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带积分边界条件的奇异方程组边值问题的正解

摘要

考虑了一类具 p-Laplacian 算子型带积分边界条件的三点奇异方程组边值问题正解的存在性.通过使用锥上的不动点定理,在适当的条件下,建立了这类方程组边值问题存在一个或多个正解的充分条件,并给出两个例子来验证主要结果.

关键词

p-Laplacian 算子;方程组;不动点定理;边值问题;正解

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0 引言

奇异边值问题具有广泛的数学与物理学应用背景,它的研究一直受到广大学者的注意.近年来,对多点奇异边值问题[1-4]、带积分边界条件的奇异边值问题[5-6]有很多研究结果,也有很多关于具 p-Laplacian 算子型微分方程边值问题[7-8]的讨论.

文献[1]讨论了三点奇异边值问题:

$$\begin{cases}(\phi_p(u''(t)))'+a(t)f(t,u(t))=0, & 0<t<1,\\ u(0)=u(1)=\alpha u(\eta), & u''(0)=0,\end{cases}\tag{1}$$

得到了存在一个或两个正解的充分条件.

文献[5]研究了 Banach 空间中带积分边值条件的二阶微分方程边值问题:

$$\begin{cases}u''(t)+f(t,u)=0, \quad 0<t<1,\\ u(0)=\int_0^1 g(t)u(t)\mathrm{d}t, u(1)=0,\text{或}\\ \qquad u(0)=0, u(1)=\int_0^1 h(t)u(t)\mathrm{d}t\end{cases}\tag{2}$$

的正解的存在性和不存在性.

文献[7]利用不动点指数定理讨论了一类具 p-Laplacian 算子型方程组边值问题:

$$\begin{cases}(\varphi_p(x'))'+a_1(t)f(x(t),y(t))=0,\\ (\varphi_p(y'))'+a_2(t)g(x(t),y(t))=0,\\ x(0)-\beta_1x'(0)=0, \quad x(1)+\delta_1x'(1)=0,\\ y(0)-\beta_2y'(0)=0, \quad y(1)+\delta_2y'(1)=0\end{cases}\tag{3}$$

(其中 $\varphi_p(x)=|x|^{p-2}x, p>1$)的正解的存在性.

而在非牛顿流体力学、多孔介质中的气体湍流理论等实际问题中,需要借助方程组边值问题来解决.因此,本文致力于研究如下一类具 p-Laplacian 算子型带积分边界条件的奇异方程组边值问题(简记为 BVP):

$$\begin{cases}(\phi_p(u''(t)))'+a_1(t)f(u(t),v(t))=0, & 0<t<1,\\ (\phi_p(v''(t)))'+a_2(t)g(u(t),v(t))=0, & 0<t<1,\\ u(0)=\int_0^1 h_1(s)u(s)\mathrm{d}s, \quad u(1)=\alpha_1u(\eta_1), \quad u''(0)=0,\\ v(0)=\int_0^1 h_2(s)v(s)\mathrm{d}s, \quad v(1)=\alpha_2v(\eta_2), \quad v''(0)=0\end{cases}\tag{4}$$

(其中 $\phi_p(s)=|s|^{p-2}s,p>1,\phi_q=(\phi_p)^{-1},\frac{1}{p}+\frac{1}{q}=1,0<\alpha_i,\eta_i<1(i=1,2)$)的正解的存在性.本文主要利用锥上的不动点定理建立了这类方程组边值问题存在一个或多个正解的充分条件.

1 预备知识

引理1[9-10] 设 X 是赋范实线性空间,$K\subset X$ 是锥,令 $\Omega_1,\Omega_2\subset K$ 为非空相对开集,且 $0\in\Omega_1\subset\bar{\Omega}_1\subset\Omega_2$,设 $F:\bar{\Omega}_2\to K$ 为全连续算子,满足

1) $\|F(x)\|\leqslant\|x\|,\forall x\in\partial\Omega_1;\|F(x)\|\geqslant\|x\|,\forall x\in\partial\Omega_2$,或

2) $\|F(x)\|\geqslant\|x\|,\forall x\in\partial\Omega_1;\|F(x)\|\leqslant\|x\|,\forall x\in\partial\Omega_2$,

则 F 在 $\bar{\Omega}_2\backslash\Omega_1$ 上有不动点.

引理2[9] 设 X 是赋范实线性空间,$K\subset X$ 是锥,记 $K_i=\{x\in K:\|x\|<r_i\}$,其中 $i=1,2,3,r_3>r_2>r_1>0$,设 $F:\bar{K}_3\to K$ 为全连续算子,如果

1) $\|F(x)\|<r_1,\forall x\in\partial K_1;\|F(x)\|>r_2,\forall x\in\partial K_2;\|F(x)\|\leqslant r_3,\forall x\in\partial K_3$,则 F 在 $\bar{K}_3$ 中至少有3个不动点 x_1,x_2,x_3,$\|x_1\|<r_1<\|x_2\|<r_2<\|x_3\|\leqslant r_3$.

2) $\|F(x)\|\geqslant r_1,\forall x\in\partial K_1;\|F(x)\|<r_2,\forall x\in\partial K_2;\|F(x)\|\geqslant r_3,\forall x\in\partial K_3$,则 F 在 $\bar{K}_3$ 中至少有两个不动点 x_2,x_3,$r_1\leqslant\|x_2\|<r_2<\|x_3\|\leqslant r_3$.

引理3[9] 设 $X=C([0,1],R)$,范数由 $\|x\|=\max\limits_{0\leqslant t\leqslant 1}|x(t)|$ 定义,$K=\{x\in X:x(t)\geqslant 0$ 为凹函数$\}$,则对 $\forall\delta\in\left(0,\frac{1}{2}\right)$,当 $u\in K$ 时有 $u(t)\geqslant\delta\|u\|$,$t\in[\delta,1-\delta]$.

令 $E=C[0,1]\times C[0,1]$ 是一个 Banach 空间,定义其范数 $\|(u,v)\|=\max\{\|u\|,\|v\|\}$,其中 $\|u\|=\max\limits_{t\in[0,1]}|u(t)|$,$\|v\|=\max\limits_{t\in[0,1]}|v(t)|$,令 $P=\{(u,v)\in E:u(t),v(t)$ 是非负凹的$\}$,则 P 是 E 中的一个锥.

