任意选取 $q_l(l=1,\cdots,n_1)$ 和 $p_{im_i}(m_i=1,\cdots,n_i)$ 个切换变量且 $q_1+\cdots+q_{n_1}=n_1,p_{i1}+\cdots+p_{in_i}=n_i$,同时

$$\begin{cases}s_{1jl_1},\cdots,s_{1jl_{q_l}}\neq 0,s_{1jl'_{ql}}=0,l'_{ql}\notin\{l_1,\cdots,l_{q_1}\},\\ \qquad s_{1j'l_{q_l}l'_q}=0,j'\neq j,\\ s_{ik_im_1},\cdots,s_{ik_im_{p_{imi}}}\neq 0,s_{ik_im'_{p_{im_i}}}=0,\\ \qquad m'_{p_{im_i}}\notin\{m_1,\cdots,m_{pimi}\},s_{ik'_im_{p_{im_i}}}=0,\end{cases}.$$

由此,基于多切换模式的组合同步的误差系统为

$$\dot{e}_{1j}=\sum_{d_j=1}^{p_{2m_2}}s_{2jm_{d_j}}\dot{x}_{2m_{d_j}}-\sum_{r=1}^{q_l}s_{1jl_r}\dot{x}_{1l_r},\tag{5}$$

$$\dot{e}_{2k_2}=\sum_{g_{k_2}=1}^{p_{3m_3}}s_{3k_3m_{g_{k_2}}}\dot{x}_{3m_{g_{k_2}}}-\sum_{d=1}^{p_{2m_2}}s_{2k_2m_d}\dot{x}_{2m_d}.\tag{6}$$

定义 1 考虑误差系统(5)和(6),在满足预设的切换规则的基础上,如果存在 $T=\max\{T_1,T_2\}$,使得:

$$\lim_{t\to T_1}\|e_{1j}\|=\lim_{t\to T_1}\left\|\sum_{d_j=1}^{p_{2m_2}}s_{2jm_{d_j}}\dot{x}_{2m_{d_j}}-\sum_{r=1}^{q_l}s_{1jl_r}\dot{x}_{1l_r}\right\|=0,$$

$$\left\|\sum_{d_j=1}^{p_{2m_2}}s_{2jm_{d_j}}\dot{x}_{2m_{d_j}}-\sum_{r=1}^{q_l}s_{1jl_r}\dot{x}_{1l_r}\right\|\equiv 0,t>T_1,$$

以及

$$\begin{cases}\lim\limits_{t\to T_2}\|e_{2k_2}\|=\lim\limits_{t\to T_1}\left\|\sum\limits_{g_{k_2}=1}^{p_{3m_3}}s_{3k_3m_{g_{k_2}}}\dot{x}_{3m_{g_{k_2}}}-\sum\limits_{d_j=1}^{p_{2m_2}}s_{2jm_{d_j}}\dot{x}_{2m_{d_j}}\right\|=0,\\ \left\|\sum\limits_{g_{k_2}=1}^{p_{3m_3}}s_{3k_3m_{g_{k_2}}}\dot{x}_{3m_{g_{k_2}}}-\sum\limits_{d_j=1}^{p_{2m_2}}s_{2jm_{d_j}}\dot{x}_{2m_{d_j}}\right\|\equiv 0,t>T_2\end{cases}$$

成立,则误差系统是有限时渐近稳定的,即系统(1)—(3)之间的有限时多切换组合同步被实现.

2 主要结果

考虑系统的阶数满足:$n_2-n_3=n_2-n_1\geqslant 2$,同时设定 $q_l=p_{3m_3}=1$,则误差系统(5)和(6)可改写为

$$\begin{cases}\dot{e}_{1j}=\sum\limits_{d_j=1}^{p_{2m_2}}\dot{x}_{2m_{d_j}}-\dot{x}_{1j},j=1,\cdots,n_1,\\ \dot{e}_{2k_3}=\dot{x}_{3k_3}-\sum\limits_{d_j=1}^{p_{2m_2}}\dot{x}_{2m_{d_j}},k_3=1,\cdots,n_3,\end{cases}\tag{7}$$

将式(1)和(2)代入式(7),可得:

$$\dot{e}_{1j}=\sum_{d_j=1}^{p_{2m_2}}[\boldsymbol{A}_{2m_{d_j}}\boldsymbol{x}_2(t)+\boldsymbol{f}_{2m_{d_j}}(x_{21}(t),\cdots,x_{2n_2}(t))+u_{1m_d}]-(\boldsymbol{A}_{1j}\boldsymbol{x}_1(t)+\boldsymbol{f}_{1j}(x_{11}(t),\cdots,x_{1n_1}(t))),\tag{8}$$

$$\dot{e}_{2k_3}=(\boldsymbol{A}_{3k_3}\boldsymbol{x}_3(t)+\boldsymbol{f}_{3k_3}(x_{31}(t),\cdots,x_{3n_3}(t))+u_{3k_3})-\sum_{d_j=1}^{p_{2m_2}}[\boldsymbol{A}_{2m_{d_j}}\boldsymbol{x}_2(t)+\boldsymbol{f}_{2m_{d_j}}(x_{21}(t),\cdots,x_{2n_2}(t))+u_{1m_{d_j}}].\tag{9}$$

