



# 带有驱动器冗余的欠驱动船舶的自适应容错控制

## 摘要

在本文中,一种非线性自适应控制容错策略的开发使得具有推力器冗余的欠驱动船舶可以遵循预定的路径行驶,尽管存在未知的系统参数以及由波浪、风和洋流引起的环境干扰.在设计和分析中涉及的技术包括使用反步法、参数投影技术和横向函数.结果表明,本文所提出的控制器,参考路径可以用任意小的跟踪误差进行全局跟踪.仿真结果证明了该控制器的有效性.

## 关键词

欠驱动船舶; 执行器故障; 冗余性

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## 0 引言

非完整约束欠驱动系统的控制在过去几十年中受到了控制领域的广泛关注.这类系统的典型例子包括非完整移动机器人、欠驱动船舶、水下航行器和飞机的垂直起降飞机(VTOL)等.由于 Brockett 必要条件不能满足<sup>[1]</sup>,所以连续时不变反馈控制器不能保证该类系统的稳定性.文献[2]采用坐标变换的方法,确保了系统的指数稳定性,并且可以使船舶的位置和航向跟踪上给定的目标.文献[3-4]给出一种连续时变跟踪控制器,将船舶跟踪系统转化为斜对称形式,设计了时变动态振荡器,解决了跟踪问题.利用一个简单的状态反馈控制律可以使跟踪误差达到指数稳定<sup>[5]</sup>.利用航天器的姿态稳定<sup>[6]</sup>,表面只有4个执行机构的自主水下航行器的指数稳定是可能的<sup>[7]</sup>.文献[8]提出了一种全局鲁棒自适应控制器,考虑恒定扰动和时变干扰,可以使船舶跟随给出的路径.文献[9]通过全局非线性坐标变换将船舶动力学转化为仿射速度,提出了一种用于欠驱动水面船舶跟踪控制的全局部分状态反馈和输出反馈控制方案.文献[10]基于李雅普诺夫函数,针对欠驱动船舶提出了2种跟踪方案.

在控制欠驱动的船舶时,执行机构的故障<sup>[11-12]</sup>是一个很关键的问题,它可能导致控制失效甚至更大事故.为了可靠性和安全性,驱动器故障补偿受到了广泛关注,并基于鲁棒控制<sup>[13]</sup>、多模型<sup>[14]</sup>、滑模控制<sup>[15-16]</sup>等多种方法已取得了相当大的进展.值得注意的是推力-力分配问题是克服推力器失效的重要因素<sup>[17]</sup>.文献[18]提出了一种方法,能同时容纳推力器故障和饱和的自主水下航行器的推力分配.文献[19]提出了一种利用加权伪逆的故障诊断和调节系统.文献[20]给出了引入加权伪逆和量子粒子群优化的混合容错控制.文献[21]针对全驱动表面容器的跟踪控制,采用了反步法和模糊容错控制相结合的方法.不过,这些研究都集中在完全或过度驱动的系统上.由于设备的增多,意味着系统的成本和质量增加.此外,如果一个完全驱动的系统被损坏,系统将会产生一个欠驱动的控制<sup>[22]</sup>.因此,对欠驱动系统开发一个容错控制器是必要的.

对于带有外部干扰的欠驱动船舶,本文提出了一种自适应容错控制设计方法.与上述结果相反,所有的系统参数都是未知的.通过引入导线函数法<sup>[23]</sup>,即引入了一个“辅助操纵变量”,可以克服控制欠驱动系统所遇到的困难.

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### 1 船舶模型与控制目标

与文献[2]类似,船舶的模型如图1所示.

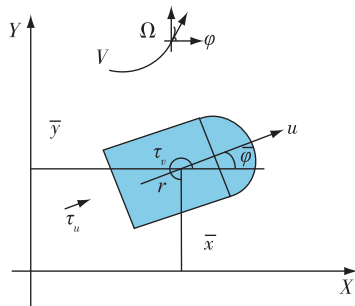


图1 船舶模型

Fig.1 Coordinates of an underacted ship

船舶的运动学和动力学模型为

$$\begin{bmatrix} \dot{\bar{x}} \\ \dot{\bar{y}} \\ \dot{\bar{\varphi}} \end{bmatrix} = \begin{bmatrix} \cos \bar{\varphi} & -\sin \bar{\varphi} & 0 \\ \sin \bar{\varphi} & \cos \bar{\varphi} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ r \end{bmatrix}, \quad (1)$$

$$\begin{aligned} \dot{u} &= \frac{m_{22}}{m_{11}}vr - \frac{d_{11}}{m_{11}}u - \sum_{i=2}^3 \frac{d_{ui}}{m_{11}}|u|^{i-1}u + \frac{1}{11} \sum_{k=1}^{n_u} \tau_{u,k} + \frac{1}{m_{11}}\tau_{wu}(t), \\ \dot{v} &= -\frac{m_{11}}{m_{22}}ur - \frac{d_{22}}{m_{22}}v - \sum_{i=2}^3 \frac{d_{vi}}{m_{22}}|v|^{i-1}v + \frac{1}{m_{22}}\tau_{wv}(t), \\ \dot{r} &= \frac{m_{11} - m_{22}}{m_{33}}uv - \frac{d_{33}}{m_{33}}r - \sum_{i=2}^3 \frac{d_{ri}}{m_{33}}|r|^{i-1}r + \frac{1}{m_{33}} \sum_{k=1}^{n_r} \tau_{r,k} + \frac{1}{m_{33}}\tau_{wr}(t), \end{aligned} \quad (2)$$

式中,  $\bar{x}, \bar{y}$  分别表示船舶在坐标轴上的位置,  $\bar{\varphi}$  表示船航行的角度,  $u, v$  和  $r$  分别表示浪涌速度、摆动速度和偏航速度.  $\tau_{u,k}$  和  $\tau_{r,k}$  表示执行器,  $n_u$  和  $n_r$  表示执行器的个数. 当第  $k$  个执行器发生故障时, 可以表示为

$$\begin{aligned} \tau_{u,k}(t) &= \delta_{k,h} \bar{\tau}_{u,k}(t) + \chi_{u,k,h}, \\ \tau_{r,k}(t) &= \eta_{k,h} \bar{\tau}_{r,k}(t) + \chi_{r,k,h}, \\ t &\in [T_{k,h}^s, T_{k,h}^e), \quad h = 1, 2, 3, \dots, \end{aligned} \quad (3)$$

式中,  $0 \leq \delta_{k,h} < 1, 0 \leq \eta_{k,h} < 1, 0 \leq T_{k,1}^s < T_{k,1}^e \leq T_{k,2}^s < T_{k,2}^e \leq \dots \leq \infty$  且它们均是未知的  $\chi_{u,k,h}, \chi_{r,k,h}$  是连续分段的有界未知信号. 式(3) 涵盖以下2种类型的故障:

1) 当  $0 < \delta_{k,h} < 1$  或者  $0 < \eta_{k,h} < 1$ , 部分执行器故障. 此时执行器的增益下降到区间  $(0, 1)$  中, 同时执行器可能受到来自  $\chi_{u,k,h}$  或者  $\chi_{r,k,h}$  的附加故障.

