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具有时变时滞的数据采样无人船舵减摇系统稳定性分析

摘要

本文研究了基于数据采样的无人船舵减摇闭环控制系统的稳定性.考虑到采样过程存在延迟现象,引用动态时滞区间的方法,构造相应的 Lyapunov-Krasovskii 泛函(LKF).此方法将固定时滞区间扩展成为动态时滞区间,不仅放宽了时滞区间上界和下界的限制,还能同时获得基于线性矩阵不等式(LMI)的更小保守性的相关稳定性判据.最终可以获得一个更宽松的标准来分析基于数据采样的无人船舵减摇闭环控制系统的稳定性.最后,举例说明了所提出方法的有效性.

关键词

数据采样系统;无人船;时变时滞;动态时滞区间方法

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0 引言

近年来,多无人船系统作为一类分布式复杂系统,在环境检测、资源勘探、科学研究等领域发展迅速.无人船(USV)被设计为智能运动平台,可以在恶劣的海洋环境中工作并完成各种任务[1-4],它是造船业和海洋产业的下一个发展方向,具有重要的科学和社会意义.因此,无人船已成为当前研究和应用的热点之一[5-6].在理论研究和实际应用中,由于对安全性和舒适性的需求不断增加,船舶舵减摇控制系统的稳定性开始受到人们的关注[7].所以,研究如何同时实现船舶的精确航向控制[8]和舵减摇[9-10]的性能非常重要.

在工程实际中,无人船的航向角、横摇角和艏摇角速度等状态都是由采样器采样并传输到控制器,控制器构建控制指令并将其发送到舵机(图 1 描述了受数据采样延迟影响的无人船)[11].很明显,由于采样过程的存在,不可避免地产生延迟,这可能导致系统振荡、发散或不稳定[12-14].因此,在控制设计时必须充分考虑采样引起的时滞.

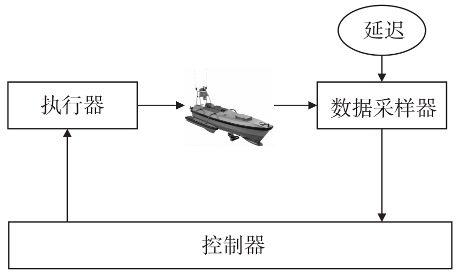

图 1　带有数据采样延迟约束的无人船
Fig. 1　The USV subject to sampled-data delay

目前,研究人员已经找到了一些方法和技术来处理时滞系统的稳定性问题.相关的研究结果可以分为两类:频域方法和时域方法.频域方法已经出现了很长时间,通常通过分析时滞系统的特征根是否具有负实部来判断系统的稳定性.它们主要包括扫频法、直接法、图解法、分析法、数值计算法等[15-16].然而,频域方法对于处理具有时变时滞的系统是不可行的.目前时滞系统研究的稳定性主要集中在时域方法上,主要方法是构建 Lyapunov-Krasovskii 泛函(LKF)并使用处理

LKF 导数的数学技术来得到相应的稳定性判据定理[17-19].在时变时滞分析方法中,Park[20]提出了具有向量交叉内积的边界方法;Fridman[21]提出了模型转换方法;将具有向量交叉内积的边界方法与模型转换方法相结合,Fridman 等[22]得到了一系列保守性较低的稳定性标准.为了进一步降低保守性,Wu 等[23]提出了一种自由加权矩阵方法,通过采用 Newton-Leibniz 公式,引入了一些自由加权矩阵.之后,Park 等[24]通过构建新的 Lyapunov 函数,进一步改进了自由加权矩阵方法;Han 等[25]提出了离散 Lyapunov 函数方法,以得到更小的保守性.然而,自由加权矩阵方法和离散 Lyapunov 函数方法都将引入大量计算,这使得稳定性条件变得复杂.Shao[26]和 Seuret 等[27]采用了积分不等式方法,这种方法无需引入过多的自由权矩阵参数,由此降低了计算量,但是对于分析时滞稳定性仍存在一定的保守性.最近,Zhang 等[28]提出了动态时滞区间方法,此方法改变固定的时滞区间为动态的时滞区间,并且更准确地分析了 LKF 的导数项,这对于获得一个更大的允许时滞上界更为有效.

本文研究了一种具有时变时滞的基于数据采样的无人船舵减摇闭环控制系统稳定性.本文引用动态时滞区间方法[28]的思想,针对所研究系统,获得了基于线性矩阵不等式(LMI)的更小保守性的相关稳定性判据.本方法允许有更多的能力来应对采样引起的延迟,这将对降低船舶设备的故障率有重要意义.

在本文中:对于矩阵 $\boldsymbol{A}$,$\boldsymbol{A}^{\mathrm{T}}$ 表示其转置;$\boldsymbol{I}$ 表示具有适当维度的单位矩阵;对于对称矩阵 $\boldsymbol{A}$,$\boldsymbol{A}>0$ 和 $\boldsymbol{A}<0$ 分别表示正定和负定;$\mathrm{He}(\boldsymbol{A})=\boldsymbol{A}+\boldsymbol{A}^{\mathrm{T}}$,$\mathbf{R}^n$ 表示 n 维欧氏空间.

1 无人船舵减摇数学模型介绍

船舶具有 6 个自由度,即横荡、艏摇、横摇、纵荡、垂荡、纵摇.然而,在本文中,仅考虑 y 方向上的不对称运动,它包括横荡、艏摇、横摇.由牛顿定律,可以得到相关运动的基本方程[29-30]:

$$\begin{cases} m_y \dfrac{\mathrm{d}^2 y}{\mathrm{d}t^2}=F_y, & \text{横荡}, \\ I_{zz}\dfrac{\mathrm{d}^2 e}{\mathrm{d}t^2}=N, & \text{艏摇}, \\ I_{xx}\dfrac{\mathrm{d}^2 f}{\mathrm{d}t^2}=M, & \text{横摇}, \end{cases} \tag{1}$$

其中 x,y 和 z 表示空间固定坐标系的轴,m_y 表示船舶在 y 方向上的有效质量,F_y 表示 y 方向的力,y 表示航向,e 表示航向角,f 表示横摇角,I_{zz} 和 I_{xx} 分别表示相对于 z 轴和 x 轴的惯性矩,N,M 分别表示相对于 z 轴和 x 轴的力矩.

