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# 具有混合延时和不同时标的混沌忆阻竞争神经网络的自适应同步

**摘要**

本文考虑了具有混合时变延时和不同时标的混沌忆阻竞争神经网络的自适应同步问题.使用 Lyapunov 泛函方法和不等式分析技术,设计了一类新的具有反馈控制律的自适应控制器以取得网络同步及指数同步目的,提出了不用过多计算,如求解线性矩阵不等式或复杂代数计算的保证网络同步条件;同时,所获条件也可以应用到已有文献里关于忆阻器网络不同数学模型中.最后,通过实例验证了本文获得的理论结果的有效和正确性.

**关键词**

自适应同步;忆阻器;竞争神经网络;时间延时;时标

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## 0　引言

1983 年,在文献[1]中,Cohen 和 Grossberg 为模拟神经生物学中的细胞抑制现象提出了竞争神经网络模型.随后,Meyer-Bäse 等[2]提出了具有不同时标的竞争神经网络,该网络不仅模型了神经激励层的动态行为——短时记忆,而且也模型了神经突触变化的动力学行为——长时记忆,同时,该网络系统的状态以两个不同时标在进行变化,一个与神经网络状态的快速变化有关,另一个与外部刺激下突触的缓慢变化有关.在文献[3]中,Meyer-Bäse 等进一步研究了具有不同时标的竞争神经网络的全局指数稳定性.

为了保证相应的信息存储,需要设计大型有效的神经网络.随着维数的增加,这将会占用大量的计算机内存和硬盘空间.2008 年,惠普实验室研究人员成功研制出了一种纳米级电子设备,称之为忆阻器[4-5]. 根据数学关系,忆阻器是一个非线性时变原件,它的值(即忆导值)依赖于先前通过的电流值,因而该设备拥有记忆能力,这与神经系统中的突触具有相似性.基于此特性,忆阻器已被应用于纳米记忆、计算机逻辑等领域[6-7].

使用忆阻器代替传统神经网络中的电阻,能够设计忆阻神经网络[8-9].现有大量文献讨论了忆阻网络的同步问题.在文献[10]中,通过周期性的间断控制,得到一些新的确保基于忆阻的混沌神经网络的指数同步的充分性代数条件;文献[11]则利用广义的 Halanay 不等式和 Lyapunov-Krasovskii 泛函方法,提出了耦合忆阻神经网络弱的、修正的和泛函投影同步条件;基于极值分析理论,文献[12]证明了具有延时的忆阻神经网络的周期解的存在性.

另一方面,具有不同时标的竞争神经网络的同步问题也得到了研究.基于设计的反馈控制器,文献[13]提出了代数和线性矩阵不等式形式的同步条件;文献[14]针对具有混合时滞和不确定混合扰动的竞争神经网络,设计了一种简单鲁棒自适应控制器,该控制器具有较好的抗干扰能力.

在本文中,我们研究具有混合时变延时和不同时标的忆阻神经网络的同步问题.利用 Lyapunov 泛函方法与不等式分析技术,通过设计两个新的简单有效的自适应控制器,给出了完全同步与指数同步

的条件.所设计的自适应控制器能够适用于其他具有不同数学模型的忆阻神经网络.与文献[15-21]的结果相比较,本文建立的同步条件的优点是不需求解线性矩阵不等式或计算代数方程等过多的复杂计算.

## 1 模型描述及预备

考虑具有混合延时和不同时标的忆阻神经网络:

$$\begin{cases}\varepsilon\dot{x}_i(t)=-d_i(x_i(t))x_i(t)+\sum\limits_{j=1}^{n}a_{ij}(x_j(t))f_j(x_j(t))+\\ \quad\sum\limits_{j=1}^{n}b_{ij}(x_j(t-\tau(t)))f_j(x_i(t-\tau(t)))+\\ \quad\sum\limits_{j=1}^{n}c_{ij}(x_j(t))\int_{t-\mu(t)}^{t}f_j(x_j(s))\mathrm{d}s+\\ \quad H_i\sum\limits_{l=1}^{p}m_{il}(t)F_l,\\ \dot{m}_{il}(t)=-\alpha_i m_{il}(t)+\beta_i F_l f_j(x_j(t)),\\ \quad i=1,2,\cdots,n,\quad l=1,2,\cdots,p.\end{cases}\tag{1}$$

在上述方程组中,第一个方程表示短时记忆,第二个方程表示长时记忆,$x_i(t)$ 表示第 $i$ 个神经元的状态,$f_j(x_j(t))$ 是第 $j$ 个神经元输出,$m_{il}$ 表示突触效率,$F_l$ 表示恒定外部刺激,$H_i$ 表示外部刺激强度,$\alpha_i$ 和 $\beta_i$ 是任意常数且 $\alpha_i>0$,$\varepsilon$ 是短时记忆状态的时标,$\tau(t)$ 和 $\mu(t)$ 分别表示离散与分布延时,$d_i(\cdot)>0$,$a_{ij}(\cdot)$,$b_{ij}(\cdot)$,$c_{ij}(\cdot)$ 是忆阻器连接权重,且满足:

$$d_i(x_i(t))=\begin{cases}d_i',\ |x_i(t)|\leqslant\chi_i;\\ d_i'',\ |x_i(t)|>\chi_i;\end{cases}\tag{2}$$

$$a_{ij}(x_j(t))=\begin{cases}a_{ij}',\ |x_j(t)|\leqslant\chi_j;\\ a_{ij}'',\ |x_j(t)|>\chi_j;\end{cases}\tag{3}$$

$$b_{ij}(x_j(t-\tau(t)))=\begin{cases}b_{ij}',\ |x_j(t-\tau(t))|\leqslant\chi_j;\\ b_{ij}'',\ |x_j(t-\tau(t))|>\chi_j;\end{cases}\tag{4}$$

$$c_{ij}(x_j(t))=\begin{cases}c_{ij}',\ |x_j(t)|\leqslant\chi_j;\\ c_{ij}'',\ |x_j(t)|>\chi_j.\end{cases}\tag{5}$$

在这里,$\chi_i>0$ 表示转换跳跃,$d_i'$,$d_i''$,$a_{ij}'$,$a_{ij}''$,$b_{ij}'$,$b_{ij}''$,$c_{ij}'$ 和 $c_{ij}''$ 是常数.记 $\hat{d}_i=\min\{d_i',d_i''\}$,$\tilde{a}_{ij}=\max\{|a_{ij}'|,|a_{ij}''|\}$,$\tilde{b}_{ij}=\max\{|b_{ij}'|,|b_{ij}''|\}$,$\tilde{c}_{ij}=\max\{|c_{ij}'|,|c_{ij}''|\}$.