2) 当  $\delta_{k,h} = 0$  或者  $\eta_{k,h} = 0$ , 此时  $\tau_{u,k} = \chi_{u,k,h}$  或  $\tau_{r,k} = \chi_{r,k,h}$ , 执行器完全失去作用. 这时的输出  $\tau_{u,k}, \tau_{r,k}$  不再受控制输入  $\bar{\tau}_{u,k}, \bar{\tau}_{r,k}$  的影响.

式(3) 中,  $T_{k,h}^s$  与  $T_{k,h}^e$  分别表示第  $k$  个驱动器的  $h$  故障开始和结束时的时刻. 如果  $T_{k,h+1}^s > T_{k,h}^e$ , 则表示执行机构在下次故障发生时恢复正常工作. 如果  $T_{k,h+1}^s = T_{k,h}^e$ , 说明在  $T_{k,h}^e$  这个时刻故障  $\delta_{k,h}$  或者  $\eta_{k,h}$  跳转到故障  $\delta_{k,h+1}$  或  $\eta_{k,h+1}$  的过程中没有恢复. 式(3) 中, 当时间趋于无穷大时,  $h$  也是趋于无穷大的.

引入4个分段连续函数:

$$\begin{aligned} \delta_k(t) &= \begin{cases} \delta_{k,h}, & t \in [T_{k,h}^s, T_{k,h}^e), \\ 1, & t \in [T_{k,h}^e, T_{k,h+1}^s), \end{cases} \\ \chi_{u,k}(t) &= \begin{cases} \chi_{u,k,h}, & t \in [T_{k,h}^s, T_{k,h}^e), \\ 1, & t \in [T_{k,h}^e, T_{k,h+1}^s), \end{cases} \\ \eta_k(t) &= \begin{cases} \eta_{k,h}, & t \in [T_{k,h}^s, T_{k,h}^e), \\ 1, & t \in [T_{k,h}^e, T_{k,h+1}^s), \end{cases} \\ \chi_{r,k}(t) &= \begin{cases} \chi_{r,k,h}, & t \in [T_{k,h}^s, T_{k,h}^e), \\ 1, & t \in [T_{k,h}^e, T_{k,h+1}^s). \end{cases} \end{aligned}$$

当第  $k$  个执行器发生式(3) 中的未知故障时, 可以表示为

$$\begin{aligned} \tau_{u,k}(t) &= \delta_k(t) \tau_{u,k}(t) + \chi_{u,k}, \\ \tau_{r,k}(t) &= \eta_k(t) \tau_{r,k}(t) + \chi_{r,k}. \end{aligned} \quad (4)$$

由于输入量少于输出量, 所以船舶是欠驱动的. 正常数  $m_{ij}$  表示整个船舶的惯性,  $d_{ij}, d_{ui}, d_{vi}, d_{ri}$  代表流体阻尼系数, 其中  $2 \leq i \leq 3, 1 \leq j \leq 3$ . 式中所有的常数都是未知的.  $\tau_{wu}(t), \tau_{wv}(t)$  和  $\tau_{wr}(t)$  等时变项是由波浪、风和海流引起的环境扰动.

针对上述问题, 设计2个控制输入变量  $\tau_{u,k}$  和  $\tau_{r,k}$ , 解决了带有未知系统参数和环境干扰情况下的水面船舶路径跟踪控制问题.

**引理1** 对于任何标量  $\varepsilon$  与  $z > 0$ , 都可以使式(5) 成立<sup>[24]</sup>:

$$0 \leq |z| - \frac{z^2}{\sqrt{z^2 + \varepsilon^2}} - \varepsilon. \quad (5)$$

另外, 本文提出如下假设:

**假设1**  $x'_d(s)^2 + y'_d(s)^2 \geq \mu$ , 式中  $\mu$  是一个正常数.

**假设2** 路径的内切圆的最小半径大于或等于船舶的最小转弯半径.

**假设3** 以上未知的参数均在一个已知的紧凸集合中.

**假设 4** 上述所有扰动满足以下条件:

$$|\tau_{uw}(t)| < \tau_{uw\max}, |\tau_{wv}(t)| < \tau_{wv\max}, |\tau_{wr}(t)| < \tau_{wr\max},$$

式中  $\tau_{uw\max}, \tau_{wv\max}, \tau_{wr\max}$  均是未知的正常数.

**假设 5** 在任意时刻,到  $k-1$  个执行器失去作用.  $\delta_k(t) \geq \mu_k > 0, \eta_k(t) \geq \mu_k > 0, \mu_k$  为未知的常数.

**注 1** 假设 1 和假设 2 说明路径  $\Omega$  是规则的,否则可以将路径分割成规则的几段.

## 2 自适应控制器设计

### 2.1 船舶动力学模型转换

为了解决船舶欠驱动的问题,首先进行坐标变换.采用文献[23]中的横截函数法,除了实际的控制输入,还引入了“辅助操纵变量”.

坐标变换如下:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} + R(\varphi) \begin{bmatrix} f_1(\alpha) \\ f_2(\alpha) \end{bmatrix},$$

$$\varphi = \bar{\varphi} - f_3(\alpha), \quad (6)$$

式中的  $R(\varphi) = [\cos(\varphi), -\sin(\varphi), \sin(\varphi), \cos(\varphi)]$  和  $f_l(\alpha), l=1,2,3$  将在后文中给出.对  $x, y$  和  $\varphi$  进行求导可得:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = Q \begin{bmatrix} u \\ \alpha \end{bmatrix} + \begin{bmatrix} -v\sin(\bar{\varphi}) \\ v\cos(\bar{\varphi}) \end{bmatrix} + R'(\varphi) \begin{bmatrix} f_1(\alpha) \\ f_2(\alpha) \end{bmatrix} (r - f_3'(\alpha)\alpha), \quad (7)$$

$$\dot{\varphi} = r - f_3'(\alpha)\alpha, \quad (8)$$

式中  $\alpha$  是一个辅助操纵变量,  $Q = \begin{bmatrix} \cos(\bar{\varphi}) \\ \sin(\bar{\varphi}) \end{bmatrix}, R(\varphi) \begin{bmatrix} f_1'(\alpha) \\ f_2'(\alpha) \end{bmatrix}, f_l' = \frac{\partial f_l(\alpha)}{\partial \alpha}, l=1,2,3,$   $R'(\varphi) = \frac{\partial R(\varphi)}{\partial \varphi}$ . 当  $\bar{\varphi}, \alpha$  是实数时,  $f_l(\alpha)$  要使  $Q$  可逆.