本文将用到下面两个常数:

$$k=\left[1-\int_0^1\left[1-\frac{1-\alpha}{1-\alpha\eta}t\right]h(t)\mathrm{d}t\right]^{-1},$$

$$k_i=\left[1-\int_0^1\left[1-\frac{1-\alpha_i}{1-\alpha_i\eta_i}t\right]h_i(t)\mathrm{d}t\right]^{-1}(i=1,2).$$

为了方便起见,先列出本文下面用到的两个假设:

(H_1) $f,g\in C([0,+\infty)\times[0,+\infty),[0,+\infty))$,$h_i\in L^1[0,1]$ 是非负的且满足 $k_i>0(i=1,2)$.

(H_2) $a_i:(0,1)\to[0,+\infty)$ 是连续的,$0<\int_0^1 a_i(t)\mathrm{d}t<+\infty$,且 $a_i(t)(i=1,2)$ 在$(0,1)$的任何子区间上不恒为零.

2 引理

引理4 令 $0<\alpha,\eta<1,w\in L^1[0,1],h\in L^1[0,1]$ 是非负的且满足 $k>0$,则 BVP

$$\begin{cases}u''(t)+w(t)=0, & 0<t<1,\\ u(0)=\int_0^1 h(s)u(s)\mathrm{d}s, & u(1)=\alpha u(\eta)\end{cases}\tag{5}$$

有唯一解

$$u(t)=\int_0^1 H(t,s)w(s)\mathrm{d}s,\tag{6}$$

其中

$$H(t,s)=G(t,s)+k\left(1-\frac{1-\alpha}{1-\alpha\eta}t\right)\int_0^1 G(\tau,s)h(\tau)\mathrm{d}\tau,\tag{7}$$

$$G(t,s)=l(t,s)+\frac{\alpha t}{1-\alpha\eta}l(\eta,s),\tag{8}$$

$$l(t,s)=\begin{cases}t(1-s), & 0\leqslant t\leqslant s\leqslant 1,\\ s(1-t), & 0\leqslant s\leqslant t\leqslant 1.\end{cases}\tag{9}$$

证明 首先对方程积分两次可得:

$$u'(t)=-\int_0^t w(s)\mathrm{d}s+c_1,$$

$$u(t)=-\int_0^t\int_0^\tau w(s)\mathrm{d}s\mathrm{d}\tau+c_1t+c_2.\tag{10}$$

结合边值条件有

$$c_1=\frac{1}{1-\alpha\eta}\int_0^1\int_0^\tau w(s)\mathrm{d}s\mathrm{d}\tau-\frac{\alpha}{1-\alpha\eta}\int_0^\eta\int_0^\tau w(s)\mathrm{d}s\mathrm{d}\tau+\frac{\alpha-1}{1-\alpha\eta}\int_0^1 h(s)u(s)\mathrm{d}s,\tag{11}$$

$$c_2=\int_0^1 h(s)u(s)\mathrm{d}s.\tag{12}$$

将式(11),(12)代入式(10),有

$$u(t)=-\int_0^t\int_0^\tau w(s)\mathrm{d}s\mathrm{d}\tau+\frac{t}{1-\alpha\eta}\int_0^1\int_0^\tau w(s)\mathrm{d}s\mathrm{d}\tau-\frac{\alpha t}{1-\alpha\eta}\int_0^\eta\int_0^\tau w(s)\mathrm{d}s\mathrm{d}\tau+\left(1-\frac{1-\alpha}{1-\alpha\eta}t\right)\int_0^1 h(s)u(s)\mathrm{d}s=$$
$$-\int_0^t\int_0^\tau w(s)\mathrm{d}s\mathrm{d}\tau+t\int_0^1\int_0^\tau w(s)\mathrm{d}s\mathrm{d}\tau+\frac{\alpha t}{1-\alpha\eta}\left(-\int_0^\eta\int_0^\tau w(s)\mathrm{d}s\mathrm{d}\tau+\eta\int_0^1\int_0^\tau w(s)\mathrm{d}s\mathrm{d}\tau\right)+$$

$$\left(1-\frac{1-\alpha}{1-\alpha\eta}t\right)\int_0^1 h(s)u(s)\mathrm{d}s=$$
$$\int_0^t s(1-t)w(s)\mathrm{d}s+\int_t^1 t(1-s)w(s)\mathrm{d}s+$$
$$\frac{\alpha t}{1-\alpha\eta}\left(\int_0^\eta s(1-\eta)w(s)\mathrm{d}s+\int_\eta^1 \eta(1-s)w(s)\mathrm{d}s\right)+$$
$$\left(1-\frac{1-\alpha}{1-\alpha\eta}t\right)\int_0^1 h(s)u(s)\mathrm{d}s.$$

结合式(8) 和(9),可得:

$$u(t)=\int_0^1 G(t,s)w(s)\mathrm{d}s+\left(1-\frac{1-\alpha}{1-\alpha\eta}t\right)\int_0^1 h(s)u(s)\mathrm{d}s. \quad (13)$$

在式(13)的两边同乘以 $h(t)$,并对 t 在[0,1]上进行积分可以解得:

$$\int_0^1 h(s)u(s)\mathrm{d}s=k\int_0^1\int_0^1 G(\tau,s)h(\tau)\mathrm{d}\tau w(s)\mathrm{d}s. \quad (14)$$

将式(14)代入式(13),并结合式(7)有

$$u(t)=\int_0^1 H(t,s)w(s)\mathrm{d}s.$$

证毕.

定义算子 $T:P\to P$ 使

$$T(u,v)(t)=(T_1(u,v)(t),T_2(u,v)(t)), \quad (15)$$

其中

$$T_1(u,v)(t)=\int_0^1 H_1(t,s)\phi_q\left(\int_0^s a_1(\tau)f(u(\tau),v(\tau))\mathrm{d}\tau\right)\mathrm{d}s, \quad (16)$$

$$T_2(u,v)(t)=\int_0^1 H_2(t,s)\phi_q\left(\int_0^s a_2(\tau)g(u(\tau),v(\tau))\mathrm{d}\tau\right)\mathrm{d}s, \quad (17)$$

$$H_i(t,s)=G_i(t,s)+k_i\left(1-\frac{1-\alpha_i}{1-\alpha_i\eta_i}t\right)\cdot\int_0^1 G_i(\tau,s)h_i(\tau)\mathrm{d}\tau, \quad (18)$$

$$G_i(t,s)=l(t,s)+\frac{\alpha_i t}{1-\alpha_i\eta_i}l(\eta_i,s), \quad i=1,2. \quad (19)$$

易证 BVP(4) 的解等价于算子 T 在 P 中的不动点.