令 $r_i(t)=\sum\limits_{l=1}^{p}m_{il}(t)F_l$,式(1)可转化为

$$\begin{cases}\varepsilon\dot{x}_i(t)=-d_i(x_i(t))x_i(t)+\sum\limits_{j=1}^{n}a_{ij}(x_j(t))f_j(x_j(t))+\\ \quad\sum\limits_{j=1}^{n}b_{ij}(x_j(t-\tau(t)))f_j(x_i(t-\tau(t)))+\\ \quad\sum\limits_{j=1}^{n}c_{ij}(x_j(t))\int_{t-\mu(t)}^{t}f_j(x_j(s))\mathrm{d}s+H_ir_i,\\ \dot{r}_i(t)=-\alpha_ir_i(t)+\beta_i|F|^2f_i(x_i(t)),\\ \quad i=1,2,\cdots,n.\end{cases}\tag{6}$$

其中 $|F|^2=F_1^2+F_2^2+\cdots+F_p^2$ 是常数.不失一般性,假设 $F$ 满足 $|F|^2=1$,则系统(6)可简化为

$$\begin{cases}\varepsilon\dot{x}_i(t)=-d_i(x_i(t))x_i(t)+\sum\limits_{j=1}^{n}a_{ij}(x_j(t))f_j(x_j(t))+\\ \quad\sum\limits_{j=1}^{n}b_{ij}(x_j(t-\tau(t)))f_j(x_i(t-\tau(t)))+\\ \quad\sum\limits_{j=1}^{n}c_{ij}(x_j(t))\int_{t-\mu(t)}^{t}f_j(x_j(s))\mathrm{d}s+H_ir_i,\\ \dot{r}_i(t)=-\alpha_ir_i(t)+\beta_if_i(x_i(t)),\\ \quad i=1,2,\cdots,n.\end{cases}\tag{7}$$

为了得到本文的结果,作如下假设:

(A1)延时 $\tau(t)$ 和 $\mu(t)$ 是可微的,且存在常数 $\tau_M$,$\tau_D$,$\mu_M$,$\mu_D$ 满足:

$0<\tau(t)<\tau_M$, $\dot{\tau}(t)\leqslant\tau_D<1$;

$0<\mu(t)<\mu_M\leqslant\tau_M$, $\dot{\mu}(t)\leqslant\mu_D<1$.

(A2)激励函数 $f_j(\cdot)$ 在 $\mathbf{R}$ 上是全局 Lipschitz 连续的,即存在常数 $l_j>0$,使得:

$|f_j(\nu)-f_j(\vartheta)|\leqslant l_j|\nu-\vartheta|$,

$\forall\nu,\vartheta\in\mathbf{R}$, $j=1,2,\cdots,n$.

(A3)激励函数 $f_j(\cdot)$ 是有界的,即存在常数 $m_j>0$,使得:

$|f_j(\vartheta)|\leqslant m_j$, $j=1,2,\cdots,n$.

(A4)激励函数 $f_j(\cdot)$ 满足 $f_j(\pm\chi_j)=0$,$j=1,2,\cdots,n$.

考虑到系统(7)中短时记忆的右端是不连续的,对系统(7)的解,连续情况下微分方程解的定义已不再适合.下面,我们引进系统(7)的 Fillipov 形式解的定义.

**定义1** 函数 $[x(t),r(t)]:[-\tau_M,T)\to\mathbf{R}^n\times\mathbf{R}^n$,$T\in(0,+\infty]$ 是(1)在 $[-\tau_M,T)$ 上的解,如果:

1)$[x(t),r(t)]$ 在 $[-\tau_M,T)$ 是连续的,且在 $[0,T)$ 上是绝对连续的;

2)存在 $\gamma_i(t)\in\mathrm{co}[d_i(x_j(t))]$,$\delta_{ij}(t)\in\mathrm{co}[a_{ij}(x_j(t))]$,$\theta_{ij}(t)\in\mathrm{co}[b_{ij}(x_j(t-\tau(t)))]$ 和 $\lambda_{ij}(t)\in\mathrm{co}[c_{ij}(x_j(t))]$,使得

$$\begin{cases}\varepsilon\dot{x}_i(t)=-\gamma_i(t)x_i(t)+\sum\limits_{j=1}^{n}\delta_{ij}(t)f_j(x_j(t))+\\ \quad\sum\limits_{j=1}^{n}\theta_{ij}(t)f_j(x_i(t-\tau(t)))_i+\\ \quad\sum\limits_{j=1}^{n}\lambda_{ij}(t)\int_{t-\mu(t)}^{t}f_j(x_j(s))\mathrm{d}s+H_ir,\\ \dot{r}_i(t)=-\alpha_ir_i(t)+\beta_if_i(x_i(t)),\quad i=1,2,\cdots,n.\end{cases}\tag{8}$$

其中 $[x_i(t),r_i(t)]$ 是 $[x(t),r(t)]$ 的第 $i$ 个元素, $\mathrm{co}[\cdot]$ 是凸算子.

