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徐炳吉[1]

具有不确定参数的滞后型 Lurie 控制系统鲁棒绝对稳定性的 LMI 方法

摘要

分别针对具有结构参数和范数有界参数的滞后型 Lurie 控制系统的鲁棒绝对稳定性问题,利用 Lyapunov 方法给出了系统鲁棒绝对稳定的时滞无关条件及时滞相关条件.得到的结果用线性矩阵不等式(LMI)表示,易于利用 MATLAB 工具箱求得保守性较低的条件.

关键词

Lurie 控制系统;鲁棒绝对稳定性;线性矩阵不等式(LMI)

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作者简介

徐炳吉,男,博士,教授,研究方向为系统分析与智能控制.bjxu@ cugb.edu.cn

1 中国地质大学(北京)信息工程学院,北京,100083

0 引言

近年来,对具有参数或非参数不确定性的动态系统的鲁棒稳定性问题的研究已取得了较大的进展.Lurie 控制系统是一类重要的非线性控制系统,对无时滞或含有时滞的 Lurie 控制系统的绝对稳定性问题的研究也得到了许多很好的结果[1-5].本文对具有不确定性参数的滞后型 Lurie 控制系统的鲁棒绝对稳定性问题,利用 Lyapunov 方法就不确定性的以下两种情况进行研究:1)满足结构条件;2)范数有界.

1 系统描述及引理

考虑如下具有不确定参数的滞后型 Lurie 直接控制系统:

$$\begin{cases}\dot{\boldsymbol{x}}(t)=(\boldsymbol{A}+\Delta\boldsymbol{A}(\theta))\boldsymbol{x}(t)+(\boldsymbol{B}+\Delta\boldsymbol{B}(\theta))\boldsymbol{x}(t-\tau)+(\boldsymbol{D}+\\ \quad\Delta\boldsymbol{D}(\theta))f(\boldsymbol{\sigma}(t))+(\boldsymbol{E}+\Delta\boldsymbol{E}(\theta))f(\boldsymbol{\sigma}(t-h)),\\ \dot{\boldsymbol{\sigma}}(t)=\boldsymbol{C}^{\mathrm{T}}\boldsymbol{x}(t)-\boldsymbol{J}f(\boldsymbol{\sigma}(t)),\\ \boldsymbol{x}(t)=\boldsymbol{\varphi}(t),\quad t\in[-T,0],\end{cases}\tag{1}$$

式中 $\boldsymbol{x}(t)\in\mathbf{R}^n$ 为状态向量,$\boldsymbol{A},\boldsymbol{B}\in\mathbf{R}^{n\times n},\boldsymbol{D},\boldsymbol{E}\in\mathbf{R}^{n\times m},\boldsymbol{C}_i\in\mathbf{R}^n,i=1,2,\cdots,m,\boldsymbol{J}\in\mathbf{R}^{m\times m},\boldsymbol{C}=[\boldsymbol{C}_1,\boldsymbol{C}_2,\cdots,\boldsymbol{C}_m]\in\mathbf{R}^{n\times m}$ 为常数矩阵.$\tau>0,h_i>0,i=1,2,\cdots,m$,为常数时滞,$\boldsymbol{\varphi}(t)$ 为连续的初值函数向量.$\boldsymbol{\sigma}(t)=[\sigma_1(t),\sigma_2(t),\cdots,\sigma_m(t)]^{\mathrm{T}}\in\mathbf{R}^m$,

$f(\boldsymbol{\sigma}(t))=[f_1(\sigma_1(t)),f_2(\sigma_2(t)),\cdots,f_m(\sigma_m(t))]^{\mathrm{T}}$,

$f(\boldsymbol{\sigma}(t-h))=[f_1(\sigma_1(t-h_1)),f_2(\sigma_2(t-h_2)),\cdots,f_m(\sigma_m(t-h_m))]^{\mathrm{T}}$,

$f_i(\cdot)\in K_{[0,k_i]}=\{f_i(\cdot)\,|\,f_i(0)=0,0<\sigma_if_i(\sigma_i)\leqslant k_i\sigma_i^2,\sigma_i\neq0\},k_i>0$,或

$f_i(\cdot)\in K_{[0,\infty)}=\{f_i(\cdot)\,|\,f_i(0)=0,\sigma_if_i(\sigma_i)>0,\sigma_i\neq0\},i=1,2,\cdots,m.$

$\Delta\boldsymbol{A}(\theta),\Delta\boldsymbol{B}(\theta),\Delta\boldsymbol{D}(\theta)$ 为表示不确定性参数 θ 的函数矩阵,θ 属于某闭集.本文假定它们具有如下形式:

$$\Delta\boldsymbol{A}(\theta)=\boldsymbol{G}_1\boldsymbol{F}_1(\theta)\boldsymbol{H}_1,\quad\Delta\boldsymbol{B}(\theta)=\boldsymbol{G}_2\boldsymbol{F}_2(\theta)\boldsymbol{H}_2,$$
$$\Delta\boldsymbol{D}(\theta)=\boldsymbol{G}_3\boldsymbol{F}_3(\theta)\boldsymbol{H}_3,\quad\Delta\boldsymbol{E}(\theta)=\boldsymbol{G}_4\boldsymbol{F}_4(\theta)\boldsymbol{H}_4,$$

式中 $\boldsymbol{G}_i,\boldsymbol{H}_i,i=1,2,3,4$,为适当维数的常数矩阵.$\boldsymbol{F}_i(\theta),i=1,2,3,4$,为元素 Lebesgue 可测的未知参数 θ 的实函数矩阵,满足 $\boldsymbol{F}_i^{\mathrm{T}}(\theta)\boldsymbol{F}_i(\theta)\leqslant\boldsymbol{I}(i=1,2,3,4)$,$\boldsymbol{I}$ 为单位矩阵.

为简洁,记

$\bar{A}=A+\Delta A(\theta)$,
$\bar{B}=B+\Delta B(\theta)$,
$\bar{D}=D+\Delta D(\theta)$,
$\bar{E}=E+\Delta E(\theta)$.