$$f_1(\alpha) = \varepsilon_1 \sin(\alpha) \frac{\sin(f_3)}{f_3},$$

$$f_2(\alpha) = \varepsilon_1 \sin(\alpha) \frac{1 - \cos(f_3)}{f_3},$$

$$f_3(\alpha) = \varepsilon_2 \cos(\alpha), \quad (9)$$

式(9)中,  $0 < \varepsilon_1, 0 < \varepsilon_2 < \frac{\pi}{2}$ , 且

$$|f_1| < \varepsilon_1, \quad |f_2| < \varepsilon_1, \quad |f_3| < \varepsilon_2,$$

$$\det(Q) = \frac{\varepsilon_1 \varepsilon_2}{(\varepsilon_2 \cos(\alpha))} (\cos(\varepsilon_2 \cos(\alpha)) - 1) \leq -\frac{\varepsilon_1}{\varepsilon_2} (1 - \cos(\varepsilon_2)) < 0. \quad (10)$$

### 2.2 控制器设计

系统(2)、(7)和(8)明显是一个严格反馈系统,因此将使用反步法来设计控制器.控制器的设计分为2步<sup>[25]</sup>.第1步,设计虚拟控制信号  $u_d, r_d$  和辅助变量  $\alpha$  使得船舶可以跟随预先设计的路径;第2步,设计实际控制信号  $\tau_u$  和  $\tau_r$  使得  $u$  和  $r$  分别逼近虚拟控制器  $u_d$  和  $r_d$ .

1) 第1步.路径跟踪误差定义如下:

$$x_e = x - x_d, \quad y_e = y - y_d, \quad \varphi_e = \varphi - \varphi_d, \quad (11)$$

式中参考角度  $\varphi_d = \arctan\left(\frac{y'}{x'}\right)$ .选取李雅普诺夫函数如下:

$$V_1 = \frac{1}{2} q_e^T q_e + \frac{1}{2} \varphi_e^2, \quad (12)$$

式中  $q_e = [x_e, y_e]^T$ .对  $V_1$  进行求导可得:

$$\dot{V}_1 = q_e^T \left( Q \begin{bmatrix} u \\ \alpha \end{bmatrix} + \begin{bmatrix} -v\sin(\bar{\varphi}) \\ v\cos(\bar{\varphi}) \end{bmatrix} + R'(\varphi) \begin{bmatrix} f_1(\alpha) \\ f_2(\alpha) \end{bmatrix} \right) \times (r - f_3'(\alpha)\alpha) - \dot{q}_d + \varphi_e (r - f_3'(\alpha)\alpha - \dot{\varphi}_d),$$

上式中,  $q_d = [x_d, y_d]^T, \dot{q}_d = [x_d'(s)s, y_d'(s)s],$

$$\dot{\varphi}_d = \frac{x_d'(s)y_d''(s) - x_d''(s)y_d'(s)}{x_d'(s)^2 + y_d'(s)^2}.$$

$s$  的表达式为

$$s = \frac{u^* (1 - \varepsilon_3 e^{-\varepsilon_4 t}) e^{-\varepsilon_5 \|q - q_d\|}}{\sqrt{x_d'^2 + y_d'^2}}, \quad (13)$$

$u^*$  是一个非零的给定速度.式(13)中,  $\varepsilon_i > 0, i=3, 4, 5$  且  $\varepsilon_3 < 1, q = [x, y]^T$ .

**注 2** 在跟踪问题中,跟踪速率是可以自由选择的,例如式(13)中的  $s$ .参数  $\varepsilon_3$  和  $\varepsilon_4$  保证了船舶开始以一个较低速度运行并且慢慢加速.在实践中这也是合理的,同时它也避免了当  $\|q - q_d\|$  太大时要使用高增益的输出.

在这里引入2个新的误差变量:

$$u_e = u - u_d, \quad r_e = r - r_d, \quad (14)$$

式中,  $u_d$  和  $r_d$  分别是  $u$  和  $r$  的虚拟输入.  $u_d, \tau_d$  与  $\alpha$  的取值如下:

$$\begin{bmatrix} u_d \\ \alpha \end{bmatrix} = Q^{-1} \left( -k_1 q_e - \begin{bmatrix} -v\sin(\bar{\varphi}) \\ v\cos(\bar{\varphi}) \end{bmatrix} - r'(\varphi) \begin{bmatrix} f_1(\alpha) \\ f_2(\alpha) \end{bmatrix} \right) \times (-k_2 \varphi_e + \dot{\varphi}_d) + \dot{q}_d, \quad (15)$$

$$r_d = -k_2 \varphi_e + f_3'(\alpha)\alpha + \dot{\varphi}_d, \quad (16)$$

$k_1$  和  $k_2$  是2个正常数.

根据式(16),可以得到:

$$\dot{V}_1 = -k_1 q_e^T q_e - k_2 \varphi_e^2 + \eta_u u_e + \eta_r r_e,$$

式中

$$\eta_u = q_e^T Q [1, 0]^T,$$

$$\eta_r = q_e^T R'(\varphi) [f_1(\alpha), f_2(\alpha)]^T + \varphi_e.$$

2) 第2步. 对式(14) 进行求导可得:

$$\begin{aligned} \dot{u}_e &= \frac{m_{11}}{m_{22}} vr - \frac{d_{11}}{m_{11}} u - \sum_{i=2}^3 \frac{d_{ui}}{m_{11}} |u|^{i-1} u + \frac{1}{m_{11}} \sum_{k=1}^{n_u} \delta_k(t) \bar{\tau}_{u,k} + \\ &\frac{1}{m_{11}} \bar{\tau}_{uu}(t) - \frac{\partial u_d}{\partial x_e} \dot{x}_e - \frac{\partial u_d}{\partial y_e} \dot{y}_e - \frac{\partial u_d}{\partial s} \dot{s} - \frac{\partial u_d}{\partial \varphi} \dot{\varphi} - \\ &\frac{\partial u_d}{\partial \bar{\varphi}} \dot{\bar{\varphi}} - \frac{\partial u_d}{\partial \alpha} \dot{\alpha} - \frac{\partial u_d}{\partial v} \left( -\frac{m_{11}}{m_{22}} ur - \frac{d_{22}}{m_{22}} v - \right. \\ &\left. \sum_{i=2}^3 \frac{d_{vi}}{m_{22}} |v|^{i-1} v + \frac{1}{m_{22}} \tau_{uv}(t) \right) - \frac{\partial u_d}{\partial x'_d} \ddot{x}_d - \\ &\frac{\partial u_d}{\partial y'_d} \ddot{y}_d - \frac{\partial u_d}{\partial \varphi'_d} \ddot{\varphi}_d, \\ \dot{r}_e &= \frac{m_{11} - m_{22}}{m_{33}} uv - \frac{d_{33}}{m_{33}} r - \sum_{i=2}^3 \frac{d_{ri}}{m_{33}} |r|^{i-1} r + \\ &\frac{1}{m_{33}} \sum_{k=1}^{n_r} \eta_k(t) \bar{\tau}_{r,k} + \frac{1}{m_{33}} \bar{\tau}_{wr}(t) - \frac{\partial r_d}{\partial x_e} \dot{x}_e - \frac{\partial r_d}{\partial y_e} \dot{y}_e - \\ &\frac{\partial r_d}{\partial s} \dot{s} - \frac{\partial r_d}{\partial \varphi} \dot{\varphi} - \frac{\partial r_d}{\partial \bar{\varphi}} \dot{\bar{\varphi}} - \frac{\partial r_d}{\partial \alpha} \dot{\alpha} - \frac{\partial r_d}{\partial \alpha} \left( \frac{\partial \alpha}{\partial x_e} \dot{x}_e + \frac{\partial \alpha}{\partial y_e} \dot{y}_e + \right. \\ &\left. \frac{\partial \alpha}{\partial s} \dot{s} + \frac{\partial \alpha}{\partial \varphi} \dot{\varphi} + \frac{\partial \alpha}{\partial \bar{\varphi}} \dot{\bar{\varphi}} \right) - \frac{\partial r_d}{\partial v} \left( -\frac{m_{11}}{m_{22}} ur - \frac{d_{22}}{m_{22}} v - \right. \\ &\left. \sum_{i=2}^3 \frac{d_{vi}}{m_{22}} |v|^{i-1} v + \frac{1}{m_{22}} \tau_{uv}(t) \right) - \frac{\partial r_d}{\partial x'_d} \ddot{x}_d - \\ &\frac{\partial r_d}{\partial y'_d} \ddot{y}_d - \frac{\partial r_d}{\partial \varphi'_d} \ddot{\varphi}_d, \end{aligned}$$

式中,  $\bar{\tau}_{uu}(t) = \tau_{uu}(t) + \sum_{k=1}^{n_u} \chi_{u,k}$ ,  $\bar{\tau}_{wr}(t) = \tau_{wr}(t) + \sum_{k=1}^{n_r} \chi_{r,k}$ . 根据假设5, 可以得到  $\sum_{k=1}^{n_u} \delta_k(t) > \mu_k > 0$ , 易得  $\inf_{t \geq 0} \sum_{k=1}^{n_u} \delta_k(t) > 0$ . 为了解决未知的执行器故障, 定义以下变量:

$$l_u = \inf_{t > 0} \sum_{k=1}^{n_u} \delta_k(t), \quad p_u = \frac{1}{l_u},$$

$$l_r = \inf_{t > 0} \sum_{k=1}^{n_r} \eta_k(t), \quad p_r = \frac{1}{l_r}. \quad (17)$$

选取李雅普诺夫函数如下:

$$V_2 = V_1 + \frac{m_{11}}{2} u_e^2 + \frac{m_{33}}{2} r_e^2 + \frac{1}{2} \sum_{i=1}^3 \tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i, \quad (18)$$

式中  $\Gamma_i = \text{diag}(\delta_{ij})$  是一个正定矩阵,  $\tilde{\theta}_i$  表示估计误差, 表达式为  $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ ,  $\hat{\theta}_i$  是  $\theta_i$  的估计值, 且

$$\theta_1 = \left[ m_{22}, d_u, d_{u2} \cdot d_{u3}, m_{11}, \frac{m_{11}^2}{m_{22}}, \frac{d_v m_{11}}{m_{22}}, \frac{d_{v2} m_{11}}{m_{22}}, \frac{d_{v3} m_{11}}{m_{22}} \right],$$

$$\varphi_2 = \left[ (m_{11} - m_{22}), d_r, d_{r2}, d_{r3}, m_{33}, \frac{m_{11} m_{33}}{m_{22}}, \frac{d_v m_{33}}{m_{22}}, \frac{d_{v2} m_{33}}{m_{22}}, \frac{d_{v3} m_{33}}{m_{22}} \right],$$

$$\theta_3 = \left[ \tau_{u \max}, \frac{m_{11}}{m_{22}} \tau_{uv \max}, \tau_{uv \max}, \frac{m_{33}}{m_{22}} \tau_{wr \max} \right].$$

当存在阻尼时, 控制器设计如下:

$$\alpha_u = -k_3 u_e - \hat{\theta}_1^T \beta_1 + \eta_u - \hat{\theta}_{31} \tanh\left(\frac{u_e \hat{\theta}_{31}}{\varepsilon_1}\right) -$$

$$\hat{\theta}_{32} \tanh\left(\frac{\partial u_d}{\partial v} \frac{u_e \hat{\theta}_{32}}{\varepsilon_2}\right)$$

$$\alpha_r = -k_4 r_e - \hat{\theta}_2^T \beta_2 + \eta_r - \hat{\theta}_{33} \tanh\left(\frac{r_e \hat{\theta}_{33}}{\varepsilon_3}\right) -$$

$$\hat{\theta}_{34} \tanh\left(\frac{\partial r_d}{\partial v} \frac{r_e \hat{\theta}_{34}}{\varepsilon_4}\right),$$

$$\bar{\tau}_{u,k} = -\frac{u_e \hat{p}_u^2 \alpha_u^2}{\sqrt{u_e^2 \hat{p}_u^2 \alpha_u^2 + \zeta(t)^2}}, \quad k = 1, \dots, n_u,$$

$$\bar{\tau}_{r,m} = -\frac{r_e \hat{p}_r^2 \alpha_r^2}{\sqrt{r_e^2 \hat{p}_r^2 \alpha_r^2 + \zeta(t)^2}}, \quad m = 1, \dots, n_r, \quad (19)$$