根据定义1,针对基于多切换模式的组合同步误差系统(8)和(9),可实现其有限时渐近稳定的控制器设计如下:

$$\sum_{d_j=1}^{p_{2m_j}} u_{1m_d} = \sum_{d_j=1}^{p_{2m_j}} (-\boldsymbol{A}_{2m_{d_j}}\boldsymbol{x}_2(t) - \boldsymbol{f}_{2m_{d_j}}(x_{21}(t),\cdots,x_{2n_2}(t))) + \boldsymbol{A}_{1j}\boldsymbol{x}_1(t) + \boldsymbol{f}_{1j}(x_{11}(t),\cdots,x_{1n_1}(t)) - \omega_{1j}e_{1j} - e_{1j}^{\beta_1}, \tag{10}$$

$$u_{3k_3} = -\boldsymbol{A}_{3k_3}\boldsymbol{x}_3(t) - \boldsymbol{f}_{3k_3}(x_{31}(t),\cdots,x_{3n_3}(t)) + \sum_{d_j=1}^{p_{2m_j}} (\boldsymbol{A}_{2m_{d_j}}\boldsymbol{x}_2(t) + \boldsymbol{f}_{2m_{d_j}}(x_{21}(t),\cdots,x_{2n_2}(t))) - \omega_{2k_3}e_{2k_3} - e_{2k_3}^{\beta_{k_3}}, \tag{11}$$

其中,$\omega_{1j},\omega_{2k_3} > 0,\beta_1 = \frac{c_1}{y_1},\beta_2 = \frac{c_2}{y_2},c_1 > y_1,c_2 > y_2$,且 c_1,y_1,c_2,y_2 是奇数.

定理1 考虑系统(1)和(2),如果存在控制器(10)和(11)使得基于多切换模式的组合同步误差系统(8)和(9)在有限时间 T_1,T_2 渐近稳定,其中

$$T_1 = \frac{1}{1-\beta_1}\left\| \sum_{j=1}^{n_1} e_{1,j}^2(0) \right\|^{\frac{1-\beta_1}{2}},$$

$$T_2 = \frac{1}{1-\beta_2}\left\| \sum_{k_3=1}^{n_3} e_{2k_3}^2(0) \right\|^{\frac{1-\beta_2}{2}},$$

则系统(1)和(2)可在有限时间 $T=\max\{T_1,T_2\}$ 内实现系统的多切换同步.

证明 首先证明误差系统(8)的有限时渐近稳定性.考虑如下李雅普诺夫函数:

$$V_1 = \frac{1}{2}\sum_{j=1}^{n} e_{1j}^2 = \frac{1}{2}(e_{11}^2 + e_{12}^2 + \cdots + e_{1n_1}^2),$$

对其求导,可得:

$$\dot{V}_1 = \dot{e}_{11}e_{11} + \dot{e}_{12}e_{12} + \cdots + \dot{e}_{1n_1}e_{1n_1} = \left(\sum_{d_1=1}^{p_{2m_1}} \dot{x}_{2m_{d_1}} - \dot{x}_{11}(t)\right)e_{11} + \cdots + \left(\sum_{d_{n1}=1}^{p_{2m_{n_1}}} \dot{x}_{2m_{d_{n1}}} - \dot{x}_{1n_1}(t)\right)e_{1n_1},$$

进而可得

$$\dot{V}_1 = \left(\sum_{d_1=1}^{p_{2m_1}} [\boldsymbol{A}_{2m_{d_1}}\boldsymbol{x}_2(t) + \boldsymbol{f}_{2m_{d_1}} + u_{1m_{d_1}}] - (\boldsymbol{A}_{11}\boldsymbol{x}_1(t) + \boldsymbol{f}_{11})\right)e_{11} + \cdots + \left(\sum_{d_{n_1}=1}^{p_{2m_{n_1}}} [\boldsymbol{A}_{2m_{n_1}}\boldsymbol{x}_2(t) + \boldsymbol{f}_{2m_{n_1}} + u_{1m_{n_1}}] - (\boldsymbol{A}_{1n_1}\boldsymbol{x}_1(t) + \boldsymbol{f}_{1n_1})\right)e_{1n_1}.$$