引理 5 假设(H_1) 与(H_2) 成立,则

$$m_i e(s)\leqslant H_i(t,s)\leqslant M_i e(s), \quad i=1,2, \quad (20)$$

其中函数 $e(s)=s(1-s)$,常数

$$m_i=\frac{\alpha_i(1-\eta_i)(1-\alpha_i\eta_i^2)}{(1-\alpha_i\eta_i)^2}k_i\int_0^1 e(\tau)h_i(\tau)\mathrm{d}\tau,$$

$$M_i=\frac{1+\alpha_i-\alpha_i\eta_i}{1-\alpha_i\eta_i}\left(1+k_i\int_0^1 h_i(\tau)\mathrm{d}\tau\right), \quad i=1,2.$$

证明 首先,由式(8) 和(9) 有

$$G_i(t,s)=l(t,s)+\frac{\alpha_i t}{1-\alpha_i\eta_i}l(\eta_i,s)\geqslant$$
$$e(s)e(t)+\frac{\alpha_i}{1-\alpha_i\eta_i}e(t)e(s)e(\eta_i)\geqslant$$
$$\frac{1-\alpha_i\eta_i^2}{1-\alpha_i\eta_i}e(s)e(t), \quad (21)$$

结合式(21)和(7)可以得到

$$H_i(t,s)=G_i(t,s)+k_i\left(1-\frac{1-\alpha_i}{1-\alpha_i\eta_i}t\right)\int_0^1 G_i(\tau,s)h_i(\tau)\mathrm{d}\tau\geqslant$$
$$k_i\frac{\alpha_i(1-\eta_i)(1-\alpha_i\eta_i^2)}{(1-\alpha_i\eta_i)^2}e(s)\int_0^1 e(\tau)h_i(\tau)\mathrm{d}\tau=$$
$$m_i e(s); \quad (22)$$

其次,由式(8)和(9)有

$$G_i(t,s)=l(t,s)+\frac{\alpha_i t}{1-\alpha_i\eta_i}l(\eta_i,s)\leqslant$$
$$l(t,s)+\frac{\alpha_i}{1-\alpha_i\eta_i}l(\eta_i,s)\leqslant$$
$$e(s)+\frac{\alpha_i}{1-\alpha_i\eta_i}e(s)=\frac{1+\alpha_i-\alpha_i\eta_i}{1-\alpha_i\eta_i}e(s), \quad (23)$$

结合式(23)和(7)可以得到

$$H_i(t,s)=G_i(t,s)+k_i\left(1-\frac{1-\alpha_i}{1-\alpha_i\eta_i}t\right)\int_0^1 G_i(\tau,s)h_i(\tau)\mathrm{d}\tau\leqslant$$
$$G_i(t,s)+k_i\int_0^1 G_i(\tau,s)h_i(\tau)\mathrm{d}\tau\leqslant$$
$$\frac{1+\alpha_i-\alpha_i\eta_i}{1-\alpha_i\eta_i}e(s)+k_i\frac{1+\alpha_i-\alpha_i\eta_i}{1-\alpha_i\eta_i}e(s)\int_0^1 h_i(\tau)\mathrm{d}\tau=$$
$$\frac{1+\alpha_i-\alpha_i\eta_i}{1-\alpha_i\eta_i}\left(1+k_i\int_0^1 h_i(\tau)\mathrm{d}\tau\right)e(s)=$$
$$M_i e(s). \quad (24)$$

因此,结合式(22)和(24),引理 5 证完.

引理 6 假设(H_1)与(H_2)成立,则 $T:P\to P$ 是全连续算子.

证明 对每个 $(u,v)\in P$,由 $T_i(i=1,2)$ 的定义及假设(H_1) 与(H_2),有 $T_i(u,v)\geqslant 0, t\in[0,1]$,且

$$T_1(u,v)'(t)=\int_0^1\frac{\partial}{\partial t}H_1(t,s)\phi_q\left(\int_0^s a_1(\tau)f(u(\tau),v(\tau))\mathrm{d}\tau\right)\mathrm{d}s=$$
$$\int_0^t\left(-s+\frac{\alpha_1}{1-\alpha_1\eta_1}l(\eta_1,s)-k_1\frac{1-\alpha_1}{1-\alpha_1\eta_1}\int_0^1 G(\tau,s)h_1(\tau)\mathrm{d}\tau\right)\cdot$$
$$\phi_q\left(\int_0^s a_1(\tau)f(u(\tau),v(\tau))\mathrm{d}\tau\right)\mathrm{d}s+$$
$$\int_t^1\left(1-s+\frac{\alpha_1}{1-\alpha_1\eta_1}l(\eta_1,s)-k_1\frac{1-\alpha_1}{1-\alpha_1\eta_1}\int_0^1 G(\tau,s)h_1(\tau)\mathrm{d}\tau\right)\cdot$$
$$\phi_q\left(\int_0^s a_1(\tau)f(u(\tau),v(\tau))\mathrm{d}\tau\right)\mathrm{d}s,$$

$$T_1(u,v)''(t)=-\phi_q\left(\int_0^t a_1(\tau)f(u(\tau),v(\tau))\mathrm{d}\tau\right)\leqslant 0,$$
$$t\in[0,1],$$

则 $T_1(u,v)$ 在$[0,1]$ 上是凹的,类似可证 $T_2(u,v)$ 在$[0,1]$ 上也是凹的.因此$T(u,v)\in P$,从而有$T(P)\subseteq P$.

下面证明 T 是全连续的.