将式(1)中的方程转化到图 2 坐标系,引入泰勒展式和拉普拉斯变换,忽略一些水动力效应及波浪对航向角和横摇角的影响,最后利用拉普拉斯逆变换,可以得到以下横荡-艏摇子系统和横摇子系统的状态空间模型[29-30]:

$$\begin{cases} \dot{\boldsymbol{x}}(t)=\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{B}\boldsymbol{\delta}(t), \\ \boldsymbol{x}(t_0)=\boldsymbol{x}_0, \end{cases} \tag{2}$$

其中,$\boldsymbol{x}(t)=[v(t),r(t),\psi(t),p(t),\varphi(t)]^{\mathrm{T}}\in\mathbf{R}^n$ 并且 $v(t)$、$r(t)$、$\psi(t)$、$p(t)$ 和 $\varphi(t)$ 分别表示仅由舵运动引起的横荡速度、艏摇角速度、航向角、横摇角速度和横摇角,$\boldsymbol{\delta}(t)$ 代表舵角,$\boldsymbol{x}_0\in\mathbf{R}^n$ 表示初始条件.$\boldsymbol{A}$,$\boldsymbol{B}$ 定义如下:

$$\boldsymbol{A}=\begin{bmatrix} -\dfrac{1}{T_v} & 0 & 0 & 0 & 0 \\ \dfrac{K_{vr}}{T_r} & -\dfrac{1}{T_r} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \omega_n^2 K_{vp} & 0 & 0 & -2\zeta\omega_n & -\omega_n^2 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\boldsymbol{B}=\left[\frac{K_{dv}}{T_v},\frac{K_{dr}}{T_r},0,\omega_n^2 K_{dp},0\right]^{\mathrm{T}},$$

T_v 和 T_r 表示传递函数的时间常数,ζ 和 ω_n 分别表示阻尼比和无阻尼下的固有频率,K_{vr},K_{vp},K_{dv},K_{dr} 和 K_{dp} 表示给定的增益.

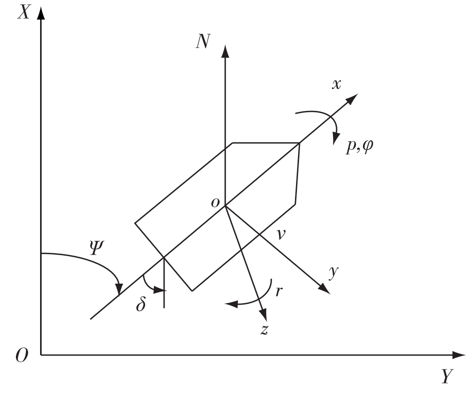

图 2 船舶平面运动变量描述

Fig. 2 The coordinate frames to describe marine vehicle motion

系统(2)中的横荡-艏摇子系统和横摇子系统模型可用于描述无人船的相应动态情况.

设 $t_k, t_{k+1}, \cdots (k=0,1,2,\cdots)$ 表示数据采样器的采样时刻.假设在时刻 $t_k, t_{k+1}, \cdots$ 采样的数据成功获得,而时刻 t_k 和 $t_{k+1}(k=0,1,2,\cdots)$ 之间的数据丢失.那么,无人船的采样过程可以如图 3 所示,其中虚线表示数据丢失.

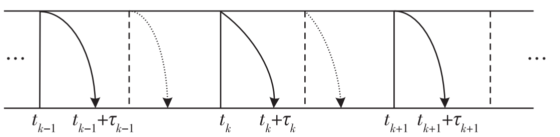

图 3　USV 采样过程

Fig. 3　Sampling process for the USV

当时间 $t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1})$ 时,选取控制输入为

$$\boldsymbol{\delta}(t) = \boldsymbol{K}\boldsymbol{x}(t_k), \tag{3}$$

因此,式(2)中提出的 USV 的状态方程被转换成:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{K}\boldsymbol{x}(t_k).$$

通过定义 $t_k = t - (t - t_k)$ 和 $\tau(t) = t - t_k$,可以建立以下闭环系统[29-30]:

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{K}\boldsymbol{x}(t - \tau(t)), \\ \boldsymbol{x}(t) = \boldsymbol{\Phi}(t), \quad t \in (-\tau_2, 0), \end{cases} \tag{4}$$

初始条件 $\boldsymbol{\Phi}(t)$ 是连续可微的向量值函数.

很明显 $\tau(t) \in [\tau_k, t_{k+1} - t_k + \tau_{k+1})$,这也可以描述为 $\tau(t) \in [\tau_1, \tau_2)$.在传统的分析中,人们经常直接分析时变时滞的固定区间$[\tau_1, \tau_2)$,本文把固定时滞区间转换成动态时滞区间$[(1-\alpha)\tau_1 + \alpha\tau(t), \alpha\tau(t) + (1-\alpha)\tau_2)$,其中 $\alpha \in [0,1)$,并且定义 $a(t) = (1-\alpha)\tau_1 + \alpha\tau(t)$、$b(t) = \alpha\tau(t) + (1-\alpha)\tau_2$.下一节阐述了具有时变时滞的基于数据采样的无人船舵减摇闭环控制系统的稳定性.