将控制器(10)代入上式,可得

$$\dot{V}_1 = -\omega_{11}e_{11}^2 - e_{11}^{\beta_1+1} - \omega_{12}e_{12}^2 - e_{12}^{\beta_1+1} - \cdots - \omega_{1n_1}e_{1n_1}^2 - e_{1n_1}^{\beta_1+1},$$

进而,

$$\dot{V}_1 \leqslant -e_{11}^{\beta_1+1} - e_{12}^{\beta_1+1} - \cdots - e_{1n_1}^{\beta_1+1} = -2^{\frac{\beta_1+1}{2}}\left(\left(\frac{1}{2}e_{11}^2\right)^{\frac{\beta_1+1}{2}} + \left(\frac{1}{2}e_{12}^2\right)^{\frac{\beta_1+1}{2}} + \cdots + \left(\frac{1}{2}e_{1n_1}^2\right)^{\frac{\beta_1+1}{2}}\right) \leqslant -2^{\frac{\beta_1+1}{2}}\left(\frac{1}{2}e_{11}^2 + \frac{1}{2}e_{12}^2 + \cdots + \frac{1}{2}e_{1n_1}^2\right)^{\frac{\beta_1+1}{2}} \leqslant -2^{\frac{\beta_1+1}{2}}V_1^{\frac{\beta_1+1}{2}} \leqslant 0,$$

可知,式(8)在有限时间 T_1 渐近稳定,其中,$T_1 = \frac{1}{1-\beta_1}\left\| \sum_{j=1}^{n_1} e_{1,j}^2(0) \right\|^{\frac{1-\beta_1}{2}}$.

同理,针对误差系统(9),取李雅普诺夫函数为

$$V_2 = \frac{1}{2}\sum_{k_3=1}^{n} e_{2k_3}^2 \geqslant 0,$$

对其求导可得:

$$\dot{V}_2 = \sum_{i=1}^{n} e_{2k_3}\dot{e}_{2k_3} = \sum_{i=1}^{n} e_{2k_3}(-w_{2k_3}e_{2k_3} - e_{2k_3}^{\beta_2}),$$

进而

$$\dot{V}_2 \leqslant -2^{\frac{\beta_2+1}{2}}\left(\left(\frac{1}{2}e_{21}^2\right)^{\frac{\beta_2+1}{2}} + \left(\frac{1}{2}e_{22}^2\right)^{\frac{\beta_2+1}{2}} + \cdots + \left(\frac{1}{2}e_{2n_3}^2\right)^{\frac{\beta_2+1}{2}}\right) \leqslant -2^{\frac{\beta_2+1}{2}}\left(\frac{1}{2}e_{21}^2 + \frac{1}{2}e_{22}^2 + \cdots + \frac{1}{2}e_{2n_3}^2\right)^{\frac{\beta_2+1}{2}} \leqslant -2^{\frac{\beta_2+1}{2}}V_2^{\frac{\beta_2+1}{2}} \leqslant 0.$$

由此,误差系统(9)在有限时间 T_2 渐近稳定,其中 $T_2 = \frac{1}{1-\beta_2}\left\| \sum_{k_3=1}^{n_3} e_{2k_3}^2(0) \right\|^{\frac{1-\beta_2}{2}}$.基于上述证明,系统(1)和(2)可在有限时间 $T=\max\{T_1,T_2\}$ 内实现系统的多切换同步.

3 数值仿真与分析

基于上述研究结果,考虑如下3个不同阶混沌系统:

$$\begin{cases} \dot{x}_{11} = x_{12}, \\ \dot{x}_{12} = x_{11} - x_{11}^3 - x_{12} + 0.8\cos t, \end{cases} \tag{12}$$

$$\begin{cases} \dot{x}_{21} = -x_{22} - x_{23} + u_{11}, \\ \dot{x}_{22} = x_{21} + 0.25x_{22} + x_{24} + u_{12}, \\ \dot{x}_{13} = x_{21}x_{23} + 3 + u_{13}, \\ \dot{x}_{14} = -0.5x_{23} + 0.05x_{24} + u_{14}, \end{cases} \tag{13}$$

$$\begin{cases} \dot{x}_{31} = x_{32} + u_{21}, \\ \dot{x}_{32} = x_{31} - x_{31}^3 - x_{32} + 0.8\cos t + u_{22}. \end{cases} \tag{14}$$

切换变量设置如下(对应的多切换传输模式如图 1 所示):

$$\begin{cases} s_{311},s_{322}\neq 0,s_{312},s_{321}=0, \\ s_{211},s_{212},s_{223},s_{224}\neq 0,s_{213},s_{214},s_{221},s_{222}=0, \\ s_{121},s_{112}\neq 0,s_{122},s_{111}=0. \end{cases} \quad (15)$$