基于驱动-响应同步概念,将(8)作为驱动系统,并设计如下响应系统:

$$\begin{cases}\varepsilon\dot{y}_i(t)=-\bar{\gamma}_i(t)y_i(t)+\sum\limits_{j=1}^{n}\bar{\delta}_{ij}(t)f_j(y_j(t))+\\ \quad\sum\limits_{j=1}^{n}\bar{\theta}_{ij}(t)f_j(y_i(t-\tau(t)))+\\ \quad\sum\limits_{j=1}^{n}\bar{\lambda}_{ij}(t)\int_{t-\mu(t)}^{t}f_j(y_j(s))\mathrm{d}s+\\ \quad H_is_i+u_i(t),\\ \dot{s}_i(t)=-\alpha_is_i(t)+\beta_if_i(y_i(t)),\quad i=1,2,\cdots,n.\end{cases}\tag{9}$$

在这里, $\bar{\gamma}_i(t)\in\mathrm{co}[d_i(y_j(t))]$, $\bar{\delta}_{ij}(t)\in\mathrm{co}[a_{ij}(y_j(t))]$, $\bar{\theta}_{ij}(t)\in\mathrm{co}[b_{ij}(y_j(t-\tau(t)))]$ 和 $\bar{\lambda}_{ij}(t)\in\mathrm{co}[c_{ij}(y_j(t))]$, $u(t)=[u_1(t),u_2(t),\cdots,u_n(t)]^{\mathrm{T}}$ 表示控制输入向量.

本文的主要目的是通过设计适当的自适应控制器 $u(t)$,来实现系统 (8) 和(9) 的渐近同步与指数同步.为此,下面我们引进一些需要的定义和引理.

**定义 2** 如果对于任意给定的初始条件,误差纠错系统 $e(t)=y(t)-x(t)$ 和 $z(t)=s(t)-r(t)$ 分别满足:

$$\lim_{t\to\infty}\|e(t)\|^2=0,\quad \lim_{t\to\infty}\|z(t)\|^2=0,$$

在这里, $\|\cdot\|$ 表示 $\mathbf{R}^n$ 上的欧式范数,则称系统(8)和(9) 是全局渐近同步的.

**定义 3** 如果对于任意给定的初始条件,存在 $\tilde{\omega}>0$ 和 $M>0$,使得:

$$\|e(t)\|^2+\|z(t)\|^2\leqslant M\mathrm{e}^{\tilde{\omega}t},$$

则称系统(8) 和(9) 是全局指数同步的.

**定义 4** 对于任意连续函数 $f:\mathbf{R}\to\mathbf{R}$, $f(t)$ 的右上 Dini 导数定义为

$$D^+f(t)=\lim_{\theta\to0}\sup\frac{f(t+\theta)-f(t)}{\theta}.$$

**引理 1** 对于任意向量 $x,y\in\mathbf{R}^n(n\geqslant1)$ 和一个正数 $a$,下面矩阵不等式成立:

$$2x^{\mathrm{T}}y\leqslant ax^{\mathrm{T}}x+\frac{1}{a}y^{\mathrm{T}}y.$$

## 2 主要结果

**定理 1** 假设(A1)—(A3)成立,那么在控制器

$$u_i=-\xi_ie_i(t)-\eta_i\mathrm{sign}(e_i(t))-\varphi_ix_i(t)\mathrm{sign}(e_i(t)x_i(t))\tag{10}$$

的作用下,系统(8)和(9)是全局渐近同步的,其中

$$\begin{cases}\dot{\xi}_i=p_ie_i(t)^2,\\ \dot{\eta}_i=q_i|e_i(t)|,\\ \dot{\varphi}_i=\psi_i|e_i(t)x_i(t)|,\end{cases}\tag{11}$$

常数 $p_i>0$, $q_i>0$ 和 $\psi_i>0$, $i=1,2,\cdots,n$.

**证明** 式(9) 减去式(8),可以得到如下纠错系统:

$$\begin{cases}\varepsilon\dot{e}_i(t)=-\bar{\gamma}_i(t)e_i(t)-[\bar{\gamma}_i(t)-\gamma_i(t)]x_i(t)+\\ \quad\sum\limits_{j=1}^{n}\bar{\delta}_{ij}(t)g_j(e_j(t))+H_iz_i+u_i(t)+\\ \quad\sum\limits_{j=1}^{n}[\bar{\delta}_{ij}(t)-\delta_{ij}(t)]f_j(x_j(t))+\\ \quad\sum\limits_{j=1}^{n}\bar{\theta}_{ij}(t)g_j(e_j(t-\tau(t)))+\\ \quad\sum\limits_{j=1}^{n}[\bar{\theta}_{ij}(t)-\theta_{ij}(t)]f_j(x_j(t-\tau(t)))+\\ \quad\sum\limits_{j=1}^{n}\int_{t-\mu(t)}^{t}\bar{\lambda}_{ij}(t)g_j(e_j(s))\mathrm{d}s+\\ \quad\sum\limits_{j=1}^{n}\int_{t-\mu(t)}^{t}[\bar{\lambda}_{ij}(t)-\lambda_{ij}(t)]f_j(x_j(s))\mathrm{d}s,\\ \dot{z}_i(t)=-\alpha_iz_i(t)+\beta_ig_i(e_i(t)),\\ \quad i=1,2,\cdots,n.\end{cases}\tag{12}$$

其中, $g_j(e_j(t))=f_j(y_j(t))-f_j(x_j(t))$, $g_j(e_j(t-\tau(t)))=f_j(y_j(t-\tau(t)))-f_j(x_j(t-\tau(t)))$.

考虑下面的 Lyapunov 泛函:

$$V(t)=V_1(t)+V_2(t)+V_3(t)+V_4(t),\tag{13}$$

其中

$$V_1(t)=\sum_{i=1}^{n}[e_i(t)^2+z_i(t)^2],$$

$$V_2(t)=\frac{1}{1-\tau_D}\sum_{i=1}^{n}\int_{t-\tau(t)}^{t}Q_ie_i(s)^2\mathrm{d}s,$$

$$V_3(t)=\frac{1}{1-\mu_D}\sum_{i=1}^{n}\int_{-\mu(t)}^{0}\int_{t+\nu}^{t}R_ie_i(s)^2\mathrm{d}s\mathrm{d}\nu,$$

$$V_4(t)=\frac{1}{\varepsilon}\sum_{i=1}^{n}\left[\frac{1}{p_i}(\xi_i(t)-k_i)^2+\frac{1}{q_i}(\eta_i(t)-w_i)^2+\frac{1}{\psi_i}(\varphi_i(t)-v_i)^2\right],$$

$Q_i>0, R_i>0, k_i, w_i, v_i, i=1,2,\cdots,n$,是待定的.