$U>0$(或 $U\geqslant 0$) 表示 U 为正定矩阵(或半正定矩阵);$U<0$ 表示 U 为负定矩阵.

引理 1[6] 对任意矩阵 u,v 及任意正定矩阵 P,下式成立:

$$-u^{\mathrm{T}}v-v^{\mathrm{T}}u\leqslant u^{\mathrm{T}}Pu+v^{\mathrm{T}}P^{-1}v,$$
$$u^{\mathrm{T}}v+v^{\mathrm{T}}u\leqslant u^{\mathrm{T}}Pu+v^{\mathrm{T}}P^{-1}v.$$

引理 2[7] 对于实矩阵 A,L,E 及满足 $\|F\|\leqslant 1$ 的实矩阵 F,则有对任意实数 $\varepsilon>0$,

$$LFE+E^{\mathrm{T}}F^{\mathrm{T}}L^{\mathrm{T}}\leqslant\varepsilon^{-1}LL^{\mathrm{T}}+\varepsilon E^{\mathrm{T}}E;$$

对任意矩阵 $P>0$ 和使得 $\varepsilon I-EPE^{\mathrm{T}}>0$ 的实数 $\varepsilon>0$,下式成立

$$(A+LFE)P(A+LFE)^{\mathrm{T}}\leqslant APA^{\mathrm{T}}+APE^{\mathrm{T}}(\varepsilon I-EPE^{\mathrm{T}})^{-1}EPA^{\mathrm{T}}+\varepsilon LL^{\mathrm{T}};$$

对任意矩阵 $P>0$ 和使得 $P-\varepsilon LL^{\mathrm{T}}>0$ 的实数$\varepsilon>0$,下式成立

$$(A+LFE)^{\mathrm{T}}P^{-1}(A+LFE)\leqslant A^{\mathrm{T}}(P-\varepsilon LL^{\mathrm{T}})^{-1}A+\varepsilon^{-1}E^{\mathrm{T}}E.$$

基于以上引理,我们可得到以下主要结果.

2 主要结果

定理 1 若存在矩阵 $P>0,Q>0$,对角矩阵$\Lambda=\mathrm{diag}(\lambda_1,\lambda_2,\cdots,\lambda_m)>0,R=\mathrm{diag}(r_1,r_2,\cdots,r_m)>0$,及常数 $\varepsilon_i>0,i=1,2,3,4$,使得以下 LMI 成立,则系统(1)鲁棒绝对稳定.

$$\Phi=\begin{bmatrix} \Phi_{11} & PB & PD+C\Lambda & PE & PG_1 & PG_2 & PG_3 & PG_4 \\ B^{\mathrm{T}}P & \Phi_{22} & & & & & & \\ D^{\mathrm{T}}P+\Lambda C^{\mathrm{T}} & & \Phi_{33} & & & & & \\ E^{\mathrm{T}}P & & & \Phi_{44} & & & & \\ G_1^{\mathrm{T}}P & & & & -\varepsilon_1 I & & & \\ G_2^{\mathrm{T}}P & & & & & -\varepsilon_2 I & & \\ G_3^{\mathrm{T}}P & & & & & & -\varepsilon_3 I & \\ G_4^{\mathrm{T}}P & & & & & & & -\varepsilon_4 I \end{bmatrix}<0,$$

式中

$$\Phi_{11}=PA+A^{\mathrm{T}}P+\varepsilon_1 H_1^{\mathrm{T}}H_1+Q,$$
$$\Phi_{22}=\varepsilon_2 H_2^{\mathrm{T}}H_2-Q,$$
$$\Phi_{33}=\varepsilon_3 H_3^{\mathrm{T}}H_3-J^{\mathrm{T}}\Lambda-\Lambda J+R,$$
$$\Phi_{44}=\varepsilon_4 H_4^{\mathrm{T}}H_4-R.$$

证明 取 Lyapunov 函数

$$V(t)=x^{\mathrm{T}}(t)Px(t)+2\sum_{i=1}^{m}\lambda_i\int_0^{\sigma_i}f_i(\xi)\,\mathrm{d}\xi+\int_{t-\tau}^{t}x^{\mathrm{T}}(\xi)Qx(\xi)\,\mathrm{d}\xi+\sum_{i=1}^{m}r_i\int_{t-h_i}^{t}f_i^2(\sigma_i(\xi))\,\mathrm{d}\xi,$$

则 $V(t)$ 沿系统(1) 的导数

$$\begin{aligned}\dot{V}(t)=&2x^{\mathrm{T}}(t)P\dot{x}(t)+2\sum_{i=1}^{m}\lambda_i f_i(\sigma_i(t))\dot{\sigma}_i(t)+\\&x^{\mathrm{T}}(t)Qx(t)-x^{\mathrm{T}}(t-\tau)Qx(t-\tau)+\\&\sum_{i=1}^{m}r_i[f_i^2(\sigma_i(t))-f_i^2(\sigma_i(t-h_i))]=\\&x^{\mathrm{T}}(t)(P\bar{A}+\bar{A}^{\mathrm{T}}P+Q)x(t)+2x^{\mathrm{T}}(t)P\bar{B}x(t-\tau)+\\&2x^{\mathrm{T}}(t)P\bar{E}f(\sigma(t-h))+2x^{\mathrm{T}}(t)(P\bar{D}+C\Lambda)f(\sigma(t))-\\&x^{\mathrm{T}}(t-\tau)Qx(t-\tau)-f^{\mathrm{T}}(\sigma(t-h))Rf(\sigma(t-h))+\\&f^{\mathrm{T}}(\sigma(t))(R-J^{\mathrm{T}}\Lambda-\Lambda J)f(\sigma(t)).\end{aligned}$$