2　主要结果

对于给定的控制器增益 $\boldsymbol{K}$,设 $\boldsymbol{B}\boldsymbol{K} = \boldsymbol{A}_1$,那么可以得到如下具有时变延迟的线性系统:

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{A}_1\boldsymbol{x}(t - \tau(t)), \quad t > 0, \\ \boldsymbol{x}(t) = \boldsymbol{\Phi}(t), \quad t \in (-\tau_2, 0). \end{cases} \tag{5}$$

在分析时变时滞系统的稳定性时需要如下引理:

引理 1(Wirtinger 积分不等式[27])　给定一个矩阵 $\boldsymbol{M} > 0$,对于在$[a,b] \to \mathbf{R}^n$ 上的所有连续可微函数 g,有以下不等式成立:

$$-\int_a^b \dot{\boldsymbol{g}}^{\mathrm{T}}(s)\boldsymbol{M}\dot{\boldsymbol{g}}(s)\mathrm{d}s \leqslant -\frac{1}{b-a}(\boldsymbol{g}(b) - \boldsymbol{g}(a))^{\mathrm{T}}\boldsymbol{M}(\boldsymbol{g}(b) - \boldsymbol{g}(a)) - \frac{3}{b-a}\left(\boldsymbol{g}(b) + \boldsymbol{g}(a) - \frac{2}{b-a}\int_a^b \boldsymbol{g}(s)\mathrm{d}s\right)^{\mathrm{T}} \times \boldsymbol{M}\left(\boldsymbol{g}(b) + \boldsymbol{g}(a) - \frac{2}{b-a}\int_a^b \boldsymbol{g}(s)\mathrm{d}s\right).$$

引理 2(反凸组合不等式[31])　对于任意向量 $\boldsymbol{f}_1, \boldsymbol{f}_2$,适当维数的二阶矩阵 $\boldsymbol{S}, \boldsymbol{X}$ 和正数 α_1、α_2,如果 $\begin{bmatrix} \boldsymbol{S} & \boldsymbol{X} \\ * & \boldsymbol{S} \end{bmatrix} \geqslant 0$,$\alpha_1 + \alpha_2 = 1$,那么,有以下不等式成立:

$$\frac{1}{\alpha_1}\boldsymbol{f}_1^{\mathrm{T}}\boldsymbol{S}\boldsymbol{f}_1 + \frac{1}{\alpha_2}\boldsymbol{f}_2^{\mathrm{T}}\boldsymbol{S}\boldsymbol{f}_2 \geqslant \begin{bmatrix} \boldsymbol{f}_1 \\ \boldsymbol{f}_2 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \boldsymbol{S} & \boldsymbol{X} \\ * & \boldsymbol{S} \end{bmatrix} \begin{bmatrix} \boldsymbol{f}_1 \\ \boldsymbol{f}_2 \end{bmatrix}.$$

根据引理 1、2 及动态时滞区间思想,针对我们所考虑的基于采样控制系统的无人船,本文应用李雅普诺夫稳定性判定理论可以得到定理 1.

定理 1

如果存在一个 20 × 20 的正定对称矩阵 $\boldsymbol{P} = \begin{bmatrix} \boldsymbol{P}_{11} & \boldsymbol{P}_{12} & \boldsymbol{P}_{13} & \boldsymbol{P}_{14} \\ * & \boldsymbol{P}_{22} & \boldsymbol{P}_{23} & \boldsymbol{P}_{24} \\ * & * & \boldsymbol{P}_{33} & \boldsymbol{P}_{34} \\ * & * & * & \boldsymbol{P}_{44} \end{bmatrix}$,5 × 5 的正定对称矩阵 $\boldsymbol{R}_1$, $\boldsymbol{R}_2, \boldsymbol{R}_3, \boldsymbol{Q}_1, \boldsymbol{Q}_2$,一个 10 × 10 的矩阵 $\boldsymbol{X} = \begin{bmatrix} \boldsymbol{X}_{11} & \boldsymbol{X}_{12} \\ \boldsymbol{X}_{21} & \boldsymbol{X}_{22} \end{bmatrix}$,使得下列矩阵不等式成立:

$$\boldsymbol{\Phi}_{[35\times35]} < 0, \quad \begin{bmatrix} \boldsymbol{Q}_2 & 0 & \boldsymbol{X}_{11} & \boldsymbol{X}_{12} \\ 0 & 3\boldsymbol{Q}_2 & \boldsymbol{X}_{21} & \boldsymbol{X}_{22} \\ \boldsymbol{X}_{11}^{\mathrm{T}} & \boldsymbol{X}_{21}^{\mathrm{T}} & \boldsymbol{Q}_2 & 0 \\ \boldsymbol{X}_{12}^{\mathrm{T}} & \boldsymbol{X}_{22}^{\mathrm{T}} & 0 & 3\boldsymbol{Q}_2 \end{bmatrix} > 0, \tag{6}$$

那么,依赖采样控制器的无人船舶系统(5)在控制输入(3)的作用下是渐进稳定的.

其中,

$$\boldsymbol{\Phi}_{11} = \boldsymbol{P}_{11}\boldsymbol{A} + \boldsymbol{P}_{12} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}_{11} + \boldsymbol{P}_{12}^{\mathrm{T}} + \boldsymbol{R}_1 + \boldsymbol{R}_3 + a(t)\boldsymbol{A}^{\mathrm{T}}\boldsymbol{Q}_1\boldsymbol{A} + (b(t) - a(t))\boldsymbol{A}^{\mathrm{T}}\boldsymbol{Q}_2\boldsymbol{A} - 4\frac{1 - \alpha\dot{\tau}(t)}{\hat{a}}\boldsymbol{Q}_1,$$

$$\boldsymbol{\Phi}_{12} = -(1 - \alpha\dot{\tau}(t))\boldsymbol{P}_{12} + (1 - \alpha\dot{\tau}(t))\boldsymbol{P}_{13} - 2\frac{1 - \alpha\dot{\tau}(t)}{\hat{a}}\boldsymbol{Q}_1,$$