第 1 步.对 $\forall M>0$,令 $D=\{(u,v)\in P:\|(u,v)\|\leqslant M\}$ 为P的任意有界集.由f的连续性,记$\tilde{f}_M=\max\limits_{\|(u,v)\|\leqslant M}f(u,v)$ 为一常数,则有

$$|T_1(u,v)(t)|=\left|\int_0^1 H_1(t,s)\phi_q\left(\int_0^s a_1(\tau)f(u(\tau),v(\tau))\mathrm{d}\tau\right)\mathrm{d}s\right|\leqslant$$
$$\phi_q(\tilde{f}_M)\left|\int_0^1 H_1(t,s)\phi_q\left(\int_0^s a_1(\tau)\mathrm{d}\tau\right)\mathrm{d}s\right|\leqslant$$
$$\phi_q(\tilde{f}_M)M_1\int_0^1 e(s)\mathrm{d}s\phi_q\left(\int_0^1 a_1(\tau)\mathrm{d}\tau\right)\leqslant$$
$$\phi_q(\tilde{f}_M)\frac{M_1}{6}\phi_q\left(\int_0^1 a_1(\tau)\mathrm{d}\tau\right)=\text{常数},$$

因此,$T_1(D)$ 是有界的.类似可证$T_2(D)$ 也是有界的,因此 $T(D)$ 是有界的.

第2步.对 $\forall(u,v)\in D$,当$t_1,t_2\in[0,1]$时,记常数

$$L_M=\phi_q\left(\tilde{f}_M\int_0^1 a_1(\tau)\mathrm{d}\tau\right)\left(1+\frac{\alpha_1}{6(1-\alpha_1\eta_1)}+\frac{(1-\alpha_1)(1+\alpha_1-\alpha_1\eta_1)}{6(1-\alpha_1\eta_1)^2}k_1\int_0^1 h_1(\tau)\mathrm{d}\tau\right),$$

有

$$|T_1(u,v)(t_1)-T_1(u,v)(t_2)|=$$
$$\left|\int_0^1(H_1(t_1,s)-H_1(t_2,s))\cdot\phi_q\left(\int_0^s a_1(\tau)f(u(\tau),v(\tau))\mathrm{d}\tau\right)\mathrm{d}s\right|\leqslant$$
$$|t_1-t_2|\phi_q\left(\tilde{f}_M\int_0^1 a_1(\tau)\mathrm{d}\tau\right)\cdot\int_0^1\left(1+l(\eta_1,s)\frac{\alpha_1}{1-\alpha_1\eta_1}+k_1\frac{1-\alpha_1}{1-\alpha_1\eta_1}\int_0^1 G_1(\tau,s)h_1(\tau)\mathrm{d}\tau\right)\mathrm{d}s\leqslant$$
$$|t_1-t_2|\phi_q\left(\tilde{f}_M\int_0^1 a_1(\tau)\mathrm{d}\tau\right)\int_0^1\left(1+\frac{\alpha_1 e(s)}{1-\alpha_1\eta_1}+k_1\frac{(1-\alpha_1)(1+\alpha_1-\alpha_1\eta_1)}{(1-\alpha_1\eta_1)^2}e(s)\int_0^1 h_1(\tau)\mathrm{d}\tau\right)\mathrm{d}s\leqslant$$
$$|t_1-t_2|L_M,$$

因此,$T_1(D)$ 是等度连续的.类似可证 $T_2(D)$ 也是等度连续的,因此 $T(D)$ 是等度连续的.

第 3 步.取$(u_n,v_n)\in D$ 且一致收敛于(u_0,v_0),则由f 的连续性,对 $\forall t,s\in[0,1]$ 有

$$H_1(t,s)\phi_q\left(\int_0^s a_1(\tau)f(u_n(\tau),v_n(\tau))\mathrm{d}\tau\right)\leqslant\frac{M_1}{4}\phi_q\left(\tilde{f}_M\int_0^1 a_1(\tau)\mathrm{d}\tau\right),$$

则由 Lebesgue 控制收敛定理知:

$$\lim_{n\to\infty}T_1(u_n,v_n)(t)=$$
$$\lim_{n\to\infty}\int_0^1 H_1(t,s)\phi_q\left(\int_0^s a_1(\tau)f(u_n(\tau),v_n(\tau))\mathrm{d}\tau\right)\mathrm{d}s=$$
$$\int_0^1 H_1(t,s)\phi_q\left(\int_0^s a_1(\tau)\lim_{n\to\infty}f(u_n(\tau),v_n(\tau))\mathrm{d}\tau\right)\mathrm{d}s=$$
$$\int_0^1 H_1(t,s)\phi_q\left(\int_0^s a_1(\tau)f(u_0(\tau),v_0(\tau))\mathrm{d}\tau\right)\mathrm{d}s=$$
$$T_1(u_0,v_0)(t).$$

故 T_1 是连续的.类似可证 T_2 也是连续的,因此 T 是连续的.

从而 T 是全连续的.证完.

3 主要结果

为了叙述方便,引入以下记号:

$$f_0=\lim_{(u,v)\to(0,0)}\frac{f(u,v)}{\phi_p(u+v)},\quad f_\infty=\lim_{u+v\to+\infty}\frac{f(u,v)}{\phi_p(u+v)},$$
$$g_0=\lim_{(u,v)\to(0,0)}\frac{g(u,v)}{\phi_p(u+v)},\quad g_\infty=\lim_{u+v\to+\infty}\frac{g(u,v)}{\phi_p(u+v)},$$
$$L_i=\int_\delta^{1-\delta}e(s)\phi_q\left(\int_\delta^s a_i(\tau)\mathrm{d}\tau\right)\mathrm{d}s,\quad i=1,2,$$

其中 $\delta\in\left(0,\frac{1}{2}\right)$.

定理1 假设(H_1) 与(H_2) 成立,如果存在常数$R_1\neq R_2$ 使得下列条件成立:

(H_3) 存在常数 $R_1>0,0<b_1<\min\left\{6\left[M_i\phi_q\left(\int_0^1 a_i(\tau)\mathrm{d}\tau\right)\right]^{-1},i=1,2\right\}$,使得

$$f(u,v)\leqslant\phi_p(b_1R_1),g(u,v)\leqslant\phi_p(b_1R_1),$$
$$0\leqslant u,v\leqslant R_1;$$

(H_4) 存在常数$R_2>0,b_2>\max\{[m_iL_i]^{-1},i=1,2\}$,使得

$$f(u,v)\geqslant\phi_p(b_2R_2),g(u,v)\geqslant\phi_p(b_2R_2),$$
$$u+v\in[\delta R_2,2R_2],$$

则 BVP(4) 至少存在一个正解$(u^*,v^*)\in P$ 且满足$\min\{R_1,R_2\}\leqslant\|(u^*,v^*)\|\leqslant\max\{R_1,R_2\}$.