沿着系统(12)的解计算 $V(t)$ 的右上 Dini 导数,可得:

$$\begin{aligned}D^+V_1(t)=&2\sum_{i=1}^{n}(e_i(t)\dot{e}_i(t)+z_i(t)\dot{z}_i(t))=\\&\frac{2}{\varepsilon}\sum_{i=1}^{n}\Big\{-\bar{\gamma}_i(t)e_i(t)^2-[\bar{\gamma}_i(t)-\gamma_i(t)]e_i(t)x_i(t)+\\&\sum_{j=1}^{n}e_i(t)\bar{\delta}_{ij}(t)g_j(e_j(t))+e_i(t)H_iz_i(t)+\\&\sum_{j=1}^{n}e_i(t)[\bar{\delta}_{ij}(t)-\delta_{ij}(t)]f_j(x_j(t))+\\&\sum_{j=1}^{n}e_i(t)\bar{\theta}_{ij}(t)g_j(e_j(t-\tau(t)))+\\&\sum_{j=1}^{n}e_i(t)[\bar{\theta}_{ij}(t)-\theta_{ij}(t)]f_j(x_j(t-\tau(t)))+\\&\sum_{j=1}^{n}\int_{t-\mu(t)}^{t}e_i(t)\bar{\lambda}_{ij}(t)g_j(e_j(\zeta))\mathrm{d}\zeta+\\&\sum_{j=1}^{n}\int_{t-\mu(t)}^{t}e_i(t)[\bar{\lambda}_{ij}(t)-\lambda_{ij}(t)]f_j(x_j(\zeta))\mathrm{d}\zeta-\\&\xi_ie_i(t)^2-\eta_i|e_i(t)|-\phi_i|e_i(t)x_i(t)|\Big\}+\\&2\sum_{i=1}^{n}\{-\alpha_iz_i(t)^2+\beta_iz_i(t)g_i(e_i(t))\},\end{aligned}\tag{14}$$

$$D^+V_2(t)\leqslant\sum_{i=1}^{n}\frac{Q_i}{1-\tau_D}e_i(t)^2-\sum_{i=1}^{n}Q_ie_i(t-\tau(t))^2,\tag{15}$$

$$D^+V_3(t)\leqslant\sum_{i=1}^{n}\frac{\mu_MR_i}{1-\mu_D}e_i(t)^2-\sum_{i=1}^{n}\int_{t-\mu(t)}^{t}R_ie_i(s)^2\mathrm{d}s,\tag{16}$$

$$\begin{aligned}D^+V_4(t)=&\frac{2}{\varepsilon}\sum_{i=1}^{n}[(\xi_i(t)-k_i)e_i(t)^2+(\eta_i(t)-w_i)|e_i(t)|+\\&(\varphi_i(t)-v_i)|e_i(t)x_i(t)|].\end{aligned}\tag{17}$$

应用式(2)—(5)和假设A3,我们有

$$\sum_{i=1}^{n}\sum_{j=1}^{n}e_i(t)[\bar{\delta}_{ij}(t)-\delta_{ij}(t)]f_j(x_j(t))\leqslant\sum_{i=1}^{n}\Big(\sum_{j=1}^{n}|a'_{ij}-a''_{ij}|m_j\Big)|e_i(t)|,\tag{18}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}e_i(t)[\bar{\theta}_{ij}(t)-\theta_{ij}(t)]f_j(x_j(t-\tau(t)))\leqslant\sum_{i=1}^{n}\Big(\sum_{j=1}^{n}|b'_{ij}-b''_{ij}|m_j\Big)|e_i(t)|,\tag{19}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\int_{t-\mu(t)}^{t}e_i(t)[\bar{\lambda}_{ij}(t)-\lambda_{ij}(t)]f_j(x_j(s))\mathrm{d}s\leqslant\sum_{i=1}^{n}\Big(\sum_{j=1}^{n}\mu_M|c'_{ij}-c''_{ij}|m_j\Big)|e_i(t)|.\tag{20}$$

根据引理1,可得

$$\sum_{i=1}^{n}\sum_{j=1}^{n}e_i(t)\bar{\delta}_{ij}(t)g_j(e_j(t))\leqslant\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{\tilde{a}_{ij}^2e_i(t)^2}{2}+\frac{l_j^2e_j(t)^2}{2}\right)=\sum_{i=1}^{n}\left(\sum_{j=1}^{n}\frac{\tilde{a}_{ij}^2+l_i^2}{2}\right)e_i(t)^2,\tag{21}$$

$$\begin{aligned}&\sum_{i=1}^{n}\sum_{j=1}^{n}e_i(t)\bar{\theta}_{ij}(t)g_j(e_j(t-\tau(t)))\leqslant\\&\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{\tilde{b}_{ij}^2e_i(t)^2}{2}+\frac{l_j^2e_j(t-\tau(t))^2}{2}\right)=\\&\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\tilde{b}_{ij}^2}{2}e_i(t)^2+\sum_{i=1}^{n}\frac{nl_i^2}{2}e_i(t-\tau(t))^2,\end{aligned}\tag{22}$$

$$\begin{aligned}&\sum_{i=1}^{n}\sum_{j=1}^{n}\int_{t-\mu(t)}^{t}e_i(t)\bar{\lambda}_{ij}(t)g_j(e_j(\zeta))\mathrm{d}\zeta\leqslant\\&\sum_{i=1}^{n}\sum_{j=1}^{n}\int_{t-\mu(t)}^{t}\left(\frac{\tilde{c}_{ij}^2e_i(t)^2}{2}+\frac{l_j^2e_j(\zeta)^2}{2}\right)\mathrm{d}\zeta\leqslant\\&\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\mu_M\tilde{c}_{ij}^2}{2}e_i(t)^2+\sum_{i=1}^{n}\int_{t-\mu(t)}^{t}\frac{nl_i^2}{2}e_i(\zeta)^2\mathrm{d}\zeta.\end{aligned}\tag{23}$$

$$e_i(t)H_iz_i(t)\leqslant\frac{\hat{u}_i}{2}z_i(t)^2+\frac{H_i^2}{2\hat{u}_i}e_i(t)^2,\tag{24}$$

$$z_i(t)\beta_ig(e_i(t))\leqslant\frac{y'_i}{2}z_i(t)^2+\frac{\beta_i^2l_i^2}{2y'_i}e_i(t)^2,\tag{25}$$

其中 $\hat{u}_i$ 和 $y'_i$ 是任意正数,$i=1,2,\cdots,n$.