$$\boldsymbol{\Phi}_{13} = \boldsymbol{P}_{11}\boldsymbol{A}_1 - (1 - \dot{\tau}(t))\boldsymbol{P}_{13} + (1 - \dot{\tau}(t))\boldsymbol{P}_{14} + a(t)\boldsymbol{A}^{\mathrm{T}}\boldsymbol{Q}_1\boldsymbol{A}_1 + (b(t) - a(t))\boldsymbol{A}^{\mathrm{T}}\boldsymbol{Q}_2\boldsymbol{A}_1,$$

$$\boldsymbol{\Phi}_{14} = -(1 - \alpha\dot{\tau}(t))\boldsymbol{P}_{14},$$

$$\boldsymbol{\Phi}_{15}=a(t)\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}_{12}+a(t)\boldsymbol{P}_{22}+6\frac{1-\alpha\dot{\tau}(t)}{\hat{a}}\boldsymbol{Q}_1,$$

$$\boldsymbol{\Phi}_{16}=(\tau(t)-a(t))\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}_{13}+(\tau(t)-a(t))\boldsymbol{P}_{23}^{\mathrm{T}},$$

$$\boldsymbol{\Phi}_{17}=(b(t)-\tau(t))\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}_{14}+(b(t)-\tau(t))\boldsymbol{P}_{24}^{\mathrm{T}},$$

$$\boldsymbol{\Phi}_{22}=-(1-\alpha\dot{\tau}(t))\boldsymbol{R}_1+(1-\alpha\dot{\tau}(t))\boldsymbol{R}_2-4\frac{1-\alpha\dot{\tau}(t)}{\hat{a}}\boldsymbol{Q}_1-4\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{Q}_2,$$

$$\boldsymbol{\Phi}_{23}=-2\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{Q}_2-\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{11}-\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{12}-\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{21}-\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{22},$$

$$\boldsymbol{\Phi}_{24}=\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{11}+\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{21}-\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{12}-\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{22},$$

$$\boldsymbol{\Phi}_{25}=-(1-\alpha\dot{\tau}(t))a(t)\boldsymbol{P}_{22}+(1-\alpha\dot{\tau}(t))a(t)\boldsymbol{P}_{23}^{\mathrm{T}}+6\frac{1-\alpha\dot{\tau}(t)}{\hat{a}}\boldsymbol{Q}_1,$$

$$\boldsymbol{\Phi}_{26}=-(1-\alpha\dot{\tau}(t))(\tau(t)-a(t))\boldsymbol{P}_{23}^{\mathrm{T}}+(1-\alpha\dot{\tau}(t))(\tau(t)-a(t))\boldsymbol{P}_{33}+6\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{Q}_2,$$

$$\boldsymbol{\Phi}_{27}=-(1-\alpha\dot{\tau}(t))(b(t)-\tau(t))\boldsymbol{P}_{24}^{\mathrm{T}}+(1-\alpha\dot{\tau}(t))(b(t)-\tau(t))\boldsymbol{P}_{34}^{\mathrm{T}}+2\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{12}+2\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{22},$$

$$\begin{aligned}\boldsymbol{\Phi}_{33}=&-(1-\dot{\tau}(t))\boldsymbol{R}_2+a(t)\boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{Q}_1\boldsymbol{A}_1+(b(t)-a(t))\boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{Q}_2A_1-8\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{Q}_2+\\&\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{11}^{\mathrm{T}}+\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{12}^{\mathrm{T}}-\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{21}^{\mathrm{T}}-\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{22}^{\mathrm{T}}+\\&\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{11}+\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{12}-\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{21}-\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{22},\end{aligned}$$

$$\boldsymbol{\Phi}_{34}=-2\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{Q}_2-\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{11}+\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{12}+\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{21}-\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{22},$$

$$\boldsymbol{\Phi}_{35}=a(t)\boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{P}_{12}-(1-\dot{\tau}(t))a(t)\boldsymbol{P}_{23}^{\mathrm{T}}+(1-\dot{\tau}(t))a(t)\boldsymbol{P}_{24}^{\mathrm{T}},$$

$$\begin{aligned}\boldsymbol{\Phi}_{36}=&(\tau(t)-a(t))\boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{P}_{13}-(1-\dot{\tau}(t))(\tau(t)-a(t))\boldsymbol{P}_{33}+(1-\dot{\tau}(t))(\tau(t)-a(t))\boldsymbol{P}_{34}^{\mathrm{T}}+\\&6\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{Q}_2+2\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{21}^{\mathrm{T}}+2\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{22}^{\mathrm{T}},\end{aligned}$$

$$\begin{aligned}\boldsymbol{\Phi}_{37}=&(b(t)-\tau(t))\boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{P}_{14}-(1-\dot{\tau}(t))(b(t)-\tau(t))\boldsymbol{P}_{34}^{\mathrm{T}}+(1-\dot{\tau}(t))(b(t)-\tau(t))\boldsymbol{P}_{44}+\\&6\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{Q}_2-2\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{12}+2\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{22},\end{aligned}$$

$$\boldsymbol{\Phi}_{44}=-(1-\alpha\dot{\tau}(t))\boldsymbol{R}_3-4\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{Q}_2,$$

$$\boldsymbol{\Phi}_{45}=-(1-\alpha\dot{\tau}(t))a(t)\boldsymbol{P}_{24}^{\mathrm{T}},$$

$$\boldsymbol{\Phi}_{46}=-(1-\alpha\dot{\tau}(t))(\tau(t)-a(t))\boldsymbol{P}_{34}^{\mathrm{T}}-2\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{21}^{\mathrm{T}}+2\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{22}^{\mathrm{T}},$$

$$\boldsymbol{\Phi}_{47}=-(1-\alpha\dot{\tau}(t))(b(t)-\tau(t))\boldsymbol{P}_{44}+6\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{Q}_2,$$

$$\boldsymbol{\Phi}_{55}=-12\frac{1-\alpha\dot{\tau}(t)}{\hat{a}}\boldsymbol{Q}_1,\quad \boldsymbol{\Phi}_{66}=-12\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{Q}_2,$$

$$\boldsymbol{\Phi}_{67}=-4\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{X}_{22},\quad \boldsymbol{\Phi}_{77}=-12\frac{1-\alpha\dot{\tau}(t)}{b(t)-a(t)}\boldsymbol{Q}_2.$$

其中,$\hat{a}=(1-\alpha)\tau+\alpha\tau_2$.