由引理 2,可得

$$P\bar{A}+\bar{A}^{\mathrm{T}}P\leqslant PA+A^{\mathrm{T}}P+\frac{1}{\varepsilon_1}PG_1G_1^{\mathrm{T}}P+\varepsilon_1 H_1^{\mathrm{T}}H_1,$$

$$2x^{\mathrm{T}}(t)P\Delta B(\theta)x(t-\tau)\leqslant\frac{1}{\varepsilon_2}x^{\mathrm{T}}(t)PG_2G_2^{\mathrm{T}}Px(t)+\varepsilon_2 x^{\mathrm{T}}(t-\tau)H_2^{\mathrm{T}}H_2x(t-\tau),$$

$$2x^{\mathrm{T}}(t)P\Delta D(\theta)f(\sigma(t))\leqslant\frac{1}{\varepsilon_3}x^{\mathrm{T}}(t)PG_3G_3^{\mathrm{T}}Px(t)+\varepsilon_3 f^{\mathrm{T}}(\sigma(t))H_3^{\mathrm{T}}H_3f(\sigma(t)),$$

$$2x^{\mathrm{T}}(t)P\Delta E(\theta)f(\sigma(t-h))\leqslant\frac{1}{\varepsilon_4}x^{\mathrm{T}}(t)PG_4G_4^{\mathrm{T}}Px(t)+\varepsilon_4 f^{\mathrm{T}}(\sigma(t-h))H_4^{\mathrm{T}}H_4f(\sigma(t-h)).$$

因此,有 $\dot{V}(t)\leqslant y^{\mathrm{T}}(t)\Psi y(t)$,式中
$y(t)=[x^{\mathrm{T}}(t),x^{\mathrm{T}}(t-\tau),f^{\mathrm{T}}(\sigma(t)),f^{\mathrm{T}}(\sigma(t-h))]^{\mathrm{T}}$,

$$\Psi=\begin{bmatrix}\Phi_{11}+P\sum_{i=1}^{4}\frac{1}{\varepsilon_i}G_iG_i^{\mathrm{T}}P & PB & PD+C\Lambda & PE\\ B^{\mathrm{T}}P & \Phi_{22} & 0 & 0\\ D^{\mathrm{T}}P+\Lambda C^{\mathrm{T}} & 0 & \Phi_{33} & 0\\ E^{\mathrm{T}}P & 0 & 0 & \Phi_{44}\end{bmatrix}.$$

由 Schur 补[8],$\Phi<0$ 等价于 $\Psi<0$.证毕

定理 1 给出了系统(1)鲁棒绝对稳定的时滞无关条件,下面给出系统(1)鲁棒绝对稳定的时滞相关条件.

定理 2 若存在矩阵 $P>0,R_i>0,i=1,2,3,4$,对角矩阵 $Q=\mathrm{diag}(q_1,q_2,\cdots,q_m)>0,\Lambda=\mathrm{diag}(\lambda_1,\lambda_2,\cdots,\lambda_m)>0$,及常数 $\varepsilon_i>0,\eta_i>0,\mu_i>0,i=1,2,3,4$,使得以下 LMI 成立,则系统(1)鲁棒绝对稳定.

$$\boldsymbol{T}=\begin{bmatrix} \boldsymbol{T}_{11} & \boldsymbol{T}_{12} & \boldsymbol{T}_{13} & \boldsymbol{T}_{14} & \boldsymbol{T}_{15} & \boldsymbol{T}_{16} & \boldsymbol{T}_{17} & 0 & 0 \\ \boldsymbol{T}_{12}^{\mathrm{T}} & \boldsymbol{T}_{22} & 0 & 0 & 0 & 0 & 0 & \boldsymbol{T}_{28} & 0 \\ \boldsymbol{T}_{13}^{\mathrm{T}} & 0 & \boldsymbol{T}_{33} & 0 & 0 & 0 & 0 & 0 & \boldsymbol{T}_{39} \\ \boldsymbol{T}_{14}^{\mathrm{T}} & 0 & 0 & -\boldsymbol{T}_{44} & & & & & \\ \boldsymbol{T}_{15}^{\mathrm{T}} & 0 & 0 & & -\boldsymbol{T}_{55} & & & & \\ \boldsymbol{T}_{16}^{\mathrm{T}} & 0 & 0 & & & -\boldsymbol{T}_{66} & & & \\ \boldsymbol{T}_{17}^{\mathrm{T}} & 0 & 0 & & & & -\boldsymbol{T}_{77} & & \\ 0 & \boldsymbol{T}_{28}^{\mathrm{T}} & 0 & & & & & -\boldsymbol{T}_{88} & \\ 0 & 0 & \boldsymbol{T}_{39}^{\mathrm{T}} & & & & & & -\boldsymbol{T}_{99} \end{bmatrix}<0,$$

式中

$$\boldsymbol{T}_{11}=\sum_{i=1}^{2}(\tau\varepsilon_i+\mu_i)\boldsymbol{H}_i^{\mathrm{T}}\boldsymbol{H}_i+\tau(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{R}_1\boldsymbol{A}+\boldsymbol{B}^{\mathrm{T}}\boldsymbol{R}_2\boldsymbol{B})+\boldsymbol{P}(\boldsymbol{A}+\boldsymbol{B})+(\boldsymbol{A}+\boldsymbol{B})^{\mathrm{T}}\boldsymbol{P},$$

$\boldsymbol{T}_{12}=\boldsymbol{PD}+\boldsymbol{C\Lambda}$, $\boldsymbol{T}_{13}=\boldsymbol{PE}$,

$\boldsymbol{T}_{14}=[\boldsymbol{PG}_1,\boldsymbol{PG}_2,\boldsymbol{PG}_3,\boldsymbol{PG}_4]$,

$\boldsymbol{T}_{15}=[\sqrt{\tau}\boldsymbol{PG}_2,\sqrt{\tau}\boldsymbol{PG}_2,\sqrt{\tau}\boldsymbol{PG}_2,\sqrt{\tau}\boldsymbol{PG}_2]$,