结合式(14)—(25),可以得到:

$$\begin{aligned}D^+V(t)\leqslant&\sum_{i=1}^{n}\Big\{-\frac{2}{\varepsilon}d_i+\\&\sum_{j=1}^{n}\frac{\tilde{a}_{ij}^2+\tilde{b}_{ij}^2+\mu_M\tilde{c}_{ij}^2+l_i^2}{\varepsilon}+\frac{H_i^2}{\varepsilon\hat{u}_i}+\frac{\beta_i^2l_i^2}{y'_i}+\frac{Q_i}{1-\tau_D}+\\&\frac{\mu_MR_i}{1-\mu_D}-\frac{2k_i}{\varepsilon}\Big\}e_i(t)^2+\sum_{i=1}^{n}\Big\{\sum_{j=1}^{n}(|a'_{ij}-a''_{ij}|+\\&|b'_{ij}-b''_{ij}|+\mu_M|c'_{ij}-c''_{ij}|)\frac{2m_i}{\varepsilon}-\frac{2w_i}{\varepsilon}\Big\}|e_i(t)|+\\&\sum_{i=1}^{n}\left\{\frac{2}{\varepsilon}|d'_i-d''_i|-\frac{2}{\varepsilon}v_i\right\}|e_i(t)x_i(t)|+\\&\sum_{i=1}^{n}\left(\frac{\hat{u}_i}{\varepsilon}+y'_i-2\alpha_i\right)z_i(t)^2+\sum_{i=1}^{n}\left(\frac{nl_i^2}{\varepsilon}-Q_i\right)e_i(t-\tau(t))^2+\\&\sum_{i=1}^{n}\int_{t-\mu(t)}^{t}\left(\frac{nl_i^2}{\varepsilon}-R_i\right)e_i(\zeta)^2\mathrm{d}\zeta.\end{aligned}$$

令

$$\frac{\hat{u}_i}{\varepsilon}+y'_i=\alpha_i,\quad Q_i=\frac{nl_i^2}{\varepsilon},\quad R_i=\frac{nl_i^2}{\varepsilon},$$

$$k_i=\frac{\varepsilon}{2}\Big\{-\frac{2}{\varepsilon}d_i+\sum_{j=1}^{n}\frac{\tilde a_{ij}^2+\tilde b_{ij}^2+\mu_M\tilde c_{ij}^2+l_i^2}{\varepsilon}+\frac{H_i^2}{\varepsilon\hat u_i}+\frac{\beta_i^2l_i^2}{y_i'}+\frac{Q_i}{1-\tau_D}+\frac{\mu_MR_i}{1-\mu_D}+\alpha_i\Big\},$$

$$w_i=m_i(|a_{ij}'-a_{ij}''|+|b_{ij}'-b_{ij}''|+\mu_M|c_{ij}'-c_{ij}''|),$$

$$v_i=|d_i'-d''_i|,\quad i=1,2,\cdots,n,$$

则

$$D^+V(t)\leqslant-\min_{1\leqslant i\leqslant n}\alpha_i(\|e(t)\|^2+\|z(t)\|^2).\quad(26)$$

显然,式(26)成立当且仅当 $\|e(t)\|^2+\|z(t)\|^2=0$, i.e., $\|e(t)\|^2=0$, $\|z(t)\|^2=0$.因此,由 Lyapunov 稳定性定理可得:

$$\lim_{t\to\infty}\|e(t)\|^2=0,\quad \lim_{t\to\infty}\|z(t)\|^2=0.$$

根据定义 2,在具有自适应律(11)的控制器(10)的作用下,系统(8)和(9)是全局渐近同步的.证毕.

**定理 2** 假设(A1)—(A3)成立,则在控制器

$$u_i=-\xi_ie_i(t)-\eta_i\operatorname{sign}(e_i(t))-\varphi_ix_i(t)\operatorname{sign}(e_i(t)x_i(t))\quad(27)$$

的作用下,对于给定的指数常数 $0<\tilde\omega<2\min_{1\leqslant i\leqslant n}\alpha_i$,系统(8)和(9)是全局指数同步的.

其中

$$\begin{cases}\dot\xi_i=p_ie_i(t)^2e^{\tilde\omega t},\\ \dot\eta_i=q_i|e_i(t)|e^{\tilde\omega t},\\ \dot\varphi_i=\psi_i|e_i(t)x_i(t)|e^{\tilde\omega t},\end{cases}\quad(28)$$

$p_i>0, q_i>0, \psi_i>0, i=1,2,\cdots,n.$

**证明** 对纠错系统(12),考虑 Lyapunov 泛函:

$$V(t)=V_1(t)+V_2(t)+V_3(t)+V_4(t),\quad(29)$$

其中

$$V_1(t)=e^{\tilde\omega t}\sum_{i=1}^{n}[e_i(t)^2+z_i(t)^2],$$

$$V_2(t)=\frac{e^{\tilde\omega\tau_M}}{1-\tau_D}\sum_{i=1}^{n}\int_{t-\tau(t)}^{t}Q_ie_i(\zeta)^2e^{\tilde\omega\zeta}d\zeta,$$

$$V_3(t)=\sum_{i=1}^{n}\int_{-\mu_M}^{0}\int_{t+\nu}^{t}R_ie_i(\zeta)^2e^{\tilde\omega(\zeta-\nu)}d\zeta d\nu,$$

$$V_4(t)=\frac{1}{\varepsilon}\sum_{i=1}^{n}\Big[\frac{1}{p_i}(\xi_i(t)-k_i)^2+\frac{1}{q_i}(\eta_i(t)-w_i)^2+\frac{1}{\psi_i}(\varphi_i(t)-v_i)^2\Big].$$