证明

由于保证采样控制的有效性是设计控制器的基本前提,而时滞系统的分析又常常受限于 Jensen 不等式的限制,于是引用动态时滞区间方法[28]的思想,构建如下 Lyapunov-Krasovskii 泛函:

$$\begin{aligned}V=&\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{\xi}(t)+\int_{t-a(t)}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{R}_1\boldsymbol{x}(s)\mathrm{d}s+\\&\int_{t-\tau(t)}^{t-a(t)}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{R}_2\boldsymbol{x}(s)\mathrm{d}s+\int_{t-b(t)}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{R}_3\boldsymbol{x}(s)\mathrm{d}s+\\&\int_{-a(t)}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{Q}_1\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta+\\&\int_{-b(t)}^{-a(t)}\int_{t+\theta}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{Q}_2\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta,\end{aligned}$$

其中,$\boldsymbol{\xi}(t)=\left[\boldsymbol{x}^{\mathrm{T}}(t),\int_{t-a(t)}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\mathrm{d}s,\int_{t-\tau(t)}^{t-a(t)}\boldsymbol{x}^{\mathrm{T}}(s)\mathrm{d}s,\int_{t-b(t)}^{t-\tau(t)}\boldsymbol{x}^{\mathrm{T}}(s)\mathrm{d}s\right]^{\mathrm{T}}$,显然,由于 $\boldsymbol{P},\boldsymbol{R}_1,\boldsymbol{R}_2,\boldsymbol{R}_3,\boldsymbol{Q}_1,\boldsymbol{Q}_2$ 是正定矩阵,所以 V 是正定的.

对所设计的 V 进行求导,有:

$$\begin{aligned}\dot{V}=&2\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{P}\dot{\boldsymbol{\xi}}(t)+\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{R}_1\boldsymbol{x}(t)-(1-\dot{a}(t))\boldsymbol{x}^{\mathrm{T}}(t-a(t))\boldsymbol{R}_1\boldsymbol{x}(t-a(t))+(1-\dot{a}(t))\boldsymbol{x}^{\mathrm{T}}(t-\end{aligned}$$

$$a(t))\boldsymbol{R}_2\boldsymbol{x}(t-a(t))-(1-\dot{\tau}(t))\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{R}_2\boldsymbol{x}(t-\tau(t))+\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{R}_3\boldsymbol{x}(t)-(1-\dot{b}(t))\boldsymbol{x}^{\mathrm{T}}(t-b(t))\boldsymbol{R}_3\boldsymbol{x}(t-b(t))+a(t)\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Q}_1\dot{\boldsymbol{x}}(t)+(b(t)-a(t))\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Q}_2\dot{\boldsymbol{x}}(t)-(1-\alpha\dot{\tau}(t))\int_{t-a(t)}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Q}_1\dot{\boldsymbol{x}}(t)\mathrm{d}s-(1-\alpha\dot{\tau}(t))\int_{t-b(t)}^{t-a(t)}\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Q}_2\dot{\boldsymbol{x}}(t)\mathrm{d}s, \quad (7)$$

其中取:

$$\boldsymbol{\Psi}(t)=\Big[\boldsymbol{x}^{\mathrm{T}}(t),\boldsymbol{x}^{\mathrm{T}}(t-a(t)),\boldsymbol{x}^{\mathrm{T}}(t-\tau(t)),\boldsymbol{x}^{\mathrm{T}}(t-b(t)),\frac{1}{a(t)}\int_{t-a(t)}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\mathrm{d}s,\frac{1}{\tau(t)-a(t)}\int_{t-\tau(t)}^{t-a(t)}\boldsymbol{x}^{\mathrm{T}}(s)\mathrm{d}s,\frac{1}{b(t)-\tau(t)}\int_{t-b(t)}^{t-\tau(t)}\boldsymbol{x}^{\mathrm{T}}(s)\mathrm{d}s\Big]^{\mathrm{T}}.$$

对于 $\dot{V}$ 后两项,一方面有:

$$-(1-\alpha\dot{\tau}(t))\int_{t-a(t)}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Q}_1\dot{\boldsymbol{x}}(t)\mathrm{d}s-(1-\alpha\dot{\tau}(t))\int_{t-b(t)}^{t-a(t)}\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Q}_2\dot{\boldsymbol{x}}(t)\mathrm{d}s=$$
$$-(1-\alpha\dot{\tau}(t))\int_{t-a(t)}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Q}_1\dot{\boldsymbol{x}}(t)\mathrm{d}s-(1-\alpha\dot{\tau}(t))\int_{t-\tau(t)}^{t-a(t)}\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Q}_2\dot{\boldsymbol{x}}(t)\mathrm{d}s-(1-\alpha\dot{\tau}(t))\int_{t-b(t)}^{t-\tau(t)}\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Q}_2\dot{\boldsymbol{x}}(t)\mathrm{d}s\leqslant$$
$$-(1-\alpha\dot{\tau}(t))\frac{a(t)}{\hat{a}}\int_{t-a(t)}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Q}_1\dot{\boldsymbol{x}}(t)\mathrm{d}s-(1-\alpha\dot{\tau}(t))\frac{\tau(t)-a(t)}{\tau(t)-a(t)}\int_{t-\tau(t)}^{t-a(t)}\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Q}_2\dot{\boldsymbol{x}}(t)\mathrm{d}s-(1-\alpha\dot{\tau}(t))\frac{b(t)-\tau(t)}{b(t)-\tau(t)}\int_{t-b(t)}^{t-\tau(t)}\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Q}_2\dot{\boldsymbol{x}}(t)\mathrm{d}s=$$
$$-\frac{1-\alpha\dot{\tau}(t)}{(1-\alpha)(\tau_2-\tau_1)}\Big(\frac{(1-\alpha)(\tau_2-\tau_1)}{\hat{a}}a(t)\times\int_{t-a(t)}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Q}_1\dot{\boldsymbol{x}}(t)\mathrm{d}s-\frac{b(t)-a(t)}{\tau(t)-a(t)}(\tau(t)-a(t))\int_{t-\tau(t)}^{t-a(t)}\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Q}_2\dot{\boldsymbol{x}}(t)\mathrm{d}s-\frac{b(t)-a(t)}{b(t)-\tau(t)}(b(t)-\tau(t))\int_{t-b(t)}^{t-\tau(t)}\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Q}_2\dot{\boldsymbol{x}}(t)\mathrm{d}s\Big).$$

另一方面,应用引理 1 的 Wirtinger 积分不等式来估计上式中 $a(t)\int_{t-a(t)}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Q}_1\dot{\boldsymbol{x}}(t)\mathrm{d}s,(\tau(t)-a(t))\times\int_{t-\tau(t)}^{t-a(t)}\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Q}_2\dot{\boldsymbol{x}}(t)\mathrm{d}s,(b(t)-\tau(t))\int_{t-b(t)}^{t-\tau(t)}\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Q}_2\dot{\boldsymbol{x}}(t)\mathrm{d}s$ 三项,由于采用了动态时滞区间方法,所以当使用 Wirtinger 积分不等式的时候,所考虑的时滞区间并不是原来的固定的时滞区间,因此得到的稳定性判据定理也不再受限于固定时滞区间所带来的保守性.

接下来,应用引理 2 反凸组合方法,选取:

$$\alpha_1=\frac{\tau(t)-a(t)}{b(t)-a(t)},\alpha_2=\frac{b(t)-\tau(t)}{b(t)-a(t)},$$

$$\boldsymbol{f}_1=\Big[\boldsymbol{x}(t-a(t))-\boldsymbol{x}(t-\tau(t)),\boldsymbol{x}(t-a(t))+\boldsymbol{x}(t-\tau(t))-\frac{2}{\tau(t)-a(t)}\int_{t-\tau(t)}^{t-a(t)}\boldsymbol{x}(s)\mathrm{d}s\Big],$$

$$\boldsymbol{f}_2=\Big[\boldsymbol{x}(t-\tau(t))-\boldsymbol{x}(t-b(t)),\boldsymbol{x}(t-\tau(t))+\boldsymbol{x}(t-b(t))-\frac{2}{b(t)-\tau(t)}\int_{t-b(t)}^{t-\tau(t)}\boldsymbol{x}(s)\mathrm{d}s\Big].$$

由此,可以得到以下不等式:

$$-\frac{1-\alpha\dot{\tau}(t)}{(1-\alpha)(\tau_2-\tau_1)}\Big(\frac{(1-\alpha)(\tau_2-\tau_1)}{\hat{a}}a(t)\times\int_{t-a(t)}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Q}_1\dot{\boldsymbol{x}}(t)\mathrm{d}s-\frac{b(t)-a(t)}{\tau(t)-a(t)}(\tau(t)-a(t))\int_{t-\tau(t)}^{t-a(t)}\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Q}_2\dot{\boldsymbol{x}}(t)\mathrm{d}s-\frac{b(t)-a(t)}{b(t)-\tau(t)}(b(t)-\tau(t))\int_{t-b(t)}^{t-\tau(t)}\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Q}_2\dot{\boldsymbol{x}}(t)\mathrm{d}s\Big)\leqslant-\frac{1-\alpha\dot{\tau}(t)}{(1-\alpha)(\tau_2-\tau_1)}\times$$
$$\Big(\frac{(1-\alpha)(\tau_2-\tau_1)}{\hat{a}}\Big(4\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}_1\boldsymbol{x}(t)+2\mathrm{He}(\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}_1\boldsymbol{x}(t-a(t)))+4\boldsymbol{x}^{\mathrm{T}}(t-a(t))\boldsymbol{Q}_1\boldsymbol{x}(t-a(t))-6\mathrm{He}\Big(\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}_1\frac{1}{a(t)}\int_{t-a(t)}^{t}\boldsymbol{x}(s)\mathrm{d}s\Big)-6\mathrm{He}\Big(\boldsymbol{x}^{\mathrm{T}}(t-a(t))\boldsymbol{Q}_1\frac{1}{a(t)}\int_{t-a(t)}^{t}\boldsymbol{x}(s)\mathrm{d}s\Big)+12\frac{1}{a(t)}\int_{t-a(t)}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\mathrm{d}s\boldsymbol{Q}_1\frac{1}{a(t)}\int_{t-a(t)}^{t}\boldsymbol{x}(s)\mathrm{d}s\Big)+$$
$$4\boldsymbol{x}^{\mathrm{T}}(t-a(t))\boldsymbol{Q}_2\boldsymbol{x}(t-a(t))+8\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{Q}_2\boldsymbol{x}(t-\tau(t))+4\boldsymbol{x}^{\mathrm{T}}(t-b(t))\boldsymbol{Q}_2\boldsymbol{x}(t-b(t))+$$
$$12\frac{1}{\tau(t)-a(t)}\int_{t-\tau(t)}^{t-a(t)}\boldsymbol{x}^{\mathrm{T}}(s)\mathrm{d}s\boldsymbol{Q}_2\frac{1}{\tau(t)-a(t)}\int_{t-\tau(t)}^{t-a(t)}\boldsymbol{x}(s)\mathrm{d}s+12\frac{1}{b(t)-\tau(t)}\int_{t-b(t)}^{t-\tau(t)}\boldsymbol{x}^{\mathrm{T}}(s)\mathrm{d}s\boldsymbol{Q}_2\frac{1}{b(t)-\tau(t)}\int_{t-b(t)}^{t-\tau(t)}\boldsymbol{x}(s)\mathrm{d}s+$$
$$\mathrm{He}\Big(2\boldsymbol{x}^{\mathrm{T}}(t-a(t))\boldsymbol{Q}_2\boldsymbol{x}(t-\tau(t))-6\boldsymbol{x}^{\mathrm{T}}(t-a(t))\boldsymbol{Q}_2\frac{1}{\tau(t)-a(t)}\int_{t-\tau(t)}^{t-a(t)}\boldsymbol{x}(s)\mathrm{d}s-6\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{Q}_2\frac{1}{\tau(t)-a(t)}\int_{t-\tau(t)}^{t-a(t)}\boldsymbol{x}(s)\mathrm{d}s+$$