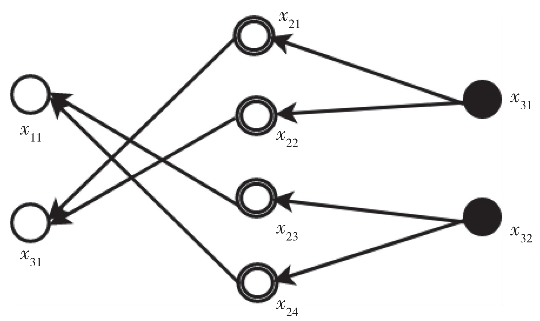

图 1　基于切换规则(15)的多切换组合同步模式框图

Fig. 1　The diagram of multi-switching combination synchronization mode in respect of (15)

考虑系统参数设置为 $\omega_{11}=\omega_{12}=1,\omega_{21}=\omega_{22}=2,\beta_1=3/5,\beta_2=5/7$,且系统的初始值设为$(x_{11}(0),x_{12}(0))=(-1,3)$,$(x_{21}(0),x_{22}(0),x_{23}(0),x_{24}(0))=(1,1,-1,2)$,$(x_{31}(0),x_{32}(0))=(-1,4)$.由此,误差系统的初始值为$(e_{11}(0),e_{12}(0),e_{21}(0),e_{22}(0))=(2,-1,3,-3)$,针对系统(12—14),基于切换规则(15)和控制器(10),(11),可得对应误差系统有限时间渐近稳定的仿真结果.

在基于预设切换规则 $(x_{31}\to\{x_{21},x_{22}\}\to x_{12})$ 和 $(x_{32}\to\{x_{23},x_{23}\}\to x_{11})$ 的前提下,图2给出了实现系统(12) 和(13) 所对应误差系统的状态变化.从图 2 可以看出,误差变量可以在有限时间趋近于 0.同理,从图 3 可以看出,实现系统(13) 和(14) 切换组合同步的误差变量可以在有限时间趋近于 0. 其中,$\{x_{21},x_{22}\}$,$\{x_{23},x_{23}\}$ 分别为组合状态变量,从而,较好地实现了系统(12)—(14)之间的有限时多切换组合同步,所设计控制方案的有效性和可行性被验证.

4　结束语

本文研究了一类混沌系统多切换模式下的有限时同步控制问题,分析了信号传输过程中的多切换同步行为,给出了有限时同步的定义.基于有限时稳定性理论,给出了一类实现误差系统快速收敛的有限时同步控制方案,通过仿真分析,该方案具有较好的有效性和可行性.如何进一步研究多复变量混沌系统的同步问题将是下一步的研究重点.

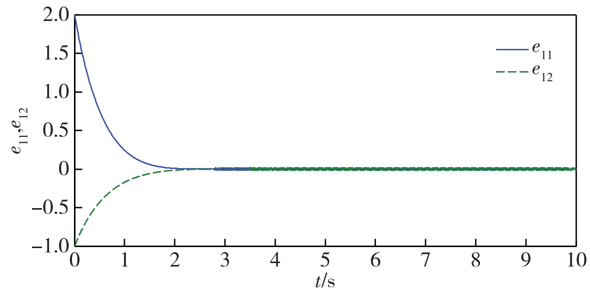

图 2　基于系统(12)和(13)的误差系统状态变化

Fig. 2　The state trajectories of the error systems between (12) and (13)

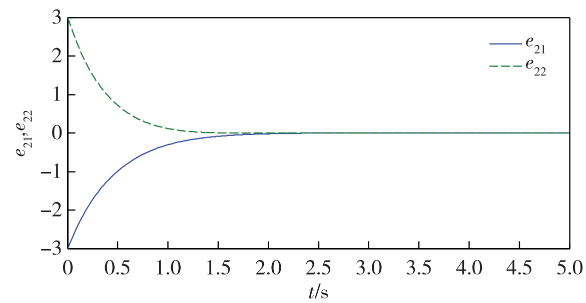

图 3　基于系统(13)和(14)的误差系统状态变化

Fig. 3　The state trajectories of the error systems between (13) and (14)

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Finite-time combination synchronization of multiple chaotic systems with multi-switching mode

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Abstract　This paper mainly investigates finite-time synchronization control of chaotic systems with multi-switching mode.For multiple real chaotic systems with different orders,its multi-switching synchronization behavior is investigated and finite-time combination multi-switching synchronization is defined,and a class of finite-time control schemes is designed,which can realize fast synchronization,when sufficient conditions for finite-time stability of error systems are provided.Finally,simulation results show that the proposed control scheme has fast convergence and reasonable validity.

Key words　chaotic systems;multi-switching mode;combination synchronization;finite-time control