显然,

$$V(t)\geqslant e^{\tilde\omega t}(\|e(t)\|^2+\|z(t)\|^2).\quad(30)$$

与定理 1 的证明方法类似,沿着轨迹(12)的解求 $V(t)$ 的右上 Dini 导数,可得:

$$D^+V_1(t)=e^{\tilde\omega t}\sum_{i=1}^{n}\{\tilde\omega[e_i(t)^2+z_i(t)^2]+2[e_i(t)\dot e_i(t)+z_i(t)\dot z_i(t)]\},\quad(31)$$

$$D^+V_2(t)=\frac{e^{\tilde\omega\tau_M}}{1-\tau_D}\sum_{i=1}^{n}[Q_ie_i(t)^2e^{\tilde\omega t}-(1-\dot\tau(t))Q_ie_i(t-\tau(t))^2e^{\tilde\omega(t-\tau(t))}]\leqslant e^{\tilde\omega t}\sum_{i=1}^{n}\Big[\frac{e^{\tilde\omega\tau_M}Q_i}{1-\tau_D}e_i(t)^2-Q_ie_i(t-\tau(t))^2\Big],\quad(32)$$

$$D^+V_3(t)=\sum_{i=1}^{n}\int_{-\mu_M}^{0}[R_ie_i(t)^2e^{\tilde\omega(t-\nu)}-R_ie_i(t+v)^2e^{\tilde\omega t}]dv=e^{\tilde\omega t}\sum_{i=1}^{n}\Big[\frac{e^{\tilde\omega\mu_M}-1}{\tilde\omega}R_ie_i(t)^2-\int_{t-\mu_M}^{t}R_ie_i(s)^2ds\Big]\leqslant e^{\tilde\omega t}\sum_{i=1}^{n}\Big[\frac{e^{\tilde\omega\mu_M}-1}{\tilde\omega}R_ie_i(t)^2-\int_{t-\mu(t)}^{t}R_ie_i(s)^2ds\Big],\quad(33)$$

$$D^+V_4(t)=\frac{2e^{\tilde\omega t}}{\varepsilon}\sum_{i=1}^{n}[(\xi_i(t)-k_i)e_i(t)^2+(\eta_i(t)-w_i)|e_i(t)|+(\varphi_i(t)-v_i)|e_i(t)x_i(t)|].\quad(34)$$

结合式(31)—(34),进一步有

$$\begin{aligned}D^+V(t)\leqslant{}&e^{\tilde\omega t}\sum_{i=1}^{n}\Big\{\tilde\omega-\frac{2}{\varepsilon}d_i+\sum_{j=1}^{n}\frac{\tilde a_{ij}^2+\tilde b_{ij}^2+\mu_M\tilde c_{ij}^2+l_i^2}{\varepsilon}+\frac{H_i^2}{\varepsilon\hat u_i}+\frac{\beta_i^2l_i^2}{y_i'}+\frac{e^{\tilde\omega\mu_M}Q_i}{1-\tau_D}+\frac{R_i(e^{\tilde\omega\mu_M}-1)}{\tilde\omega}-\frac{2k_i}{\varepsilon}\Big\}e_i(t)^2+\\&e^{\tilde\omega t}\sum_{i=1}^{n}\Big\{\sum_{j=1}^{n}(|a_{ij}'-a_{ij}''|+|b_{ij}'-b_{ij}''|+\mu_M|c_{ij}'-c_{ij}''|)\frac{2m_i}{\varepsilon}-\frac{2w_i}{\varepsilon}\Big\}|e_i(t)|+\\&e^{\tilde\omega t}\sum_{i=1}^{n}\Big\{\frac{2}{\varepsilon}|d_i'-d_i''|-\frac{2}{\varepsilon}v_i\Big\}|e_i(t)x_i(t)|+\\&e^{\tilde\omega t}\sum_{i=1}^{n}\Big(\tilde\omega+\frac{\hat u_i}{\varepsilon}+y_i'-2\alpha_i\Big)z_i(t)^2+\\&e^{\tilde\omega t}\sum_{i=1}^{n}\Big(\frac{nl_i^2}{\varepsilon}-Q_i\Big)e_i(t-\tau(t))^2+\\&e^{\tilde\omega t}\sum_{i=1}^{n}\int_{t-\mu(t)}^{t}\Big(\frac{nl_i^2}{\varepsilon}-R_i\Big)e_i(\zeta)^2d\zeta.\end{aligned}$$

令 $\Pi_i=\alpha_i-\frac{\tilde\omega}{2}$.由定理 2 的假设可知 $\Pi_i>0$.令

$$\frac{\hat u_i}{\varepsilon}+y_i'=\Pi_i,\quad Q_i=\frac{nl_i^2}{\varepsilon},\quad R_i=\frac{nl_i^2}{\varepsilon},$$

$$k_i=\frac{\varepsilon}{2}\Big\{\tilde\omega-\frac{2}{\varepsilon}d_i+\sum_{j=1}^{n}\frac{\tilde a_{ij}^2+\tilde b_{ij}^2+\mu_M\tilde c_{ij}^2+l_i^2}{\varepsilon}+\frac{H_i^2}{\varepsilon\hat u_i}+\frac{\beta_i^2l_i^2}{y_i'}+\frac{e^{\tilde\omega\mu_M}Q_i}{1-\tau_D}+\frac{R_i(e^{\tilde\omega\mu_M}-1)}{\tilde\omega}+\Pi_i\Big\},$$

$$w_i = m_i(|a'_{ij} - a''_{ij}| + |b'_{ij} - b''_{ij}| + \mu_M |c'_{ij} - c''_{ij}|),$$
$$v_i = |d'_i - d''_i|, \quad i = 1,2,\cdots,n,$$

则有

$$D^+ V(t) \leqslant - \min_{1 \leqslant i \leqslant n} \prod_i (\| e(t) \|^2 + \| z(t) \|^2) \mathrm{e}^{\tilde{\omega} t} \leqslant 0. \tag{35}$$

由式(35)可知

$$V(t) \leqslant V(0), \forall t \geqslant 0. \tag{36}$$

结合式(30)和(36)可得,$\| e(t) \|^2 + \| z(t) \|^2 \leqslant V(t) \mathrm{e}^{-\tilde{\omega} t} \leqslant V(0) \mathrm{e}^{-\tilde{\omega} t}$,证毕.