$\boldsymbol{T}_{16}=[\sqrt{\tau}\boldsymbol{PB},\sqrt{\tau}\boldsymbol{PB},\sqrt{\tau}\boldsymbol{PB},\sqrt{\tau}\boldsymbol{PB}]$,

$\boldsymbol{T}_{17}=[\sqrt{\tau}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{R}_1\boldsymbol{G}_1,\sqrt{\tau}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{R}_2\boldsymbol{G}_2]$,

$\boldsymbol{T}_{22}=\tau\boldsymbol{D}^{\mathrm{T}}\boldsymbol{R}_3\boldsymbol{D}+(\mu_3+\tau\varepsilon_3)\boldsymbol{H}_3^{\mathrm{T}}\boldsymbol{H}_3+\boldsymbol{Q}-\boldsymbol{\Lambda J}-\boldsymbol{J}^{\mathrm{T}}\boldsymbol{\Lambda}$,

$\boldsymbol{T}_{28}=\sqrt{\tau}\boldsymbol{D}^{\mathrm{T}}\boldsymbol{R}_3\boldsymbol{G}_3$,

$\boldsymbol{T}_{33}=\tau\boldsymbol{E}^{\mathrm{T}}\boldsymbol{R}_4\boldsymbol{E}+(\mu_4+\tau\varepsilon_4)\boldsymbol{H}_4^{\mathrm{T}}\boldsymbol{H}_4-\boldsymbol{Q}$,

$\boldsymbol{T}_{39}=\sqrt{\tau}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{R}_4\boldsymbol{G}_4$, $\boldsymbol{T}_{44}=\mathrm{diag}(\mu_1\boldsymbol{I},\mu_2\boldsymbol{I},\mu_3\boldsymbol{I},\mu_4\boldsymbol{I})$,

$\boldsymbol{T}_{55}=\mathrm{diag}(\eta_1\boldsymbol{I},\eta_2\boldsymbol{I},\eta_3\boldsymbol{I},\eta_4\boldsymbol{I})$,

$\boldsymbol{T}_{66}=\mathrm{diag}(\boldsymbol{R}_1-\eta_1\boldsymbol{H}_2^{\mathrm{T}}\boldsymbol{H}_2,\boldsymbol{R}_2-\eta_2\boldsymbol{H}_2^{\mathrm{T}}\boldsymbol{H}_2,\boldsymbol{R}_3-\eta_3\boldsymbol{H}_2^{\mathrm{T}}\boldsymbol{H}_2,\boldsymbol{R}_4-\eta_4\boldsymbol{H}_2^{\mathrm{T}}\boldsymbol{H}_2)$,

$\boldsymbol{T}_{77}=\mathrm{diag}(\varepsilon_1\boldsymbol{I}-\boldsymbol{G}_1^{\mathrm{T}}\boldsymbol{R}_1\boldsymbol{G}_1,\varepsilon_2\boldsymbol{I}-\boldsymbol{G}_2^{\mathrm{T}}\boldsymbol{R}_2\boldsymbol{G}_2)$,

$\boldsymbol{T}_{88}=\varepsilon_3\boldsymbol{I}-\boldsymbol{G}_3^{\mathrm{T}}\boldsymbol{R}_3\boldsymbol{G}_3$, $\boldsymbol{T}_{99}=\varepsilon_4\boldsymbol{I}-\boldsymbol{G}_4^{\mathrm{T}}\boldsymbol{R}_4\boldsymbol{G}_4$.

证明 令 $\boldsymbol{\varphi}(t)=\boldsymbol{\varphi}(-T)$,$t\in[-T-\tau,-T]$.因当 $t\geqslant\tau$ 时,

$$\boldsymbol{x}(t-\tau)=\boldsymbol{x}(t)-\int_{t-\tau}^{t}\dot{\boldsymbol{x}}(\xi)\mathrm{d}\xi=\boldsymbol{x}(t)-\int_{t-\tau}^{t}[\bar{\boldsymbol{A}}\boldsymbol{x}(\xi)+\bar{\boldsymbol{B}}\boldsymbol{x}(\xi-\tau)+\bar{\boldsymbol{D}}f(\boldsymbol{\sigma}(\xi))+\bar{\boldsymbol{E}}f(\boldsymbol{\sigma}(\xi-h))]\mathrm{d}\xi.$$

考虑如下系统:

$$\begin{cases}\dot{\boldsymbol{x}}(t)=(\bar{\boldsymbol{A}}+\bar{\boldsymbol{B}})\boldsymbol{x}(t)+\bar{\boldsymbol{D}}f(\boldsymbol{\sigma}(t))+\bar{\boldsymbol{E}}f(\boldsymbol{\sigma}(t-h))-\bar{\boldsymbol{B}}\int_{t-\tau}^{t}[\bar{\boldsymbol{A}}\boldsymbol{x}(\xi)+\bar{\boldsymbol{B}}\boldsymbol{x}(\xi-\tau)+\bar{\boldsymbol{D}}f(\boldsymbol{\sigma}(\xi))+\bar{\boldsymbol{E}}f(\boldsymbol{\sigma}(\xi-h))]\mathrm{d}\xi,\\ \dot{\boldsymbol{\sigma}}(t)=\boldsymbol{C}^{\mathrm{T}}\boldsymbol{x}(t)-\boldsymbol{J}f(\boldsymbol{\sigma}(t)),\\ \boldsymbol{x}(t)=\boldsymbol{\varphi}(t),\quad t\in[-T-\tau,0],\end{cases}\qquad(2)$$

由文献[9]可知,若系统(2)全局一致渐近稳定,则系统(1)全局一致渐近稳定.