式中  $k_3, k_4, \varepsilon_i (1 \leq i \leq 4)$  均为正常数,  $\zeta(t) = v e^{-\kappa t}$ ,  $v$  和  $\kappa$  为正常数, 并且

$$\beta_1 = \left[ vr, -u_d, -|u| u_d, -u^2 u_d, \gamma_1, \frac{\partial u_d}{\partial v} ur, \frac{\partial u_d}{\partial v} v, \frac{\partial u_d}{\partial v} |v| v, \frac{\partial u_d}{\partial v} v^3 \right],$$

$$\beta_2 = \left[ vu, -r_d, -|r| r_d, -r^2 r_d, \gamma_2, \frac{\partial r_d}{\partial v} ur, \frac{\partial r_d}{\partial v} v, \frac{\partial r_d}{\partial v} |v| v, \frac{\partial r_d}{\partial v} v^3 \right],$$

$$\gamma_1 = -\frac{\partial u_d}{\partial x_e} \dot{x}_e - \frac{\partial u_d}{\partial y_e} \dot{y}_e - \frac{\partial u_d}{\partial s} \dot{s} - \frac{\partial u_d}{\partial \varphi} \dot{\varphi} - \frac{\partial u_d}{\partial \bar{\varphi}} \dot{\bar{\varphi}} - \frac{\partial u_d}{\partial \alpha} \dot{\alpha} -$$

$$\frac{\partial u_d}{\partial x'_d} \ddot{x}_d - \frac{\partial u_d}{\partial y'_d} \ddot{y}_d - \frac{\partial u_d}{\partial \varphi'_d} \ddot{\varphi}_d,$$

$$\gamma_2 = -\frac{\partial r_d}{\partial x_e} \dot{x}_e - \frac{\partial r_d}{\partial y_e} \dot{y}_e - \frac{\partial r_d}{\partial s} \dot{s} - \frac{\partial r_d}{\partial \varphi} \dot{\varphi} - \frac{\partial r_d}{\partial \bar{\varphi}} \dot{\bar{\varphi}} - \frac{\partial r_d}{\partial \alpha} \dot{\alpha} -$$

$$\frac{\partial r_d}{\partial \alpha} \left( \frac{\partial \alpha}{\partial x_e} \dot{x}_e + \frac{\partial \alpha}{\partial y_e} \dot{y}_e + \frac{\partial \alpha}{\partial s} \dot{s} + \frac{\partial \alpha}{\partial \varphi} \dot{\varphi} + \frac{\partial \alpha}{\partial \bar{\varphi}} \dot{\bar{\varphi}} \right) - \frac{\partial r_d}{\partial x'_d} \ddot{x}_d -$$

$$\frac{\partial r_d}{\partial y'_d} \ddot{y}_d - \frac{\partial r_d}{\partial \varphi'_d} \ddot{\varphi}_d.$$

自适应律设计如下:

$$\begin{aligned}
\dot{\hat{\theta}}_{1j} &= \delta_{1j} \text{Proj}(u_e \beta_{1j}, \hat{\theta}_{1j}), \quad 1 \leq j \leq 9, \\
\dot{\hat{\theta}}_{2j} &= \delta_{2j} \text{Proj}(r_e \beta_{2j}, \hat{\theta}_{2j}), \quad 1 \leq j \leq 9, \\
\dot{\hat{\theta}}_{31} &= \delta_{31} \text{Proj}(|u_e|, \hat{\theta}_{31}), \\
\dot{\hat{\theta}}_{32} &= \delta_{32} \text{Proj}\left(\left|u_e \frac{\partial u_d}{\partial v}\right|, \hat{\theta}_{32}\right), \\
\dot{\hat{\theta}}_{33} &= \delta_{33} \text{Proj}(|r_e|, \hat{\theta}_{33}), \\
\dot{\hat{\theta}}_{34} &= \delta_{34} \text{Proj}\left(\left|r_e \frac{\partial r_d}{\partial v}\right|, \hat{\theta}_{34}\right), \quad (20)
\end{aligned}$$

式中  $\dot{\hat{\theta}}_{ij}$  表示第  $j$  项  $\dot{\hat{\theta}}_i$ ,  $\delta_{1j}$ ,  $\delta_{2j}$  和  $\delta_{3j}$  均为正常数, 且  $q \leq i \leq 4, 1 \leq j \leq 9$ .  $\beta_{1j}$  和  $\beta_{2j}$  分别是  $\beta_1$  和  $\beta_2$  第  $j$  个元素.

$$\dot{\hat{p}}_u = \gamma_u u_e \alpha_u, \quad \dot{\hat{p}}_r = \gamma_r r_e \alpha_r, \quad (21)$$

式中  $\gamma_u$  和  $\gamma_r$  为正常数. 根据引理 1, 可得:

$$\begin{aligned}
\sum_{k=1}^{n_u} u_e \delta_k \bar{\tau}_{u,k} &= - \sum_{k=1}^{n_u} \delta_k \frac{u_e^2 \hat{p}_u^2 \alpha_u^2}{\sqrt{u_e^2 \hat{p}_u^2 \alpha_u^2 + \zeta(t)^2}} \leq \\
&- \frac{l_u u_e^2 \hat{p}_u^2 \alpha_u^2}{\sqrt{u_e^2 \hat{p}_u^2 \alpha_u^2 + \zeta(t)^2}} \leq l_u \zeta(t) - u_e l_u \hat{p}_u \alpha_u, \quad (22)
\end{aligned}$$

$$\begin{aligned}
\sum_{k=1}^{n_r} r_e \delta_k \bar{\tau}_{r,k} &= - \sum_{k=1}^{n_r} \delta_k \frac{r_e^2 \hat{p}_r^2 \alpha_r^2}{\sqrt{r_e^2 \hat{p}_r^2 \alpha_r^2 + \zeta(t)^2}} \leq \\
&- \frac{l_r r_e^2 \hat{p}_r^2 \alpha_r^2}{\sqrt{r_e^2 \hat{p}_r^2 \alpha_r^2 + \zeta(t)^2}} \leq l_r \zeta(t) - r_e l_r \hat{p}_r \alpha_r. \quad (23)
\end{aligned}$$

Proj 是 Lipschitz 连续投影算法<sup>[26]</sup>, 定义如下:

$$\text{Proj}(a, \hat{b}) = \begin{cases} a, & \mu(\hat{b}) \leq 0, \\ a, & \mu(\hat{b}) \geq 0, \mu(\hat{b})a \leq 0, \\ (1 - \mu(\hat{b}))a, & \mu(\hat{b}) > 0, \mu(\hat{b}) > 0, \end{cases} \quad (24)$$

式(24)中,  $\mu(\hat{b}) = \frac{\hat{b}^2 - b_M^2}{\zeta^2 + 2\zeta b_M}$ ,  $\mu'(\hat{b}) = \frac{\partial \mu(\hat{b})}{\partial \hat{b}}$ ,  $\zeta$  是一个任意小的正常数,  $b_M$  是一个正常数且满足  $|b| < b_M$ . 由上可以得到以下结果:

**引理 2** 如果  $|\hat{b}(t_0)| \leq b_M$ , 则算法具有以下属性:

- 1) 当  $0 \leq t_0 \leq t \leq \infty$  时, 均有  $|\hat{b}(t)| \leq b_M + \varepsilon$ ;
- 2)  $\text{Proj}(a, \hat{b})$  是 Lipschitz 连续的;
- 3)  $|\text{Proj}(a, \hat{b})| \leq |a|$ ;
- 4)  $\hat{b} \text{Proj} \geq \tilde{b} a$ , 其中  $\tilde{b} = b - \hat{b}$ .