## 3 数值仿真

在这一部分,我们通过数值仿真来验证上述所提出的理论结果的有效性.

**例 1** 考虑下面不具有不同时标的忆阻竞争神经网络:

$$\begin{cases} \varepsilon \dot{x}_i(t) = -d_i(x_i(t))x_i(t) + \sum_{j=1}^{2} a_{ij}(x_j(t)) f_j(x_j(t)) + \\ \quad \sum_{j=1}^{2} b_{ij}(x_j(t - \tau(t))) f_j(x_j(t - \tau(t))) + \\ \quad \sum_{j=1}^{2} c_{ij}(x_j(t)) \int_{t-\mu(t)}^{t} f_j(x_j(\zeta)) \mathrm{d}\zeta + H_i r_i, \\ \dot{r}_i(t) = -\alpha_i r_i(t) + \beta_i f_i(x_i(t)), \quad i = 1,2. \end{cases} \tag{37}$$

其中

$$\varepsilon = 2.5, \quad \alpha_1 = 2, \quad \alpha_2 = 1.5, \quad \beta_1 = 0.5,$$
$$\beta_2 = -0.3, \quad H_1 = 0.5, \quad H_2 = 1.5,$$
$$\tau(t) = \sin^2 0.45t, \quad \mu(t) = \cos^2 0.45t,$$
$$d_1 = \begin{cases} 7.5, |x_1(t)| \leqslant \chi_1, \\ 5, |x_1(t)| > \chi_1, \end{cases}$$
$$d_2 = \begin{cases} 3, |x_2(t)| \leqslant \chi_2, \\ 2.5, |x_2(t)| > \chi_2, \end{cases}$$
$$a_{11} = \begin{cases} -1, |x_1(t)| \leqslant \chi_1, \\ -0.8, |x_1(t)| > \chi_1, \end{cases}$$
$$a_{12} = \begin{cases} 2, |x_2(t)| \leqslant \chi_2, \\ 1.6, |x_2(t)| > \chi_2, \end{cases}$$
$$a_{21} = \begin{cases} 2, |x_1(t)| \leqslant \chi_1, \\ 1.8, |x_1(t)| > \chi_1, \end{cases}$$
$$a_{22} = \begin{cases} 1, |x_2(t)| \leqslant \chi_2, \\ 1.5, |x_2(t)| > \chi_2, \end{cases}$$
$$c_{11} = \begin{cases} -2.6, |x_1(t)| \leqslant \chi_1, \\ 2, |x_1(t)| > \chi_1, \end{cases}$$
$$c_{12} = \begin{cases} 1.3, |x_2(t)| \leqslant \chi_2, \\ -3, |x_2(t)| > \chi_2, \end{cases}$$
$$c_{21} = \begin{cases} -3.2, |x_1(t)| \leqslant \chi_1, \\ 2.3, |x_1(t)| > \chi_1, \end{cases}$$
$$c_{22} = \begin{cases} 0.2, |x_2(t)| \leqslant \chi_2, \\ -1, |x_2(t)| > \chi_2, \end{cases}$$
$$b_{11} = \begin{cases} -3, |x_1(t - \tau(t))| \leqslant \chi_1, \\ 2, |x_1(t - \tau(t))| > \chi_1, \end{cases}$$
$$b_{12} = \begin{cases} 1.6, |x_2(t - \tau(t))| \leqslant \chi_2, \\ 0.7, |x_2(t - \tau(t))| > \chi_2, \end{cases}$$
$$b_{21} = \begin{cases} 0.8, |x_1(t - \tau(t))| \leqslant \chi_1, \\ -1.5, |x_1(t - \tau(t))| > \chi_1, \end{cases}$$
$$b_{22} = \begin{cases} -0.5, |x_2(t - \tau(t))| \leqslant \chi_2, \\ 0.8, |x_2(t - \tau(t))| > \chi_2. \end{cases}$$

容易验证假设A1成立,且$\tau_M = \mu_M = 1, \tau_D = \mu_D = 0.9$.赋予系统(37)的初始条件为:$x_1(t) = \sin 2t + 0.5, x_2(t) = \cos t + 0.4, r_1(t) = \sin 2t + 0.5, r_2(t) = \cos 5t, t \in [-1,0]$.

选取系统(37)为驱动系统,并设计如下响应系统:

$$\begin{cases} \varepsilon \dot{y}_i(t) = -d_i(y_i(t))y_i(t) + \sum_{j=1}^{2} a_{ij}(y_j(t)) f_j(y_j(t)) + \\ \quad \sum_{j=1}^{2} b_{ij}(y_j(t - \tau(t))) f_j(y_j(t - \tau(t))) + \\ \quad \sum_{j=1}^{2} c_{ij}(y_j(t)) \int_{t-\mu(t)}^{t} f_j(y_j(\zeta)) \mathrm{d}\zeta + H_i s_i + u_i(t), \\ \dot{s}_i(t) = -\alpha_i s_i(t) + \beta_i f_i(y_i(t)), \quad i = 1,2. \end{cases} \tag{38}$$

选择系统(38)的初始值为$y_1(t) = \cos 2t, y_2(t) = \sin t, s_1(t) = \mathrm{e}^{2t}, s_2(t) = \mathrm{e}^t + 1, t \in [-1,0]$.选择转换跳跃$\chi_j$,激励函数$f_j(\cdot)$和自适应控制器为$\chi_j = 1$, $f_j(\cdot) = \tanh(\cdot), j = 1,2$.显然,假设A1和假设A2成立.