取 $V_1(t)=\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Px}(t)$,则 $V_1(t)$ 沿系统(2)求导得:

$$\dot{V}_1(t)=2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\dot{\boldsymbol{x}}(t)=2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}(\bar{\boldsymbol{A}}+\bar{\boldsymbol{B}})\boldsymbol{x}(t)-2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\bar{\boldsymbol{B}}\int_{t-\tau}^{t}\dot{\boldsymbol{x}}(\xi)\mathrm{d}\xi+2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\bar{\boldsymbol{D}}f(\boldsymbol{\sigma}(t))+2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\bar{\boldsymbol{E}}f(\boldsymbol{\sigma}(t-h)).$$

由引理 1 可得:

$$-2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\bar{\boldsymbol{B}}\int_{t-\tau}^{t}\dot{\boldsymbol{x}}(\xi)\mathrm{d}\xi=-2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\bar{\boldsymbol{B}}\int_{t-\tau}^{t}[\bar{\boldsymbol{A}}\boldsymbol{x}(\xi)+\bar{\boldsymbol{B}}\boldsymbol{x}(\xi-\tau)+\bar{\boldsymbol{D}}f(\boldsymbol{\sigma}(\xi))+\bar{\boldsymbol{E}}f(\boldsymbol{\sigma}(\xi-h))]\mathrm{d}\xi\leqslant$$

$$\tau\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\bar{\boldsymbol{B}}(\boldsymbol{R}_1^{-1}+\boldsymbol{R}_2^{-1}+\boldsymbol{R}_3^{-1}+\boldsymbol{R}_4^{-1})\bar{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{Px}(t)+\int_{t-\tau}^{t}\boldsymbol{x}^{\mathrm{T}}(\xi)\bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{R}_1\bar{\boldsymbol{A}}\boldsymbol{x}(\xi)\mathrm{d}\xi+\int_{t-\tau}^{t}\boldsymbol{x}^{\mathrm{T}}(\xi-\tau)\bar{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{R}_2\bar{\boldsymbol{B}}\boldsymbol{x}(\xi-\tau)\mathrm{d}\xi+$$

$$\int_{t-\tau}^{t}f^{\mathrm{T}}(\boldsymbol{\sigma}(\xi))\bar{\boldsymbol{D}}^{\mathrm{T}}\boldsymbol{R}_3\bar{\boldsymbol{D}}f(\boldsymbol{\sigma}(\xi))\mathrm{d}\xi+\int_{t-\tau}^{t}f^{\mathrm{T}}(\boldsymbol{\sigma}(\xi-h))\bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{R}_4\bar{\boldsymbol{E}}f(\boldsymbol{\sigma}(\xi-h))\mathrm{d}\xi.$$

由引理 2 可得:

$$2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\Delta\boldsymbol{A}(\theta)\boldsymbol{x}(t)\leqslant\boldsymbol{x}^{\mathrm{T}}(t)(\mu_1^{-1}\boldsymbol{PG}_1\boldsymbol{G}_1^{\mathrm{T}}\boldsymbol{P}+\mu_1\boldsymbol{H}_1^{\mathrm{T}}\boldsymbol{H}_1)\boldsymbol{x}(t),$$

$$2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\Delta\boldsymbol{B}(\theta)\boldsymbol{x}(t)\leqslant\boldsymbol{x}^{\mathrm{T}}(t)(\mu_2^{-1}\boldsymbol{PG}_2\boldsymbol{G}_2^{\mathrm{T}}\boldsymbol{P}+\mu_2\boldsymbol{H}_2^{\mathrm{T}}\boldsymbol{H}_2)\boldsymbol{x}(t),$$

$$2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\Delta\boldsymbol{D}(\theta)f(\boldsymbol{\sigma}(t))\leqslant\mu_3^{-1}\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{PG}_3\boldsymbol{G}_3^{\mathrm{T}}\boldsymbol{Px}(t)+\mu_3f^{\mathrm{T}}(\boldsymbol{\sigma}(t))\boldsymbol{H}_3^{\mathrm{T}}\boldsymbol{H}_3f(\boldsymbol{\sigma}(t)),$$

$$2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\Delta\boldsymbol{E}(\theta)f(\boldsymbol{\sigma}(t-h))\leqslant\mu_4^{-1}\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{PG}_4\boldsymbol{G}_4^{\mathrm{T}}\boldsymbol{Px}(t)+\mu_4f^{\mathrm{T}}(\boldsymbol{\sigma}(t-h))\boldsymbol{H}_4^{\mathrm{T}}\boldsymbol{H}_4f(\boldsymbol{\sigma}(t-h)).$$

由 $\boldsymbol{T}<0$ 知,$\boldsymbol{R}_i-\eta_i\boldsymbol{H}_2^{\mathrm{T}}\boldsymbol{H}_2>0$,$i=1,2,3,4$.因此,由引理 2 得:

$$\bar{\boldsymbol{B}}\boldsymbol{R}_i^{-1}\bar{\boldsymbol{B}}^{\mathrm{T}}\leqslant\eta_i^{-1}\boldsymbol{G}_2\boldsymbol{G}_2^{\mathrm{T}}+\boldsymbol{B}(\boldsymbol{R}_i-\eta_i\boldsymbol{H}_2^{\mathrm{T}}\boldsymbol{H}_2)^{-1}\boldsymbol{B}^{\mathrm{T}},\quad i=1,2,3,4.$$

所以有

$$\dot{V}_1(t)\leqslant\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\Big(\sum_{i=1}^{4}\mu_i^{-1}\boldsymbol{G}_i\boldsymbol{G}_i^{\mathrm{T}}+\tau\sum_{i=1}^{4}\eta_i^{-1}\boldsymbol{G}_2\boldsymbol{G}_2^{\mathrm{T}}+\tau\boldsymbol{B}\sum_{i=1}^{4}(\boldsymbol{R}_i-\eta_i\boldsymbol{H}_2^{\mathrm{T}}\boldsymbol{H}_2)^{-1}\boldsymbol{B}^{\mathrm{T}}\Big)\boldsymbol{Px}(t)+\boldsymbol{x}^{\mathrm{T}}(t)[\mu_1\boldsymbol{H}_1^{\mathrm{T}}\boldsymbol{H}_1+\mu_2\boldsymbol{H}_2^{\mathrm{T}}\boldsymbol{H}_2+\boldsymbol{P}(\boldsymbol{A}+\boldsymbol{B})+(\boldsymbol{A}+\boldsymbol{B})^{\mathrm{T}}\boldsymbol{P}]\boldsymbol{x}(t)+$$