$$
\begin{aligned}
&2\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{Q}_2 x(t-b(t)) - \\
&6\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{Q}_2 \frac{1}{b(t)-\tau(t)}\int_{t-b(t)}^{t-\tau(t)}\boldsymbol{x}^{\mathrm{T}}(s)\,\mathrm{d}s - \\
&6\boldsymbol{x}^{\mathrm{T}}(t-b(t))\boldsymbol{Q}_2 \frac{1}{b(t)-\tau(t)}\int_{t-b(t)}^{t-\tau(t)}\boldsymbol{x}^{\mathrm{T}}(s)\,\mathrm{d}s + \\
&\boldsymbol{x}^{\mathrm{T}}(t-a(t))\boldsymbol{X}_{11}\boldsymbol{x}(t-\tau(t)) - \\
&\boldsymbol{x}^{\mathrm{T}}(t-a(t))\boldsymbol{X}_{11}\boldsymbol{x}(t-b(t)) - \\
&\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{X}_{11}\boldsymbol{x}(t-\tau(t)) + \\
&\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{X}_{11}\boldsymbol{x}(t-b(t)) + \\
&\boldsymbol{x}^{\mathrm{T}}(t-a(t))\boldsymbol{X}_{21}\boldsymbol{x}(t-\tau(t)) - \\
&\boldsymbol{x}^{\mathrm{T}}(t-a(t))\boldsymbol{X}_{21}\boldsymbol{x}(t-b(t)) + \\
&\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{X}_{21}\boldsymbol{x}(t-\tau(t)) - \\
&\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{X}_{21}\boldsymbol{x}(t-b(t)) + \\
&\boldsymbol{x}^{\mathrm{T}}(t-a(t))\boldsymbol{X}_{12}\boldsymbol{x}(t-\tau(t)) + \\
&\boldsymbol{x}^{\mathrm{T}}(t-a(t))\boldsymbol{X}_{12}\boldsymbol{x}(t-b(t)) - \\
&\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{X}_{12}\boldsymbol{x}(t-\tau(t)) - \\
&\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{X}_{12}\boldsymbol{x}(t-b(t)) + \\
&\boldsymbol{x}^{\mathrm{T}}(t-a(t))\boldsymbol{X}_{22}\boldsymbol{x}(t-\tau(t)) + \\
&\boldsymbol{x}^{\mathrm{T}}(t-a(t))\boldsymbol{X}_{22}\boldsymbol{x}(t-b(t)) + \\
&\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{X}_{22}\boldsymbol{x}(t-\tau(t)) + \\
&\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{X}_{22}\boldsymbol{x}(t-b(t)) - \\
&2\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{X}_{21}^{\mathrm{T}}\frac{1}{\tau(t)-a(t)}\int_{t-\tau(t)}^{t-a(t)}\boldsymbol{x}(s)\,\mathrm{d}s + \\
&2\boldsymbol{x}^{\mathrm{T}}(t-b(t))\boldsymbol{X}_{21}^{\mathrm{T}}\frac{1}{\tau(t)-a(t)}\int_{t-\tau(t)}^{t-a(t)}\boldsymbol{x}(s)\,\mathrm{d}s - \\
&2\boldsymbol{x}^{\mathrm{T}}(t-a(t))\boldsymbol{X}_{12}\frac{1}{b(t)-\tau(t)}\int_{t-b(t)}^{t-\tau(t)}\boldsymbol{x}^{\mathrm{T}}(s)\,\mathrm{d}s + \\
&2\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{X}_{12}\frac{1}{b(t)-\tau(t)}\int_{t-b(t)}^{t-\tau(t)}\boldsymbol{x}^{\mathrm{T}}(s)\,\mathrm{d}s - \\
&2\boldsymbol{x}^{\mathrm{T}}(t-a(t))\boldsymbol{X}_{22}\frac{1}{b(t)-\tau(t)}\int_{t-b(t)}^{t-\tau(t)}\boldsymbol{x}^{\mathrm{T}}(s)\,\mathrm{d}s - \\
&2\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{X}_{22}\frac{1}{b(t)-\tau(t)}\int_{t-b(t)}^{t-\tau(t)}\boldsymbol{x}^{\mathrm{T}}(s)\,\mathrm{d}s + \\
&4\frac{1}{\tau(t)-a(t)}\int_{t-\tau(t)}^{t-a(t)}\boldsymbol{x}(s)\mathrm{d}s\boldsymbol{X}_{22}\frac{1}{b(t)-\tau(t)}\int_{t-b(t)}^{t-\tau(t)}\boldsymbol{x}^{\mathrm{T}}(s)\mathrm{d}s - \\
&2\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{X}_{22}^{\mathrm{T}}\frac{1}{\tau(t)-a(t)}\int_{t-\tau(t)}^{t-a(t)}\boldsymbol{x}(s)\,\mathrm{d}s - \\
&2\boldsymbol{x}^{\mathrm{T}}(t-b(t))\boldsymbol{X}_{22}^{\mathrm{T}}\frac{1}{\tau(t)-a(t)}\int_{t-\tau(t)}^{t-a(t)}\boldsymbol{x}(s)\,\mathrm{d}s\Big)\Big).
\end{aligned}
\tag{8}
$$