根据定理1,在控制器

$$u_i = -\xi_i e_i(t) - \eta_i \mathrm{sign}(e_i(t)) - \varphi_i x_i(t) \mathrm{sign}(e_i(t) x_i(t)), \quad i = 1,2 \tag{39}$$

作用下,系统(37)与(38)取得全局渐进同步.
其中

$$\begin{cases} \dot{\xi}_i = p_i e_i(t)^2, i = 1,2, \\ \dot{\eta}_i = q_i |e_i(t)|, i = 1,2, \\ \dot{\varphi}_i = \psi_i |e_i(t) x_i(t)|, i = 1,2. \end{cases} \tag{40}$$

在仿真中,令$p_i = q_i = \psi_i = 0.2$,$\xi_i(t)$,$\eta_i(t)$和$\varphi_i(t)$的初始值均为0,$i = 1,2$.图1和图2分别描述了纠错系统的状态轨迹和控制增益的演变过程.

根据定理2,对给出的指数常数$0 < \tilde{\omega} <$

$2\min_{1\leqslant i\leqslant n}\alpha_i = 3$,在控制器

$$u_i = -\xi_i e_i(t) - \eta_i \operatorname{sign}(e_i(t)) - \varphi_i x_i(t)\operatorname{sign}(e_i(t)x_i(t)), \quad i = 1,2, \tag{41}$$

$$\begin{cases}\dot{\xi}_i = p_i e_i(t)^2 \mathrm{e}^{\tilde{\omega}t}, i = 1,2, \\ \dot{\eta}_i = q_i\,|\,e_i(t)\,|\,\mathrm{e}^{\tilde{\omega}t}, i = 1,2, \\ \dot{\varphi}_i = \psi_i\,|\,e_i(t)x_i(t)\,|\,\mathrm{e}^{\tilde{\omega}t}, i = 1,2\end{cases} \tag{42}$$

的作用下,系统(37)和(38)达到全局指数同步.

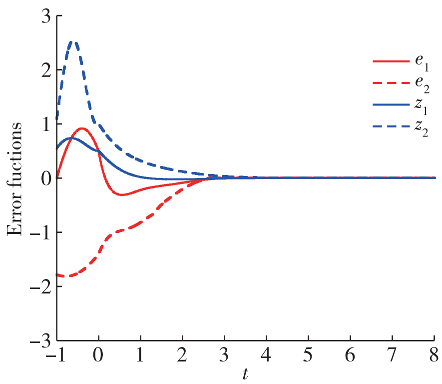


图 1 误差系统的状态轨线

Fig. 1 Trajectories of error states

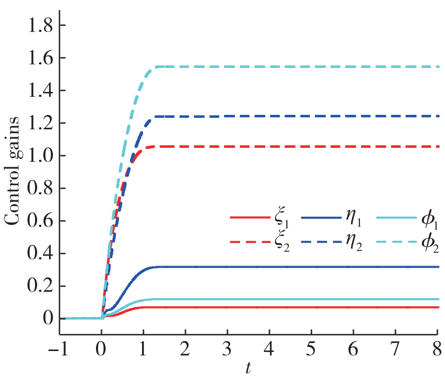


图 2 控制增益的演变

Fig. 2 Evolutions of control gains

在仿真中,令 $\tilde{\omega} = 2$,$p_i = q_i = \psi_i = 0.2$,且 $\xi_i(t)$,$\eta_i(t)$ 和 $\varphi_i(t)$ 的初始值均为 0,$i = 1,2$. 图 3 和图 4 分别描述了误差系统的状态轨迹和控制增益的演变过程.

从图 1 和图 3 的误差纠错系统的状态轨迹比较中可以看出,指数自适应控制器具有更好的收敛性. 另外,也可以从图 2 和图 4 的比较中发现,指数自适应控制增益收敛到更高的水平.

## 4 结束语

本文研究了具有混合时变延时和不同时标的忆阻竞争神经网络的自适应同步问题,一个新的自适应控制器被设计以确保网络的全局同步与指数同步.同时,我们提出了自适应同步条件,该条件能够被应用在忆阻神经网络的其他数学模型中.

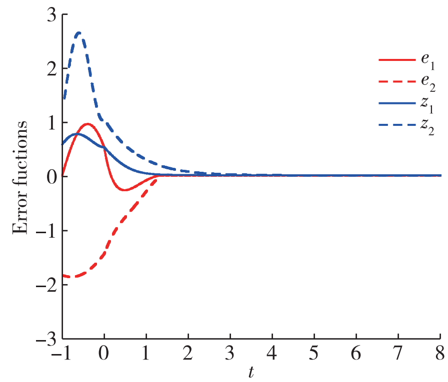


图 3 误差系统的状态轨线($\tilde{w}=2$)

Fig. 3 Trajectories of error states ($\tilde{w}=2$)

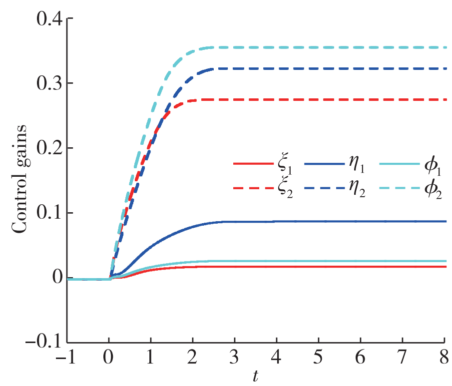


图 4 控制增益的演变($\tilde{w}=2$)

Fig. 4 Evolutions of control gains($\tilde{w}=2$)

有限时间内的收敛现象是不连续动力系统的特有功能之一,特别感兴趣的是它在现实中的应用.在今后的工作中,我们将重点研究忆阻网络的有限时间同步问题.

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# Adaptive synchronization of chaotic memristive competitive neural networks with mixed time-varying delays and different time scales

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**Abstract** This paper is concerned with the issue of adaptive synchronization for a general class of chaotic memristive competitive neural networks (MCNNs) with mixed time-varying delays and different time scales.By applying Lyapunov functional method and inequality analysis technique,a kind of novel adaptive controllers with feedback control laws are designed to achieve synchronization and exponential synchronization. The synchronization conditions,without need of excessive calculation such as solving linear matrix inequality or computing algebraic conditions,are given,which can be applied in MCNNs with different mathematical models of memristor proposed in the existing literature.Finally,an example with numerical simulations is listed to prove the efficiency and accuracy of the theoretical results.

**Key words** adaptive synchronization;memristor;competitive neural networks;time delay;time scale