$$2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{PD}f(\boldsymbol{\sigma}(t))+2x^{\mathrm{T}}(t)\boldsymbol{PE}f(\boldsymbol{\sigma}(t-h))+\mu_3f^{\mathrm{T}}(\boldsymbol{\sigma}(t))\boldsymbol{H}_3^{\mathrm{T}}\boldsymbol{H}_3f(\boldsymbol{\sigma}(t))+\mu_4f^{\mathrm{T}}(\boldsymbol{\sigma}(t-h))\boldsymbol{H}_4^{\mathrm{T}}\boldsymbol{H}_4f(\boldsymbol{\sigma}(t-h))+I_1+I_2+I_3+I_4,$$

式中

$$I_1=\int_{t-\tau}^{t}\boldsymbol{x}^{\mathrm T}(\xi)\bar{\boldsymbol{A}}^{\mathrm T}\boldsymbol{R}_1\bar{\boldsymbol{A}}\boldsymbol{x}(\xi)\mathrm{d}\xi,$$

$$I_2=\int_{t-\tau}^{t}\boldsymbol{x}^{\mathrm T}(\xi-\tau)\bar{\boldsymbol{B}}^{\mathrm T}\boldsymbol{R}_2\bar{\boldsymbol{B}}\boldsymbol{x}(\xi-\tau)\mathrm{d}\xi,$$

$$I_3=\int_{t-\tau}^{t}f^{\mathrm T}(\boldsymbol{\sigma}(\xi))\bar{\boldsymbol{D}}^{\mathrm T}\boldsymbol{R}_3\bar{\boldsymbol{D}}f(\boldsymbol{\sigma}(\xi))\mathrm{d}\xi,$$

$$I_4=\int_{t-\tau}^{t}f^{\mathrm T}(\boldsymbol{\sigma}(\xi-h))\bar{\boldsymbol{E}}^{\mathrm T}\boldsymbol{R}_4\bar{\boldsymbol{E}}f(\boldsymbol{\sigma}(\xi-h))\mathrm{d}\xi.$$

取

$$V_2(t)=\int_{-\tau}^{0}\int_{t+\eta}^{t}\boldsymbol{x}^{\mathrm T}(\xi)\bar{\boldsymbol{A}}^{\mathrm T}\boldsymbol{R}_1\bar{\boldsymbol{A}}\boldsymbol{x}(\xi)\mathrm{d}\xi\mathrm{d}\eta,$$

$$V_3(t)=\int_{-\tau}^{0}\int_{t+\eta-\tau}^{t}\boldsymbol{x}^{\mathrm T}(\xi)\bar{\boldsymbol{B}}^{\mathrm T}\boldsymbol{R}_2\bar{\boldsymbol{B}}\boldsymbol{x}(\xi)\mathrm{d}\xi\mathrm{d}\eta,$$

$$V_4(t)=\int_{-\tau}^{0}\int_{t+\eta}^{t}f^{\mathrm T}(\boldsymbol{\sigma}(\xi))\bar{\boldsymbol{D}}^{\mathrm T}\boldsymbol{R}_3\bar{\boldsymbol{D}}f(\boldsymbol{\sigma}(\xi))\mathrm{d}\xi\mathrm{d}\eta,$$

$$V_5(t)=\int_{-\tau}^{0}\int_{t+\eta}^{t}f^{\mathrm T}(\boldsymbol{\sigma}(\xi-h))\bar{\boldsymbol{E}}^{\mathrm T}\boldsymbol{R}_4\bar{\boldsymbol{E}}f(\boldsymbol{\sigma}(\xi-h))\mathrm{d}\xi\mathrm{d}\eta,$$

$$V_6(t)=\sum_{i=1}^{m}q_i\int_{t-h_i}^{t}f_i^2(\boldsymbol{\sigma}_i(\xi))\mathrm{d}\xi,$$

$$V_7(t)=2\sum_{i=1}^{m}\lambda_i\int_{0}^{\sigma_i}f_i(\xi)\mathrm{d}\xi.$$

沿系统 (2) 对 $V_i(t)$, $i=1,2,\cdots,7$ 求导,得:

$$\dot V_2(t)=\tau\boldsymbol{x}^{\mathrm T}(t)\bar{\boldsymbol{A}}^{\mathrm T}\boldsymbol{R}_1\bar{\boldsymbol{A}}\boldsymbol{x}(t)-\int_{-\tau}^{0}\boldsymbol{x}^{\mathrm T}(t+\eta)\bar{\boldsymbol{A}}^{\mathrm T}\boldsymbol{R}_1\bar{\boldsymbol{A}}\boldsymbol{x}(t+\eta)\mathrm{d}\eta=\tau\boldsymbol{x}^{\mathrm T}(t)\bar{\boldsymbol{A}}^{\mathrm T}\boldsymbol{R}_1\bar{\boldsymbol{A}}\boldsymbol{x}(t)-I_1,$$

$$\dot V_3(t)=\tau\boldsymbol{x}^{\mathrm T}(t)\bar{\boldsymbol{B}}^{\mathrm T}\boldsymbol{R}_2\bar{\boldsymbol{B}}\boldsymbol{x}(t)-\int_{-\tau}^{0}\boldsymbol{x}^{\mathrm T}(t+\eta-\tau)\bar{\boldsymbol{B}}^{\mathrm T}\boldsymbol{R}_2\bar{\boldsymbol{B}}\boldsymbol{x}(t+\eta-\tau)\mathrm{d}\eta=\tau\boldsymbol{x}^{\mathrm T}(t)\bar{\boldsymbol{B}}^{\mathrm T}\boldsymbol{R}_2\bar{\boldsymbol{B}}\boldsymbol{x}(t)-I_2,$$