结合式(7)、(8) 和 $\boldsymbol{\Psi}(t)$ 的定义,可以得出:

$$\dot{V} \leqslant \boldsymbol{\Psi}^{\mathrm{T}}(t)\boldsymbol{\Phi}_{[35\times35]}\boldsymbol{\Psi}(t). \tag{9}$$

所以,如果式(6)满足,那么由李雅普诺夫稳定性理论,依赖采样控制器的无人船系统(5)是渐进稳定的.由此,完成了证明.

注 1 在无人船控制系统中,由于船舶的控制指令依赖于数据采样系统的传输指令,所以考虑数据采样引起的时延是影响整个无人船控制系统的一个重要因素.定理 1 运用了动态时滞区间方法来分析时滞系统的稳定性.由于动态时滞区间方法的区间可调节性,在分析中,稳定性判据的保守性被大大降低了,这为分析无人船数据采样在何种程度的延迟上能够被有效控制提供了较精确的理论依据.

注 2 文献[28]将动态时滞区间方法应用于神经网络系统稳定性分析中,并得到了保守性较少的理论成果.本文简化了文献[28]的李雅普诺夫泛函,并得到了基于数据采样的无人船舵减摇闭环控制系统的稳定性判定定理.由于所考虑的自由权参数个数的减少,这种在李雅普诺夫泛函设计上的简化使得动态时滞区间方法可以更高效地提高所考虑的系统的最大允许上界.由于运用较少矩阵变量得到较少保守性的时滞系统稳定性判定定理一直是时滞理论应用的重要着眼点[32],因此,是否可以对李雅普诺夫泛函进一步简化来得到相对较好的结果依然是一个非常值得讨论的问题.

3 仿真结果

本节给出了一个例子,选用文献[26]、文献[27]和本文所用方法进行比对,最终结果表明本文采用的方法比前两者提出的方法的保守性更小.

例 1 对于式(2)中的系统矩阵 $\boldsymbol{A}$,$\boldsymbol{B}$,选择文献[11] 所用参数:

$U = 7.8$ m/s, $T_v = 1.8/U$, $T_r = 2/U$,
$K_{dr} = -0.0036U$, $K_{dp} = -0.0022U^2$,
$K_{dv} = 0.06U$, $K_{vp} = 0.16U$,
$K_{vr} = -0.58$ m/s, $\omega_n = 2.2$ rad/s,
$\zeta = 0.58 + 0.67U$,
$\boldsymbol{K} = [0.0763 \quad 0.2385 \quad 0.5374 \quad -0.0198 \quad 0.3963]$.

因为 $\tau(t) = t - t_k$,所以 $\dot{\tau}(t) = 1$,那么,对于给定的时滞下界 τ_1,使得系统可以渐近稳定的最大时滞上界 τ_2,如表 1 所示.分别选用文献[26]、文献[27]和本文所用方法进行仿真,由表 1 可以明显看出,本文的方法可以得到更大的时滞上界.

4 结束语

本文研究了具有时变时滞的基于数据采样的无人船舵减摇闭环控制系统稳定性.采用动态时滞区间

表 1　当 $\dot{\tau}(t)=1$ 时的时滞最大上界

Table 1　Maximum upper bounds of delay with given τ_1 for $\dot{\tau}(t)=1$

τ_1	方法	$\mu=1$
1	文献[26]	8.264 0
	文献[27]	10.712 4
	本文($\alpha=0.7$)	12.939 4
2	文献[26]	8.306 0
	文献[27]	10.831 1
	本文($\alpha=0.7$)	13.360 2
3	文献[26]	8.353 0
	文献[27]	10.958 2
	本文($\alpha=0.7$)	13.820 0

的方法,将固定的时滞区间扩展为动态的时滞区间,并且采用了 Wirtinger 积分不等式和反凸组合的方法处理 LKF 的导数项.最后,将本文所用方法与文献[26]、文献[27]所提方法进行比较,结果表明本文方法能够得到保守性更小的稳定性结果.这将允许有更多的能力来应对采样引起的延迟,对降低船舶设备的故障率有重要意义.

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Stability analysis for a sampled-data rudder roll system of an unmanned surface vehicle with time-varying delay

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Abstract This paper is concerned with the stabilization of sampled-data-based unmanned surface vehicle (USV) rudder roll closed-loop control system. Considering the delay in the sampling process, we discuss the dynamic delay interval method and construct a corresponding Lyapunov-Krasovskii function (LKF). This method extends the fixed interval to a dynamic interval, which not only relaxes the restriction of upper and lower bounds to the delay interval but also can obtain a much less conservative delay-dependent stability criterion based on linear matrix inequality (LMI). Therefore, one can obtain a much more relaxed criterion to analyze the stability of a sampled-data-based USV rudder roll closed-loop control system. To this end, an example is given to illustrate the effectiveness of the proposed method.

Key words sampled-data system; unmanned surface vehicle (USV); time-varying delay; dynamic delay interval method