证明 不失一般性,假设$R_1<R_2$.首先,令$\Omega_1=\{(u,v)\in P:\|(u,v)\|<R_1\}$,则对 $\forall(u,v)\in$

$\partial\Omega_1$, $\|(u,v)\| = R_1$, 从而可知 $0 \leqslant u,v \leqslant R_1$, 由 (H_3) 及引理 5 有

$$\|T_1(u,v)\| = \max_{t\in[0,1]}\left|\int_0^1 H_1(t,s)\phi_q\left(\int_0^s a_1(\tau)f(u(\tau),v(\tau))\mathrm{d}\tau\right)\mathrm{d}s\right| \leqslant M_1\int_0^1 e(s)\phi_q\left(\int_0^s a_1(\tau)f(u(\tau),v(\tau))\mathrm{d}\tau\right)\mathrm{d}s \leqslant M_1 b_1 R_1 \frac{1}{6}\phi_q\left(\int_0^1 a_1(\tau)\mathrm{d}\tau\right) < R_1 \leqslant R_1 = \|(u,v)\|, \tag{25}$$

$$\|T_2(u,v)\| = \max_{t\in[0,1]}\left|\int_0^1 H_2(t,s)\phi_q\left(\int_0^s a_2(\tau)g(u(\tau),v(\tau))\mathrm{d}\tau\right)\mathrm{d}s\right| \leqslant M_2\int_0^1 e(s)\phi_q\left(\int_0^s a_2(\tau)g(u(\tau),v(\tau))\mathrm{d}\tau\right)\mathrm{d}s \leqslant M_2 b_1 R_1 \frac{1}{6}\phi_q\left(\int_0^1 a_2(\tau)\mathrm{d}\tau\right) < R_1 \leqslant R_1 = \|(u,v)\|. \tag{26}$$

结合 $\|T(u,v)\|$ 的定义及式(25) 和(26) 有

$$\|T(u,v)\| \leqslant \|(u,v)\|, \forall (u,v) \in \partial\Omega_1. \tag{27}$$

其次,令 $\Omega_2 = \{(u,v) \in P: \|(u,v)\| < R_2\}$, 则对 $\forall (u,v) \in \partial\Omega_2$, $\|(u,v)\| = R_2$.若 $\|u\| = R_2$, 则 $\|v\| \leqslant R_2$;若 $\|v\| = R_2$,则 $\|u\| \leqslant R_2$.为了叙述方便,我们不妨假设 $\|u\| = R_2$,则根据引理 3,对 $\forall t \in [\delta, 1-\delta]$,恒有 $u \geqslant \delta\|u\| = \delta R_2$,从而可知 $\delta R_2 \leqslant u + v \leqslant 2R_2$,根据$(H_4)$ 及引理 5,有

$$\|T_1(u,v)\| = \max_{t\in[0,1]}\left|\int_0^1 H_1(t,s)\phi_q\left(\int_0^s a_1(\tau)f(u(\tau),v(\tau))\mathrm{d}\tau\right)\mathrm{d}s\right| \geqslant m_1\int_0^1 e(s)\phi_q\left(\int_0^s a_1(\tau)f(u(\tau),v(\tau))\mathrm{d}\tau\right)\mathrm{d}s \geqslant m_1\int_\delta^{1-\delta} e(s)\phi_q\left(\int_\delta^s a_1(\tau)f(u(\tau),v(\tau))\mathrm{d}\tau\right)\mathrm{d}s \geqslant m_1 b_2 R_2\int_\delta^{1-\delta} e(s)\phi_q\left(\int_\delta^s a_1(\tau)\mathrm{d}\tau\right)\mathrm{d}s > R_2 = R_2 = \|(u,v)\|, \tag{28}$$

$$\|T_2(u,v)\| = \max_{t\in[0,1]}\left|\int_0^1 H_2(t,s)\phi_q\left(\int_0^s a_2(\tau)g(u(\tau),v(\tau))\mathrm{d}\tau\right)\mathrm{d}s\right| \geqslant m_2\int_0^1 e(s)\phi_q\left(\int_0^s a_2(\tau)g(u(\tau),v(\tau))\mathrm{d}\tau\right)\mathrm{d}s \geqslant m_2\int_\delta^{1-\delta} e(s)\phi_q\left(\int_\delta^s a_2(\tau)g(u(\tau),v(\tau))\mathrm{d}\tau\right)\mathrm{d}s \geqslant m_2 b_2 R_2\int_\delta^{1-\delta} e(s)\phi_q\left(\int_\delta^s a_2(\tau)\mathrm{d}\tau\right)\mathrm{d}s > R_2 = R_2 = \|(u,v)\|. \tag{29}$$

结合 $\|T(u,v)\|$ 的定义及式(28) 和(29) 有

$$\|T(u,v)\| \geqslant \|(u,v)\|, \forall (u,v) \in \partial\Omega_2. \tag{30}$$

结合式(27)和(30),根据引理 1 及引理 6,可知 T 至少存在一个不动点 $(u^*, v^*) \in \bar{\Omega}_2\backslash\Omega_1$, 即为 BVP(4) 的解,且满足 $R_1 \leqslant \|(u^*, v^*)\| \leqslant R_2$.

定理 2 假设(H_1),(H_2) 与(H_3) 成立,且满足

(H_5) $f_0 = \infty$ 或 $g_0 = \infty$;

(H_6) $f_\infty = \infty$ 或 $g_\infty = \infty$.

则 BVP(4) 至少存在两个正解.