$$\dot V_4(t)=\tau f^{\mathrm T}(\boldsymbol{\sigma}(t))\bar{\boldsymbol{D}}^{\mathrm T}\boldsymbol{R}_3\bar{\boldsymbol{D}}f(\boldsymbol{\sigma}(t))-\int_{-\tau}^{0}f^{\mathrm T}(\boldsymbol{\sigma}(t+\eta))\bar{\boldsymbol{D}}^{\mathrm T}\boldsymbol{R}_3\bar{\boldsymbol{D}}f(\boldsymbol{\sigma}(t+\eta))\mathrm{d}\eta=\tau f^{\mathrm T}(\boldsymbol{\sigma}(t))\bar{\boldsymbol{D}}^{\mathrm T}\boldsymbol{R}_3\bar{\boldsymbol{D}}f(\boldsymbol{\sigma}(t))-I_3,$$

$$\dot V_5(t)=\tau f^{\mathrm T}(\boldsymbol{\sigma}(t-h))\bar{\boldsymbol{E}}^{\mathrm T}\boldsymbol{R}_4\bar{\boldsymbol{E}}f(\boldsymbol{\sigma}(t-h))-\int_{-\tau}^{0}f^{\mathrm T}(\boldsymbol{\sigma}(t+\eta-h))\bar{\boldsymbol{E}}^{\mathrm T}\boldsymbol{R}_4\bar{\boldsymbol{E}}f(\boldsymbol{\sigma}(t+\eta-h))\mathrm{d}\eta=\tau f^{\mathrm T}(\boldsymbol{\sigma}(t-h))\bar{\boldsymbol{E}}^{\mathrm T}\boldsymbol{R}_4\bar{\boldsymbol{E}}f(\boldsymbol{\sigma}(t-h))-I_4,$$

$$\dot V_6(t)=f^{\mathrm T}(\boldsymbol{\sigma}(t))\boldsymbol{Q}f(\boldsymbol{\sigma}(t))-f^{\mathrm T}(\boldsymbol{\sigma}(t-h))\boldsymbol{Q}f(\boldsymbol{\sigma}(t-h)),$$

$$\dot V_7(t)=2f^{\mathrm T}(\boldsymbol{\sigma}(t))\boldsymbol{\Lambda}[\boldsymbol{C}^{\mathrm T}\boldsymbol{x}(t)-\boldsymbol{J}f(\boldsymbol{\sigma}(t))].$$

由 $T<0$ 知, $\varepsilon_i\boldsymbol{I}-\boldsymbol{G}_i^{\mathrm T}\boldsymbol{R}_i\boldsymbol{G}_i>0$, $i=1,2,3,4$. 故由引理 2 可得:

$$\bar{\boldsymbol{A}}^{\mathrm T}\boldsymbol{R}_1\bar{\boldsymbol{A}}\leqslant\boldsymbol{A}^{\mathrm T}\boldsymbol{R}_1\boldsymbol{A}+\boldsymbol{A}^{\mathrm T}\boldsymbol{R}_1\boldsymbol{G}_1(\varepsilon_1\boldsymbol{I}-\boldsymbol{G}_1^{\mathrm T}\boldsymbol{R}_1\boldsymbol{G}_1)^{-1}\boldsymbol{G}_1^{\mathrm T}\boldsymbol{R}_1\boldsymbol{A}+\varepsilon_1\boldsymbol{H}_1^{\mathrm T}\boldsymbol{H}_1,$$

$$\bar{\boldsymbol{B}}^{\mathrm T}\boldsymbol{R}_2\bar{\boldsymbol{B}}\leqslant\boldsymbol{B}^{\mathrm T}\boldsymbol{R}_2\boldsymbol{B}+\boldsymbol{B}^{\mathrm T}\boldsymbol{R}_2\boldsymbol{G}_2(\varepsilon_2\boldsymbol{I}-\boldsymbol{G}_2^{\mathrm T}\boldsymbol{R}_2\boldsymbol{G}_2)^{-1}\boldsymbol{G}_2^{\mathrm T}\boldsymbol{R}_2\boldsymbol{B}+\varepsilon_2\boldsymbol{H}_2^{\mathrm T}\boldsymbol{H}_2,$$

$$\bar{\boldsymbol{D}}^{\mathrm T}\boldsymbol{R}_3\bar{\boldsymbol{D}}\leqslant\boldsymbol{D}^{\mathrm T}\boldsymbol{R}_3\boldsymbol{D}+\boldsymbol{D}^{\mathrm T}\boldsymbol{R}_3\boldsymbol{G}_3(\varepsilon_3\boldsymbol{I}-\boldsymbol{G}_3^{\mathrm T}\boldsymbol{R}_3\boldsymbol{G}_3)^{-1}\boldsymbol{G}_3^{\mathrm T}\boldsymbol{R}_3\boldsymbol{D}+\varepsilon_3\boldsymbol{H}_3^{\mathrm T}\boldsymbol{H}_3,$$

$$\bar{\boldsymbol{E}}^{\mathrm T}\boldsymbol{R}_4\bar{\boldsymbol{E}}\leqslant\boldsymbol{E}^{\mathrm T}\boldsymbol{R}_4\boldsymbol{E}+\boldsymbol{E}^{\mathrm T}\boldsymbol{R}_4\boldsymbol{G}_4(\varepsilon_4\boldsymbol{I}-\boldsymbol{G}_4^{\mathrm T}\boldsymbol{R}_4\boldsymbol{G}_4)^{-1}\boldsymbol{G}_4^{\mathrm T}\boldsymbol{R}_4\boldsymbol{E}+\varepsilon_4\boldsymbol{H}_4^{\mathrm T}\boldsymbol{H}_4.$$