详细证明可见文献[25].

### 2.3 稳定性分析

根据引理 1, 联合式(18)—(20)和(24), 可得

$$\dot{V}_2(t) \leq -k_1 q_e^T q_e - k_2 \varphi_e^2 - (k_3 + d_u) u_e^2 -$$

$$(k_4 + d_r) r_e^2 + 0.2785 \sum_{i=1}^4 \varepsilon_i + (l_u + l_r) \zeta(t), \quad (25)$$

化简可得:

$$\dot{V}_2 \leq -\rho_1 V_2 + \rho_2. \quad (26)$$

式(26)中

$$\rho_1 = \min\left(1, 2k_1, 2k_2, \frac{2(k_3 + d_u)}{m_{11}}, \frac{2(k_4 + d_r)}{m_{33}}\right),$$

$$\rho_2 = \frac{1}{2} \sum_{i=1}^3 \tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i + 0.2785 \sum_{i=1}^4 \varepsilon_i + (l_u + l_r) \zeta(t).$$

根据式(26)可得:

$$V_2(t) \leq V_2(t_0) e^{-\rho_1(t-t_0)} + \frac{\rho_2}{\rho_1}, \quad (27)$$

根据式(27), 明显可得  $q_e, \varphi_e, u_e$  和  $r_e$  是有已知的上界的.

根据式(15)和(16),  $u_d$  和  $r_d$  可以改写为

$$u_d = \xi_u + \omega_u v, \quad r_d = \xi_r + \omega_r v, \quad (28)$$

式(28)中

$$\omega_u = [1, 0] Q^{-1} [\sin(\bar{\varphi}), 0]^T, \quad (29)$$

$$\omega_r = f_3^T [0, 1] Q^{-1} [0, -\cos(\bar{\varphi})]^T. \quad (30)$$

然后式(2)中的摆荡速度动力学可以改写为

$$\dot{v} = \tilde{\omega}_1 v + \tilde{\omega}_2 v^2 - \frac{d_{v2}}{m_{22}} |v| v - \frac{d_{v3}}{m_{22}} v^3 + \tilde{\omega}_3, \quad (31)$$

其中

$$\tilde{\omega}_1 = -\frac{m_{11}}{m_{22}} (\xi_u \omega_r + \xi_r \omega_u + r_e \omega_r + u_e \omega_r) - \frac{d_{11}}{m_{11}},$$

$$\tilde{\omega}_2 = -\frac{m_{11}}{m_{22}} \omega_u \omega_r,$$

$$\tilde{\omega}_3 = -\frac{m_{11}}{m_{22}} (\xi_u \xi_r + u_e r_e + r_e \xi_u + u_e \xi_r) + \frac{1}{m_{22}} \tau_{uv}(t),$$

当  $q_e, \varphi_e, u_e$  和  $r_e$  有已知的上界时, 易得  $\tilde{\omega}_1, \tilde{\omega}_2$  和  $\tilde{\omega}_3$  明显也是有上界的. 为了证明  $v$  是有界的, 最后一步的李雅普诺夫函数定义如下:

$$V_3 = \frac{1}{2} v^2. \quad (32)$$

式(32)的导数满足:

$$\dot{V}_3 \leq -\left(\frac{d_{v3}}{m_{22}} - \tilde{\omega}_2^M \eta\right) v^4 + \left(\tilde{\omega}_1^M + \frac{\tilde{\omega}_2^M + \tilde{\omega}_3^M}{4\eta}\right) v^2 + \tilde{\omega}_3^M \eta, \quad (33)$$

$\tilde{\omega}_i^M$  是  $\tilde{\omega}_i$  的上界,  $i = 1, 3$ .  $\tilde{\omega}_2^M$  是  $\tilde{\omega}_2 + \frac{d_{v2}}{n_{22}}$  的上界.  $\eta$  取值

满足  $\frac{d_{v3}}{m_{22}} - \tilde{\omega}_2^M \eta > \eta^* > 0$ ,  $\eta^*$  是一个正常数. 这样就

保证了摆荡速度  $v$  上限是有界的.

基于以上的分析,可以得出以下定理:

**定理 1** 在假设 1—5 前提下,考虑船舶模型、控制器以及参数自适应律,闭环系统中的所有信号都是有界的,船舶可以在误差任意小的情况下追踪上指定路径.

**证明** 根据投影运算,  $\tilde{\theta}_i$  是有界的,  $i = 1, 2, 3$ . 所以从式(27) 可得,  $V_2$  中包含的信号都是有界的. 因此,  $x_e, y_e$  和  $\varphi_e$  都是有界的. 从式(15) 和(16) 易得,  $u_d, r_d$  和  $\alpha$  是有界的, 因此  $u, v$  和  $r$  都是有界的. 根据式(19), 可知  $\tau_u$  和  $\tau_r$  是有界的.

根据式(6) 和式(9) 可得:

$$\| (x - \bar{x}, y - \bar{y}) \| \leq \sqrt{2\zeta_1^2} | \varphi - \bar{\varphi} |, \quad (34)$$

最后从式(6), 可知跟踪误差  $\bar{x} - x_d$  和  $\bar{y} - y_d$  满足:

$$\begin{aligned} | \bar{x} - x_d | &\leq | \bar{x} - x | + | x_d |, \\ | \bar{y} - y_d | &\leq | \bar{y} - y | + | y_d |, \end{aligned} \quad (35)$$

因此跟踪误差是有界的. 另一方面, 由于  $\zeta_1, \zeta_2$  和  $\varepsilon_i$  可以取到任意小的值, 通过调节  $\Gamma_j^{-1}$  的值, 式(26) 中的  $\rho_2$  也可以任意小. 根据式(27),  $x_e$  和  $y_e$  就可以任意小. 因此, 跟踪误差也可以任意小.

### 3 仿真实验

本文在 Windows 平台下利用 Matlab 搭建船舶数字仿真模型. 船舶参数如下:  $m_1 = 120 \times 10^3 \text{ kg}, m_2 = 60 \times 10^4 \text{ kg}, d_{u1} = 20 \times 10^3 \text{ kg}, d_{v1} = 140 \times 10^3 \text{ kg}, d_{r1} = 600 \times 10^3 \text{ kg}, d_{u2} = 0.2d_{u1}, d_{v2} = 0.2d_{v1}, d_{v3} = 0.1d_{v1}, d_{r2} = 0.2d_{r1}, d_{r3} = 0.1d_{r1}$ .

控制器的设计中, 假设所有参数都是未知的. 最大值和最小值设置为高于实际值的 30% 以上. 外部的扰动边界假定为 100. 设计参数如下:  $k_1 = 2k_2 = 3, k_3 = 8, k_4 = 8, \zeta_1 = 0.2, \zeta_2 = 0.2, \zeta_3 = 0.1, \zeta_4 = 4, \zeta_5 = 0.01, u^* = 1.3, \varepsilon_i = 0.5$ , 且  $i = 1, 2, 3, 4$ . 参考轨迹  $q_{od} = [s, 10\sin(0.1s)]^T$ .

