



随机非完整系统控制问题综述和展望

摘要

随着随机非线性控制的发展,随机非完整系统的控制引起了学者们的注意.本文首先探讨了随机非完整控制系统的镇定问题,涉及严反馈链式系统的反馈镇定和不满足严反馈的移动机器人镇定等;其次,介绍了该系统跟踪控制及现状;最后,在总结现有结果的基础上,分析了随机非完整系统发展的趋势,给出了6个可能的研究方向.

关键词

随机非完整系统;镇定;跟踪

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0 引言

非完整系统控制问题的研究已有30余年的历史,它的主要难点在于不存在连续的时不变纯状态反馈镇定器^[1],故需要新的控制和稳定性理论设计控制器.基于文献[1]的结论,Kolmanovsky等^[2]给出了非完整系统能够转化为链式系统的结论,文献[3]探讨了非完整系统不连续反馈控制器的设计方法,这些工作为非完整系统的快速发展打下了坚实的基础.

随机控制的概念^[4]始于1967年,可是由于随机稳定性理论和方法的匮乏,其控制问题的研究一直是一个难点.基于Backstepping方法,文献[5]首次设计了随机严反馈系统的控制器,它为随机控制的发展,特别是为随机严反馈系统控制的发展打下了坚实的基础,至此许多学者将精力投入到随机控制理论的研究中^[6-8].

近10年来,随机控制理论的发展为非完整系统和随机控制的结合起了极大的推动作用,才有了随机非完整系统控制的可行性,并引起了学者们的注意^[9-13].

1 随机非完整系统镇定问题

目前的镇定问题的研究主要为严反馈整链式系统和不满足严反馈随机非完整机器人的镇定,涉及反馈镇定和有限时间镇定等问题.

1.1 状态反馈镇定控制器设计

已有的确定性非完整系统镇定问题的结论,对解决随机非完整系统镇定问题有很大的指导意义.基于Backstepping方法,Ge等^[14]设计了带有强非线性项和不确定参数的自适应状态反馈和输出反馈镇定控制器,Hong等^[15]探讨了不确定非完整系统的有限时间镇定.

基于视觉伺服模型,文献[16]研究了移动机器人的有限时间镇定问题.基于Backstepping技术,文献[17]设计的自适应状态反馈镇定器能使系统状态以概率全局收敛.基于带有不确定参数的随机非完整系统,文献[18-21]给出了自适应律的设计方案.基于文献[22]的结论,Du等^[23]讨论了带有非线性参数的高阶非线性系统的自适应镇定反馈控制器,且该系统的第一个方程为随机微分方程.对于带有马尔科夫切换的随机非完整系统,Zhang等^[24]和Du等^[25]讨论了自适应镇定反馈控制器的设计.文献[26-28]设计了随机非完整系统的有限时间镇定器.文献[29]给出了随机非完整变时滞系

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统的镇定控制器.

1.2 输出反馈镇定控制器设计

文献[30-31]设计了非完整链式系统的输出反馈控制律,其主要原因是系统状态只有部分可测.当随机非完整系统的第一个方程为常微分方程时,文献[32]讨论了其输出反馈镇定问题.如果系统满足线性增长条件,文献[33]给出了高增益观测器,设计了输出反馈控制器.Zhang等^[34]将文献[33]的结果推广至带有马尔可夫切换的情形,设计了系统的输出反馈镇定控制器.

2 随机非完整系统跟踪问题

文献[35-40]讨论了非完整系统控制的跟踪问题.基于递归法,文献[41-42]讨论了确定性非完整链式系统跟踪问题.由上面的讨论可知,随机非完整系统镇定问题研究的结果较多,但是跟踪问题一直是一个难点,主要的原因在于现存的镇定控制器的设计,需要用到状态变换^[41].Zhang等^[43]给出了一类随机非完整动力学的模型,设计了自适应跟踪控制器,该控制器能使跟踪误差任意小,最后给出了一个实际的例子.

3 机器人镇定控制器的设计

基于文献[44]的模型,Wu等^[45]将非完整机器人推广到随机的情形并给出了反馈镇定控制器设计方法,但是此类随机非完整机器人并不满足严格的下三角结构,传统的 Backstepping 方法很难用于这类系统.在文献[45]讨论的基础上,Shang等^[46]和Gao等^[47]分别给出了随机非完整机器人的指数状态反馈控制器和鲁棒状态反馈镇定控制器的设计方法.Zhang等^[48]将基于视觉伺服的非完整机器人推广到随机的情形,给出了状态反馈镇定控制器的设计方法.Hespanha等^[49]将文献[50]中基于不确定参数的非完整移动机器人推广到随机情形,设计的自适应反馈镇定控制器和切换策略能使闭环系统镇定到原点.

4 总结与展望

综上所述,10余年来,随机非完整系统发展的较为迅速,涌现了一批结果,主要可分为镇定和跟踪两个方面.但是关于镇定的结果大都为基于不连续变换的运动学系统的控制器设计,而实际系统由于是物理驱动的,多为动力学系统,故还存在下列尚未

解决的问题.