取 Lyapunov 函数 $V(t)=\sum_{i=1}^{7}V_i(t)$,则有

$$\dot V(t)=\sum_{i=1}^{7}\dot V_i(t)\leqslant\begin{bmatrix}\boldsymbol{x}(t)\\ f(\boldsymbol{\sigma}(t))\\ f(\boldsymbol{\sigma}(t-h))\end{bmatrix}^{\mathrm T}\boldsymbol{\Xi}\begin{bmatrix}\boldsymbol{x}(t)\\ f(\boldsymbol{\sigma}(t))\\ f(\boldsymbol{\sigma}(t-h))\end{bmatrix},$$

式中

$$\boldsymbol{\Xi}=\begin{bmatrix}\boldsymbol{\Xi}_{11} & \boldsymbol{PD}+\boldsymbol{C\Lambda} & \boldsymbol{PE}\\ \boldsymbol{D}^{\mathrm T}\boldsymbol{P}+\boldsymbol{\Lambda C}^{\mathrm T} & \boldsymbol{\Xi}_{22} & 0\\ \boldsymbol{E}^{\mathrm T}\boldsymbol{P} & 0 & \boldsymbol{\Xi}_{33}\end{bmatrix},$$

$$\boldsymbol{\Xi}_{11}=\boldsymbol{T}_{11}+\boldsymbol{P}\Big(\sum_{i=1}^{4}\mu_i^{-1}\boldsymbol{G}_i\boldsymbol{G}_i^{\mathrm T}+\tau\sum_{i=1}^{4}\eta_i^{-1}\boldsymbol{G}_2\boldsymbol{G}_2^{\mathrm T}+\tau\boldsymbol{B}\sum_{i=1}^{4}(\boldsymbol{R}_i-\eta_i\boldsymbol{H}_2^{\mathrm T}\boldsymbol{H}_2)^{-1}\boldsymbol{B}^{\mathrm T}\Big)\boldsymbol{P}+\tau[\boldsymbol{A}^{\mathrm T}\boldsymbol{R}_1\boldsymbol{G}_1(\varepsilon_1\boldsymbol{I}-\boldsymbol{G}_1^{\mathrm T}\boldsymbol{R}_1\boldsymbol{G}_1)^{-1}\boldsymbol{G}_1^{\mathrm T}\boldsymbol{R}_1\boldsymbol{A}+\boldsymbol{B}^{\mathrm T}\boldsymbol{R}_2\boldsymbol{G}_2(\varepsilon_2\boldsymbol{I}-\boldsymbol{G}_2^{\mathrm T}\boldsymbol{R}_2\boldsymbol{G}_2)^{-1}\boldsymbol{G}_2^{\mathrm T}\boldsymbol{R}_2\boldsymbol{B}],$$

$$\boldsymbol{\Xi}_{22}=\boldsymbol{T}_{22}+\tau\boldsymbol{D}^{\mathrm T}\boldsymbol{R}_3\boldsymbol{G}_3(\varepsilon_3\boldsymbol{I}-\boldsymbol{G}_3^{\mathrm T}\boldsymbol{R}_3\boldsymbol{G}_3)^{-1}\boldsymbol{G}_3^{\mathrm T}\boldsymbol{R}_3\boldsymbol{D},$$

$$\boldsymbol{\Xi}_{33}=\boldsymbol{T}_{33}+\tau\boldsymbol{E}^{\mathrm T}\boldsymbol{R}_4\boldsymbol{G}_4(\varepsilon_4\boldsymbol{I}-\boldsymbol{G}_4^{\mathrm T}\boldsymbol{R}_4\boldsymbol{G}_4)^{-1}\boldsymbol{G}_4^{\mathrm T}\boldsymbol{R}_4\boldsymbol{E}.$$

由 Schur 补[8]知, $T<0$ 等价于 $\boldsymbol{\Xi}<0$. 证毕

若系统的不确定性参数项范数有界,即不确定性矩阵满足

$$\|\Delta\boldsymbol{A}(\theta)\|\leqslant\alpha,\quad \|\Delta\boldsymbol{B}(\theta)\|\leqslant\beta,$$
$$\|\Delta\boldsymbol{D}(\theta)\|\leqslant\gamma,\quad \|\Delta\boldsymbol{E}(\theta)\|\leqslant\delta,$$

则可假定系统(1)中的相关矩阵如下:

$$\boldsymbol{G}_1=\boldsymbol{H}_1=\sqrt{\alpha}\boldsymbol{I}_{n\times n},\boldsymbol{G}_2=\boldsymbol{H}_2=\sqrt{\beta}\boldsymbol{I}_{n\times n},\boldsymbol{G}_3=\sqrt{\gamma}\boldsymbol{I}_{n\times n},$$
$$\boldsymbol{H}_3=\sqrt{\gamma}\boldsymbol{I}_{m\times m},\boldsymbol{G}_4=\sqrt{\delta}\boldsymbol{I}_{n\times n},\boldsymbol{H}_4=\sqrt{\delta}\boldsymbol{I}_{m\times m}.$$

同样可分别由定理 1 和定理 2 得到 Lurie 直接控制系统(1)鲁棒绝对稳定的滞无关条件和滞相关条件.

3 结束语

本文针对具有不确定参数的滞后型 Lurie 直接控制系统,在结构参数及范数有界参数情形下,给出了系统鲁棒绝对稳定的时滞无关及时滞相关条件.这些条件用线性矩阵不等式表示,易于用 MATLAB 中的 LMI 工具箱求解.

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LMI approach for robust absolute stability of Lurie control systems with time-delay and uncertain parameters

XU Bingji[1]
1 School of Information Engineering,China University of Geosciences (Beijing),Beijing 100083

Abstract This paper deals with the problems of robust absolute stability for uncertain time-delay Lurie control systems with structured parameter perturbations and norm bound parameter perturbations respectively.Delay-dependent and delay-independent sufficient conditions for robust absolute stability of the systems are derived by using Lyapunov method.The results are presented in term of linear matrix inequality (LMI) to find the less conservative criteria,which can be easily solved by using Matlab toolbox.
Key words Lurie control systems;robust absolute stability;linear matrix inequality(LMI)