证明 首先,令 $\Omega_3 = \{(u,v) \in P: \|(u,v)\| < R_3\}$, 其中 $R_3 < \min\left\{R_1, \frac{R_*}{2}\right\}$. 根据$(H_5)$, 不妨设 $f_0 = \infty$,则对任意的 $b_3 \geqslant \max\{[m_i\delta L_i]^{-1}, i = 1,2\} > 0$,总存在充分小的正数 R_* 使得对于 $\forall 0 \leqslant u,v < R_*$ 总有 $\frac{f(u,v)}{\phi_p(u+v)} \geqslant \phi_p(b_3)$ 成立, 即 $f(u,v) \geqslant \phi_p(b_3(u+v))$.对 $\forall (u,v) \in \partial\Omega_3$, $\|(u,v)\| = R_3$, 从而对 $\forall t \in [\delta, 1-\delta]$,恒有 $\delta R_3 \leqslant u + v \leqslant 2R_3 < R_*$ 成立,根据引理 5 可证

$$\|T_1(u,v)\| = \max_{t\in[0,1]}\left|\int_0^1 H_1(t,s)\phi_q\left(\int_0^s a_1(\tau)f(u(\tau),v(\tau))\mathrm{d}\tau\right)\mathrm{d}s\right| \geqslant m_1\int_o^1 e(s)\phi_q\left(\int_0^s a_1(\tau)f(u(\tau),v(\tau))\mathrm{d}\tau\right)\mathrm{d}s \geqslant m_1\int_\delta^{1-\delta} e(s)\phi_q\left(\int_\delta^s a_1(\tau)f(u(\tau),v(\tau))\mathrm{d}\tau\right)\mathrm{d}s \geqslant m_1 b_3\delta R_3\int_\delta^{1-\delta} e(s)\phi_q\left(\int_\delta^s a_1(\tau)\mathrm{d}\tau\right)\mathrm{d}s \geqslant R_3 = \|(u,v)\|. \tag{31}$$

由 $g_0 = \infty$ 类似可证

$$\|T_2(u,v)\| \geqslant \|(u,v)\|. \tag{32}$$

结合 $\|T(u,v)\|$ 的定义及式(31) 和(32) 有

$$\|T(u,v)\| \geqslant \|(u,v)\|, \forall (u,v) \in \partial\Omega_3. \tag{33}$$

其次,令 $\Omega_4 = \{(u,v) \in P: \|(u,v)\| < R_4\}$, 其中 $R_4 > \max\left\{R_1, \frac{R^*}{\delta}\right\}$.根据$(H_6)$,不妨设 $f_\infty = \infty$, 则对任意的 $b_4 \geqslant \max\{[m_i\delta L_i]^{-1}, i = 1,2\} > 0$,总存在充分大的正数 R^* 使得对任意的 $u + v \geqslant R^*$ 恒有 $f(u,v) \geqslant \phi_p(b_4(u+v))$ 成立.对 $\forall (u,v) \in \partial\Omega_4$, $\|(u,v)\| = R_4$,从而对 $\forall t \in [\delta, 1-\delta]$,恒有 $u + v \geqslant \delta R_4 > R^*$ 成立,从而根据引理 5 可证

$$\|T_1(u,v)\| = \max_{t\in[0,1]}\left|\int_0^1 H_1(t,s)\phi_q\left(\int_0^s a_1(\tau)f(u(\tau),v(\tau))\mathrm{d}\tau\right)\mathrm{d}s\right| \geqslant m_1\int_0^1 e(s)\phi_q\left(\int_0^s a_1(\tau)f(u(\tau),v(\tau))\mathrm{d}\tau\right)\mathrm{d}s \geqslant$$

$$m_1\int_\delta^{1-\delta}e(s)\phi_q\left(\int_\delta^s a_1(\tau)f(u(\tau),v(\tau))\mathrm{d}\tau\right)\mathrm{d}s \geqslant$$
$$m_1 b_4\delta R_4\int_\delta^{1-\delta}e(s)\phi_q\left(\int_\delta^s a_1(\tau)\mathrm{d}\tau\right)\mathrm{d}s \geqslant$$
$$R_4=\|(u,v)\|. \tag{34}$$

由 $g_\infty=\infty$ 类似可证

$$\|T_2(u,v)\|\geqslant\|(u,v)\|. \tag{35}$$

结合 $\|T(u,v)\|$ 的定义及式(34)和(35)有

$$\|T(u,v)\|\geqslant\|(u,v)\|,\forall(u,v)\in\partial\Omega_4. \tag{36}$$

最后,考虑到若 (H_3) 成立,根据定理1前半部分的证明有

$$\|T(u,v)\|<\|(u,v)\|,\forall(u,v)\in\partial\Omega_1. \tag{37}$$

注意到 $R_3<R_1<R_4$,且有式(33),(37)和(36)成立,根据引理2及引理6,可知 T 在 $\overline{\Omega}_4$ 中至少存在两个不动点 (u_1,v_1),(u_2,v_2) 且满足 $R_3\leqslant\|(u_1,v_1)\|<R_1<\|(u_2,v_2)\|\leqslant R_4$,即为BVP(4)的两个正解.

定理3 假设(H_1),(H_2)与(H_4)成立,且满足

$(H_7)f_0=0$ 且 $g_0=0$;

$(H_8)f_\infty=0$ 且 $g_\infty=0$.

则BVP(4)至少存在3个正解.

证明 首先,令 $\Omega_5=\{(u,v)\in P:\|(u,v)\|<R_5\}$,其中 $R_5<\min\{R_2,r_*\}$.根据(H_7),不妨设 $f_0=0$,则对任意的 $0<b_5<\min\left\{3\left[M_i\phi_q\left(\int_0^1 a_i(\tau)\mathrm{d}\tau\right)\right]^{-1},\ i=1,2\right\}$,总存在充分小的正数 r_* 使得对 $\forall 0\leqslant u,v<r_*$ 总有 $\dfrac{f(u,v)}{\phi_p(u+v)}<\phi_p(b_5)$ 成立,即 $f(u,v)<\phi_p(b_5(u+v))$.对 $\forall(u,v)\in\partial\Omega_5$,$\|(u,v)\|=R_5$,从而 $0\leqslant u,v\leqslant R_5<r_*$,$u+v\leqslant 2R_5$,根据引理5可以证得

$$\|T_1(u,v)\|=$$
$$\max_{t\in[0,1]}\left|\int_0^1 H_1(t,s)\phi_q\left(\int_0^s a_1(\tau)f(u(\tau),v(\tau))\mathrm{d}\tau\right)\mathrm{d}s\right|\leqslant$$
$$M_1\int_0^1 e(s)\mathrm{d}s\cdot\phi_q\left(\int_0^s a_1(\tau)f(u(\tau),v(\tau))\mathrm{d}\tau\right)\leqslant$$
$$M_1 b_5 2R_5\frac{1}{6}\phi_q\left(\int_0^1 a_1(\tau)\mathrm{d}\tau\right)<$$
$$R_5=\|(u,v)\|. \tag{38}$$