4.1 动力学链式系统镇定控制器的设计

基于文献[42,51-52]的结果,全部状态可测的满足下三角结构的随机非完整动力学系统可表述为

$$\left. \begin{aligned} dx_0 &= u_0 dt + g_0^T(x_0) d\omega, \\ dx_i &= u_0 x_{i+1} dt + f_i(x_0, \bar{x}_i, \theta) dt + g_i^T(x_0, \bar{x}_i, \theta) d\omega, \\ & i = 1, \dots, n-1, \\ dx_n &= u dt + f_n(x_0, \bar{x}_i, \theta) dt + g_n^T(x_0, \bar{x}_i, \theta) d\omega, \\ \dot{u}_0 &= \tau_1, \\ \dot{u} &= \tau_2. \end{aligned} \right\} \quad (1)$$

1) 讨论不确定系统当状态全部可测时的自适应反馈镇定器;

2) 部分状态可测时,特别是系统(1)中只有状态 (x_0, x_1) 可测时,基于降阶观测器的自适应输出反馈镇定控制器的设计与稳定性分析.

4.2 机器人动力学系统的反馈镇定

首先可将随机非完整机器人运动学系统推广到动力学系统:

$$\left. \begin{aligned} d\theta &= \omega_1 dt + \omega_2 d\omega, \\ dx_c &= v_1 \cos(\theta) dt + v_2 \cos(\theta) d\omega, \\ dy_c &= v_1 \sin(\theta) dt + v_2 \sin(\theta) d\omega, \\ \dot{\omega}_1 &= \tau_1, \\ \dot{v}_1 &= \tau_2. \end{aligned} \right\} \quad (2)$$

然后探讨其状态反馈镇定控制器的设计,如果可行,则可将结果推广到视觉伺服的情形,包含不确定参数和基于力矩输入的情形.

4.3 随机非完整前馈型系统镇定控制器的设计

文献[53]设计了不含随机扰动的前馈型非完整链式系统的有限时间输出状态反馈控制器,并给出了实例;文献[54-56]给出了前馈型非完整系统的饱和镇定控制器的设计方法.但是上述文献都是确定性的系统,那么如何将上述结果推广至随机的情形,并研究其镇定问题?

1) 运动学系统的镇定问题:

$$\left. \begin{aligned} dx_0 &= u_0 dt + g_0^T(x_0) d\omega, \\ dx_1 &= x_2 u_0 dt + g_1^T(x_3, \dots, x_n, u, \theta) d\omega, \\ dx_i &= x_{i+1} u_0 dt + g_i^T(x_{i+2}, \dots, x_n, u, \theta) d\omega, \\ & 2 \leq i \leq n-1, \\ dx_n &= u dt. \end{aligned} \right\} \quad (3)$$

首先看文献[53]中的例子能否推广到随机的情形;其次,讨论系统(3)的状态反馈镇定和基于降阶观测

器的输出反馈镇定问题.

2) 动力学系统的控制问题:

针对全部状态可测的前馈型随机非完整动力学系统

$$\left. \begin{aligned} dx_0 &= u_0 dt + g_0^T(x_0) d\omega, \\ dx_1 &= x_2 u_0 dt + g_1^T(x_3, \dots, x_n, u, \theta) d\omega, \\ dx_i &= x_{i+1} u_0 dt + g_i^T(x_{i+2}, \dots, x_n, u, \theta) d\omega, \\ &2 \leq i \leq n-1, \\ dx_n &= u dt, \\ \dot{u}_0 &= \tau_1, \\ \dot{u} &= \tau_2. \end{aligned} \right\} \quad (4)$$

研究其反馈镇定问题,特别是系统包括不确定该参数和时变系数时自适应控制器的设计和稳定性分析.

3) 将系统(1)——(4)推广到含有马尔科夫切换和任意切换的情形,并讨论控制器的设计.

4.4 有限时间镇定和饱和镇定

参考文献[26-28]的结论,研究随机非完整下三角系统、随机前馈系统和随机移动机器人系统的自适应有限时间镇定问题.但是关于此系统饱和镇定问题的研究较少,基于文献[57-61]的结论,讨论上述三类系统的饱和镇定问题,包含不确定的情形.

4.5 随机非完整系统的跟踪控制问题

目前只有文献[43]给出了随机非完整系统的跟踪问题,并且第一个子系统还是确定性的,但是对于解决随机非完整系统的跟踪问题有很大的借鉴意义.那么,基于下三角结构和前馈型随机非完整系统的跟踪问题,特别是不确定系统和切换系统的跟踪问题的解决将是下一步工作的重点.

4.6 随机非完整系统新的控制方法

现存的关于随机非完整系统镇定问题的文献大都是基于切换策略的,那么能不能找到新的控制方法解决此问题?

1) 基于文献[62-64]的结果,利用滑模变结构方法讨论上述三类随机不确定非完整系统的镇定问题,特别是连续滑模理论的应用.

2) 结合文献[65-67]中光滑时变镇定控制器的设计方法,设计上述三类系统的连续时变反馈控制器.

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Overview and prospect of control for stochastic nonholonomic systems

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Abstract With the development of stochastic nonlinear control, more and more researchers have paid attention to the control of stochastic nonholonomic systems. Firstly, we give the results of stabilization for stochastic nonholonomic systems, which contain state feedback stabilization, output feedback stabilization and stabilization of mobile robots. Secondly, the tracking problems of stochastic nonholonomic systems are introduced. In the end, we analyze the prospect of the control for stochastic nonholonomic systems and give six future research challenges based on above summary.

Key words stochastic nonholonomic systems; stabilization; tracking