初始状态  $\{\bar{x}(0), \bar{y}(0), \bar{\varphi}(0), u(0), v(0), r(0)\} = \{-5, 5, 3, 0, 0, 0\}$ . 参数估计的所有初始值都是它们假设值的 80%. 假设在浪涌方向有 2 个执行器, 并且在偏航方向不存在执行器故障. 当  $t \leq 50$  时,  $\delta_1(t) = 1, \delta_2(t) = 0$ , 当  $50 \leq t \leq 100$  时,  $\delta_1(t) = 0, \delta_2(t) = 1$ . 图 2 和图 3 分别表示船舶的位置和跟踪误差随时间的变化. 图 4 表示参数  $\theta_1, \theta_2$  和  $\theta_3$  的估计值. 图 5 表示浪涌速度、摆动速度和偏航速度.

### 4 结束语

本文研究了在具有未知系统参数及受到波浪、

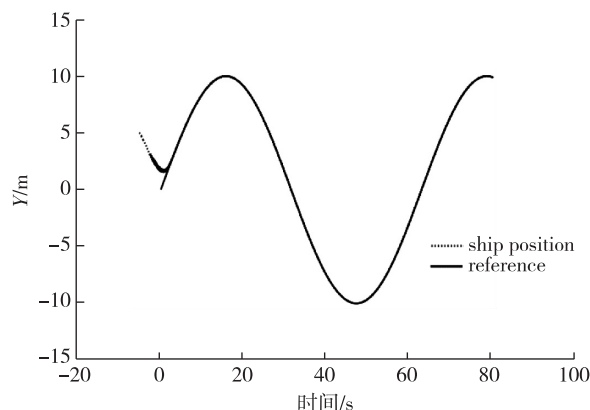


图 2 船舶位置

Fig. 2 Position of the ship in X-Y plane

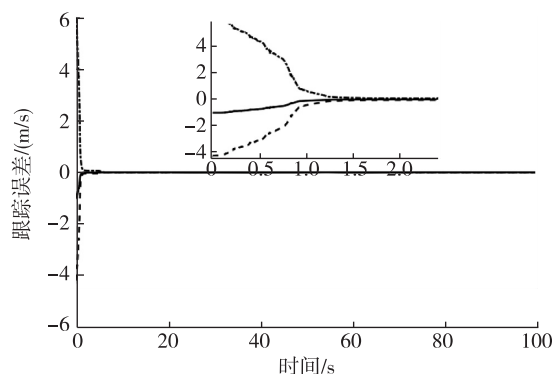


图 3 跟踪误差

Fig. 3 Tracking errors with respect to time in X

风、洋流引起的外部干扰的情况下, 欠驱动船舶的路径跟踪容错控制. 通过使用横截函数的方法, 开发了一个全局稳定的自适应控制器来实现任意小的跟踪误差. 值得注意的是横向函数方法同样可以适用在其他欠驱动的机械系统, 例如非完整的移动机器人、水下车辆以及垂直起降飞机等.

### 参考文献

#### References

- [ 1 ] Brockett R W. Asymptotic stability and feedback stabilization[ M ]. Boston: Birkhauser, 1983: 181-191
- [ 2 ] Pettersen K, Nijmeijer H. Underactuated ship tracking control: Theory and experiments[ J ]. International Journal of Control, 2001, 74( 14 ): 1435-1446
- [ 3 ] Behal A, Dawson D, Xian B, et al. Adaptive tracking control of underactuated surface vessels[ C ] // IEEE International Conference on Control Applications, 2001: 645-650
- [ 4 ] Behal A, Dawson D M, Dixon W E, et al. Tracking and regulation control of an underactuated surface vessel with