由 $g_0=0$ 类似可证

$$\|T_2(u,v)\|<\|(u,v)\|. \tag{39}$$

结合 $\|T(u,v)\|$ 的定义及式(38)和(39)有

$$\|T(u,v)\|<\|(u,v)\|,\forall(u,v)\in\partial\Omega_5. \tag{40}$$

其次,令 $\Omega_6=\{(u,v)\in P:\|(u,v)\|<R_6\}$,其中 $R_6>\max\left\{R_2,r^*,\dfrac{N}{2}\right\}$.根据$(H_8)$,不妨设 $f_\infty=0$,则对任意的 $0<b_6<\min\left\{3\left[M_i\phi_q\left(\int_0^1 a_i(\tau)\mathrm{d}\tau\right)\right]^{-1},\ i=1,2\right\}$,总存在充分大的正数 r^* 使得对 $\forall u+v>r^*$ 总有 $\dfrac{f(u,v)}{\phi_p(u+v)}<\phi_p(b_6)$ 成立,即 $f(u,v)<\phi_p(b_6(u+v))$.对 $\forall(u,v)\in\partial\Omega_6$,$\|(u,v)\|=R_6$,从而 $0\leqslant u,v\leqslant R_6$,下面分两种情形来讨论:

(ⅰ)若 f 是有界的,则存在充分大的 $N>0$ 使得 $f(u,v)\leqslant\phi_p(b_6N)$ 成立,则根据引理5有

$$\|T_1(u,v)\|=$$
$$\max_{t\in[0,1]}\left|\int_0^1 H_1(t,s)\phi_q\left(\int_0^s a_1(\tau)f(u(\tau),v(\tau))\mathrm{d}\tau\right)\mathrm{d}s\right|\leqslant$$
$$M_1\int_0^1 e(s)\mathrm{d}s\cdot\phi_q\left(\int_0^s a_1(\tau)f(u(\tau),v(\tau))\mathrm{d}\tau\right)\leqslant$$
$$M_1 b_6 N\frac{1}{6}\phi_q\left(\int_0^1 a_1(\tau)\mathrm{d}\tau\right)\leqslant\frac{N}{2}\leqslant R_6=\|(u,v)\|;$$

(ⅱ)若 f 是无界的,则由 f 的连续性,可取 $R'_6\leqslant R_6$ 使得 $f(u,v)\leqslant f(R_6,R'_6)$,对 $\forall 0\leqslant u,v\leqslant R_6$ 恒成立,则对 $0\leqslant u,v\leqslant R_6$,由于 $R_6+R'_6\geqslant R_6>r^*$,因此可知 $f(u,v)\leqslant f(R_6,R'_6)<\phi_p(2b_6R_6)$,从而根据引理5有

$$\|T_1(u,v)\|=$$
$$\max_{t\in[0,1]}\left|\int_0^1 H_1(t,s)\phi_q\left(\int_0^s a_1(\tau)f(u(\tau),v(\tau))\mathrm{d}\tau\right)\mathrm{d}s\right|\leqslant$$
$$M_1\int_0^1 e(s)\mathrm{d}s\cdot\phi_q\left(\int_0^s a_1(\tau)f(u(\tau),v(\tau))\mathrm{d}\tau\right)\leqslant$$
$$M_1 2b_6R_6\frac{1}{6}\phi_q\left(\int_0^1 a_1(\tau)\mathrm{d}\tau\right)\leqslant R_6=\|(u,v)\|.$$

因此,无论 f 是哪种情形,对 $\forall(u,v)\in\partial\Omega_6$,都可以证得

$$\|T_1(u,v)\|\leqslant\|(u,v)\|. \tag{41}$$

由 $g_\infty=0$ 类似可证

$$\|T_2(u,v)\|\leqslant\|(u,v)\|. \tag{42}$$

结合 $\|T(u,v)\|$ 的定义及式(41)和(42)有

$$\|T(u,v)\|\leqslant\|(u,v)\|,\forall(u,v)\in\partial\Omega_6. \tag{43}$$

最后,考虑到若(H_4)成立,根据定理1后半部分的证明有

$$\|T(u,v)\|>\|(u,v)\|,\forall(u,v)\in\partial\Omega_2. \tag{44}$$

注意到 $R_5<R_2<R_6$,且有式(40),(44)和(43)成立,根据引理2及引理6,可知 T 在 $\overline{\Omega_6}$ 中至少存在三个不动点 (u_1,v_1),(u_2,v_2),(u_3,v_3) 且满足 $\|(u_1,v_1)\|<R_5<\|(u_2,v_2)\|<R_2<\|(u_3,$

$v_3)\| \leqslant R_6$,即为 BVP(4) 的三个正解.