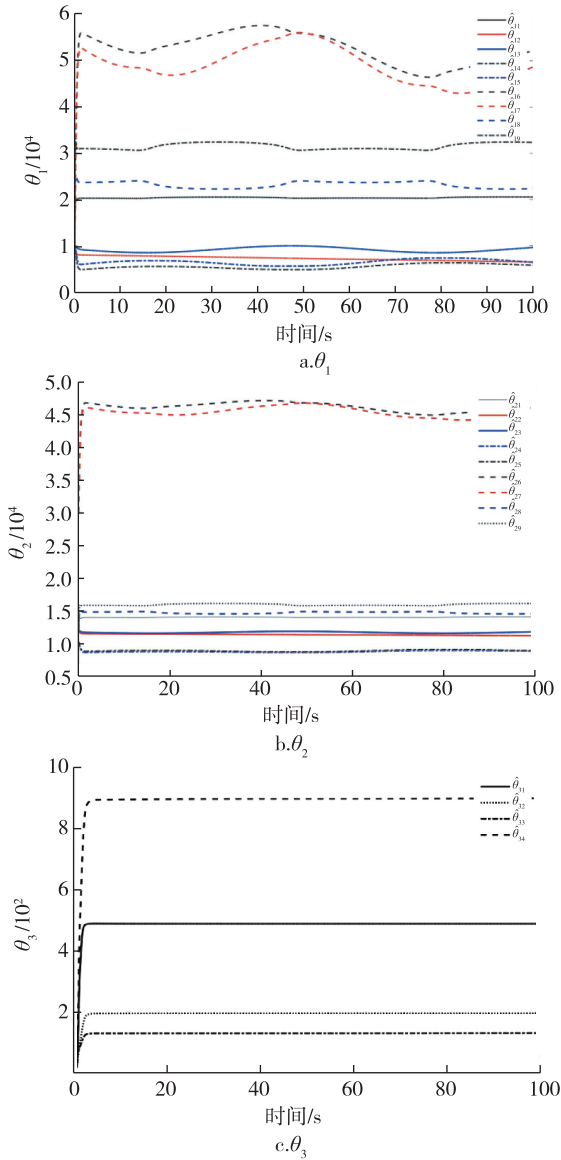


图4 参数估计  
Fig. 4 Parameter estimation

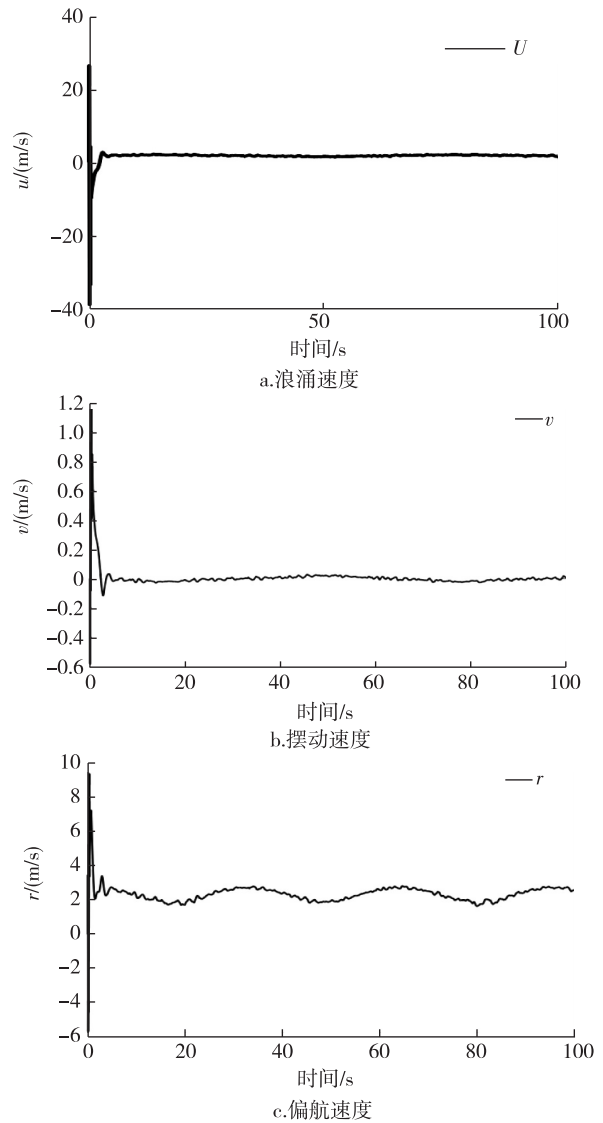


图5 速度变化轨迹  
Fig. 5 Surge velocity, sway velocity and yaw velocity of the ship

nonintegrable dynamics[J].IEEE Transactions on Automatic Control,2002,47(3):495-500

[ 5 ] Lefeber E, Pettersen K, Nijmerjer H. Tracking control of an underactuated ship[J].IEEE Transactions on Control Systems Technology,2003,11(1):52-61

[ 6 ] Morin P, Samson C. Time-varying exponential stabilization of the attitude of a rigid space craft with two controls[J]. IEEE Transactions on Automatic Control, 1997, 42(4): 528-534

[ 7 ] Pettersen K, Egeland O. Time-varying exponential stabilization of the position and attitude of an underactuated autonomous underwater vehicle[J].IEEE Transactions on Automatic Control, 1999, 44(1): 112-115

[ 8 ] Do K D, Pan J. Global robust adaptive path following of underactuated ships [J]. Automatica, 2006, 42(10): 1713-1722

[ 9 ] Do K D, Jiang Z P, Pan J. Global partial-state feedback and output feedback tracking controllers for underactuated ships [J]. Systems Control Letters, 2005, 54(10): 1015-1036

[ 10 ] Jiang Z P. Global tracking control of underactuated ships by Lyapunov's direct method [J]. Automatica, 2002, 38(2): 301-309

[ 11 ] Alessandri A, Caccia M, Veruggio G. Fault detection of actuator faults in unmanned underwater vehicles[J].Control Engineering Practice, 1999, 7(3): 357-368

[ 12 ] Blanke M, Staroswiecki M, Wu N E. Concepts and methods in fault-tolerant control [J]. American Control Conference, 2001, 4: 2606-2620

[ 13 ] Liao F, Wang J L, Yang G H. Reliable robust flight tracking control: An LMI approach [J]. IEEE Transactions on Control System Technology, 2002, 10(1): 76-89

- [14] Chen F Y, Zhang S J, Jiang B, et al. Multiple-model based fault detection and diagnosis for helicopter with actuator faults via quantum information technique [J]. *Journal of Systems and Control Engineering*, 2014, 228(3):182-190
- [15] Chen F Y, Jiang B, Tao G. An intelligent self-repairing control for nonlinear MIMO systems via adaptive sliding mode control technology [J]. *Journal of the Franklin Institute*, 2014, 351(1):399-411
- [16] Veluvolu K C, Defoort M, Soh Y C. High-gain observer with sliding mode for nonlinear state estimation and fault reconstruction [J]. *Journal of the Franklin Institute*, 2014, 351(4):1995-2014
- [17] Patton R J. Fault-tolerant control systems [C] // *Proceedings of the IFAC Symposium on Fault Detection Supervision and Safety for Technical Processes*, 1997: 1033-1054
- [18] Perez T, Donaire A. Constrained control design for dynamic positioning of marine vehicles with control allocation [J]. *Modeling, Identification and Control*, 2009, 30(2):57-70
- [19] Omerdic E, Roberts G. Thruster fault diagnosis and accommodation for open-frame underwater vehicles [J]. *Control Engineering Practice*, 2004, 12(12):1575-1598
- [20] Sun B, Zhu D Q, Sun L Y. A tracking control method with thruster fault tolerant control for unmanned underwater vehicles [C] // *Proceedings of the 25th Chinese Control and Decision Conference*, 2013:4915-4920
- [21] Chen X T, Tan W W. Tracking control of surface vessels via fault-tolerant adaptive backstepping interval type-2 fuzzy control [J]. *Ocean Engineering*, 2013, 70:97-109
- [22] Toussaint G, Basar T, Bullo F. Tracking for nonlinear underactuated surface vessels with generalized forces [C] // *IEEE International Conference on Control Applications*, 2000:355-360
- [23] Morin P, Samson C. Practical stabilization of driftless systems on Lie group [C] // *IEEE Conference on Decision and Control*, 2002:4272-4277
- [24] Wang C L, Wen C Y, Lin Y. Decentralized adaptive backstepping control for a class of interconnected nonlinear systems with unknown actuator failures [J]. *Journal of the Franklin Institute*, 2015, 352(3):835-850
- [25] Krstic M, Kanellakopoulos I, Kokotovic P. *Nonlinear and adaptive control design* [M]. New York: John Wiley & Sons, 1995
- [26] Pomet J, L Praly. Adaptive nonlinear regulation: Estimation from the Lyapunov equation [J]. *IEEE Transactions on Automatic Control*, 1992, 37(6):729-740

## Adaptive fault-tolerant control of underactuated ships with actuator redundancy

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**Abstract** In this paper, a nonlinear adaptive fault-tolerant control strategy is developed to force an underactuated ship with thruster redundancy to follow a predefined path, despite the presence of unknown system parameters and environmental disturbances induced by wave, wind and ocean current. The techniques involved in the design and analysis include the backstepping, parameter projection techniques and a traverse function. It is shown that with our proposed controller, the reference path can be tracked globally with an arbitrarily small tracking error. Simulation results demonstrate the effectiveness of our proposed controller.

**Key words** underactuated ship; actuator failure; redundancy