4 相关例子

例 1 考虑下列一类奇异方程组边值问题:

$$\begin{cases}(\phi_p(u''))' + \dfrac{1}{6}t^{-\frac{1}{2}}(u+v)e^{u+v-3} = 0, \quad 0 < t < 1, \\ (\phi_p(v''))' + \dfrac{2}{\pi\sqrt{1-t^2}}\left[\dfrac{\sqrt{2}}{20}(u+v)^{\frac{3}{2}} + \dfrac{1}{5}(u+v)^3\right] = \\ \quad 0, \quad 0 < t < 1, \\ u(0) = \int_0^1 t^2 u(t)\,dt, \quad u(1) = \dfrac{1}{2}u\left(\dfrac{2}{3}\right), \\ v(0) = \int_0^1 tu(t)\,dt, \quad v(1) = \dfrac{1}{3}v\left(\dfrac{3}{4}\right).\end{cases} \tag{45}$$

证明 考虑 $p=3$ 时的情形.不妨取 $\delta = \dfrac{1}{4}$.注意到 $\alpha_1 = \dfrac{1}{2}, \eta_1 = \dfrac{2}{3}, \alpha_2 = \dfrac{1}{3}, \eta_2 = \dfrac{3}{4}, a_1(t) = \dfrac{1}{2}t^{-\frac{1}{2}}$, $a_2(t) = \dfrac{2}{\pi}\dfrac{1}{\sqrt{1-t^2}}, h_1(t) = t^2, h_2(t) = t, f(u,v) = \dfrac{1}{3}(u+v)e^{u+v-3}, g(u,v) = \dfrac{\sqrt{2}}{20}(u+v)^{\frac{3}{2}} + \dfrac{1}{5}(u+v)^3$,经计算可得 $k_1 = \dfrac{48}{41}, k_2 = \dfrac{54}{35}, \int_0^1 a_1(t)\,dt = 1, \int_0^1 a_2(t)\,dt = 1$,因此假设($H_1$)($H_2$) 成立.另一方面,取 $b_1 = 1$, $R_1 = \dfrac{1}{4}$,对 $\forall 0 \leqslant u,v \leqslant \dfrac{1}{4}$,恒有 $f(u,v) \leqslant \dfrac{1}{6}e^{-\frac{5}{2}} < \left(\dfrac{1}{4}\right)^2 = \phi_p(b_1R_1), g(u,v) \leqslant \dfrac{1}{20} < \left(\dfrac{1}{4}\right)^2 = \phi_p(b_1R_1)$ 成立,故假设(H_3) 成立.最后,考虑到

$$f_0 = \lim_{(u,v)\to(0,0)} \frac{\frac{1}{3}(u+v)e^{u+v-3}}{(u+v)^2} = \infty;$$

$$g_0 = \lim_{(u,v)\to(0,0)} \frac{\frac{\sqrt{2}}{20}(u+v)^{\frac{3}{2}} + \frac{1}{5}(u+v)^3}{(u+v)^2} = \infty,$$

故假设(H_5) 成立.而

$$f_\infty = \lim_{u+v\to+\infty} \frac{\frac{1}{3}(u+v)e^{u+v-3}}{(u+v)^2} = \infty,$$

$$g_\infty = \lim_{u+v\to+\infty} \frac{\frac{\sqrt{2}}{20}(u+v)^{\frac{3}{2}} + \frac{1}{5}(u+v)^3}{(u+v)^2} = \infty,$$

故假设(H_6) 成立.因此,根据定理 2,BVP(45) 至少有两个正解.

例 2 考虑奇异边值问题:

$$\begin{cases}(\phi_p(u''))' + 2\,400t^{-\frac{1}{2}}(u+v)^2 e^{20-(u+v)} = 0, \\ \quad 0 < t < 1, \\ (\phi_p(v''))' + 960t^{-\frac{1}{2}}(u^2 + 2v^2)(u+v)e^{60-3(u+v)} = 0, \\ \quad 0 < t < 1, \\ u(0) = \int_0^1 tu(t)\,dt, \quad u(1) = \dfrac{1}{5}u\left(\dfrac{4}{5}\right), \\ v(0) = \int_0^1 u(t)\,dt, \quad v(1) = \dfrac{1}{3}v\left(\dfrac{2}{3}\right).\end{cases} \tag{46}$$

证明 考虑 $p=2$ 时的情形.不妨取 $\delta = \dfrac{1}{4}$.注意到 $\alpha_1 = \dfrac{1}{5}, \eta_1 = \dfrac{4}{5}, \alpha_2 = \dfrac{1}{3}, \eta_2 = \dfrac{2}{3}, a_1(t) = \dfrac{1}{2}t^{-\frac{1}{2}}$, $a_2(t) = \dfrac{1}{2}t^{-\frac{1}{2}}, h_1(t) = t, h_2(t) = 1$,经计算可知 $k_1 = \dfrac{63}{23}, k_2 = \dfrac{7}{3}, \int_0^1 a_1(t)\,dt = 1, \int_0^1 a_2(t)\,dt = 1$,而又有 $f(u,v) = 4\,800(u+v)^2 e^{20-(u+v)}, g(u,v) = 1\,920(u^2 + 2v^2)(u+v)e^{60-3(u+v)}$,故假设($H_1$)($H_2$) 成立,且存在常数 $R_2 = 10$,取 $b_2 = 3\,000$,满足对 $\forall u+v \in [\delta R_2, 2R_2]$,恒有 $f(u,v) \geqslant 4\,800(\delta R_2)^2 e^{20-2R_2} = \phi_p(b_2R_2)$, $g(u,v) \geqslant 1\,920 \times \delta^2 R_2^2 \delta R_2 e^{60-6R_2} = \phi_p(b_2R_2)$ 成立.因此假设(H_4) 成立.最后,考虑到

$$f_0 = \lim_{(u,v)\to(0,0)} \frac{4\,800(u+v)^2 e^{20-(u+v)}}{u+v} = 0,$$

$$g_0 = \lim_{(u,v)\to(0,0)} \frac{1\,920(u^2+2v^2)(u+v)e^{60-3(u+v)}}{u+v} = 0,$$

因此假设(H_7) 成立,而

$$f_\infty = \lim_{u+v\to+\infty} \frac{4\,800(u+v)^2 e^{20-(u+v)}}{u+v} = 0,$$

$$g_\infty = \lim_{u+v\to+\infty} \frac{1\,920(u^2+2v^2)(u+v)e^{60-3(u+v)}}{u+v} = 0,$$

因此可断定假设(H_8) 成立.

综上所述,根据定理 3,边值问题(46)至少存在 3 个正解.

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Positive solutions for singular boundary value systems with integral boundary condition

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Abstract In this paper, we consider the positive solutions for a class of three-point singular boundary value systems with a p-Laplacian operator under integral boundary condition.By using the fixed point theorem,under appropriate conditions,we establish the sufficient conditions for the existence of positive solution about this class of boundary value problem,and give two examples to illustrate our results.

Key words p-Laplacian operator;equation systems;fixed point theorem;boundary value problem;positive solution