文章编号:1674-7070(2015)02-0184-05

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关于随机分布参数系统的变结构控制研究

摘要

研究了一类 Itô 型随机时滞分布参数系统的滑动模控制问题,设计了该系统的变结构控制器,证明了系统的滑动模运动的存在性,并分析了在滑动切换面上滑动模控制系统关于不确定量的不变性特征及运动稳定性.与相关论文相比,所获结论与扩散系数相关,在实际应用中更具实践意义.运用文中方法,类似论文结论均可进行改进.

关键词

随机驱动;分布参数;变结构控制;扩散系数

中图分类号 TP13
文献标志码 A

收稿日期 2015-03-15
资助项目 国家自然科学基金(61340042);湖北省自然科学基金(2012FFB4102)

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0 引言

滑模变结构控制的研究起源于 20 世纪 50 年代,如今已成为相对独立的科学研究方向.变结构控制[1]可以将一个复杂的高阶系统归结成两个低阶的、相对较简单的问题,是一种比较容易实现的控制系统的综合方法.近年来,变结构控制的应用越来越广泛[2-4].阅读相关文献不难发现,关于分布参数系统的变结构控制结论大都与扩散系数相关,而随机分布参数系统的变结构控制的部分结论与分布参数系数无关[5],显然,相关更有意义.仔细分析发现,造成与扩散系数无关的主要原因是由于在运用格林公式与边界条件后,含有分布参数系数的项恒为负,通常被直接舍去,从而造成估计不精细.本文将适当改进相关证明,使得不等式估计更加精细,即可获得与扩散系数相关的结论.

1 系统描述

考虑随机时滞分布参数系统

$$\mathrm{d}\boldsymbol{v}(\boldsymbol{x},t)=[d\Delta\boldsymbol{v}(x,t)+\boldsymbol{A}\boldsymbol{v}(\boldsymbol{x},t)+\boldsymbol{E}\boldsymbol{v}(\boldsymbol{x},t-\tau)+\boldsymbol{B}\boldsymbol{u}(\boldsymbol{x},t)]\mathrm{d}t+\sum_{i=1}^{m}\sigma_i\boldsymbol{F}_i\boldsymbol{v}(\boldsymbol{x},t)\mathrm{d}W_i(t),\quad(\boldsymbol{x},t)\in\Omega\times\mathbf{R}^+,\tag{1}$$

这里 $\boldsymbol{v}(\boldsymbol{x},t)\in\mathbf{R}^n$;$\boldsymbol{u}\in\mathbf{R}^r$;$\boldsymbol{A},\boldsymbol{E},\boldsymbol{F}_i\in\mathbf{R}^{n\times n}$;$\boldsymbol{B}\in\mathbf{R}^{n\times r}$,$\boldsymbol{B}$ 是列满秩的;$\sigma_i\in\mathbf{R}$;$d>0$ 为常数;$\Delta=\sum_{i=1}^{m}\frac{\partial^2}{\partial x_i^2}$ 是 Ω 上的 Laplace 扩散算子;$\boldsymbol{W}(t)=(W_1,\cdots,W_m)^{\mathrm{T}}$ 是定义在完备的概率空间 $(\Omega,\mathscr{F},(\mathscr{F}_t)_{t\in I},\mathbf{P})$ 上具自然流 $\{\mathscr{F}_t\}_{t\geqslant0}$ 的 m 维 Brown 运动;时滞 $\tau>0$ 为常数;$\Omega=\{\boldsymbol{x},\|\boldsymbol{x}\|<l<+\infty\}\subset\mathbf{R}^s$ 是具有光滑边界 $\partial\Omega$ 的有界区域,$\mathbf{R}_\zeta^+=[\zeta,+\infty)$.

考虑初值条件

$$\boldsymbol{v}(\boldsymbol{x},t)=\varphi(\boldsymbol{x},t),\quad(\boldsymbol{x},t)\in\Omega\times[-\tau,0]\tag{2}$$

与 Neuman 型边值条件

$$\frac{\partial\boldsymbol{v}(\boldsymbol{x},t)}{\partial\boldsymbol{N}}=0,\quad(\boldsymbol{x},t)\in\partial\Omega\times\mathbf{R}_\tau^+,\tag{3}$$

或者 Dirichlet 型边值条件

$$\boldsymbol{v}(t,\boldsymbol{x})=\boldsymbol{0},\quad(t,\boldsymbol{x})\in\mathbf{R}^+\times\partial\Omega,\tag{4}$$

其中,$\varphi(\boldsymbol{x},t)$ 是适当光滑的已知函数.

关于由偏微分方程描述的随机系统的研究工作不多,文献[6-8]将偏微分方程的相关研究方法应用于随机偏微分方程,对相应的随机解域进行了定性分析,文献[9]在建立比较定理的基础上,讨论了随机偏微分方程依概率稳定等问题,文献[10]则研究了随机分布参数系统的最优控制问题.

关于系统(1),(2),(3)或(1),(2),(4)的解的定义参见文献[11],解的存在唯一性问题参见文献[6].为叙述方便,本文引入下述定义.

定义 1 称系统(1),(2),(3)或(1),(2),(4)的解 $\boldsymbol{v}(\boldsymbol{x},t)=(v_1(\boldsymbol{x},t),\cdots,v_n(\boldsymbol{x},t))^{\mathrm{T}}$ 是均方指数稳定的,如果存在常数 $\rho>0,\alpha>0$,使得 $\mathbf{E}(\|\boldsymbol{v}(\boldsymbol{x},t)\|^2)\leqslant\rho(\|\boldsymbol{\phi}(\boldsymbol{x})\|^2)\mathrm{e}^{-\alpha t}$ 依概率几乎必然成立.其中,$\boldsymbol{\phi}(\boldsymbol{x})=(\phi_1(\boldsymbol{x}),\cdots,\phi_n(\boldsymbol{x}))^{\mathrm{T}},\boldsymbol{x}\in\Omega$.

以下约定所有的等式与不等式都是依概率几乎必然成立,不再标明.

2 变结构控制器

根据已知条件,可设 $\det(\boldsymbol{B}_2)\neq0$,其中,$\boldsymbol{B}=[\boldsymbol{B}_1,\boldsymbol{B}_2]^{\mathrm{T}},\boldsymbol{B}_1\in\mathbf{R}^{(n-r)\times r},\boldsymbol{B}_2\in\mathbf{R}^{r\times r}$.

对系统(1)作非线性变换 $\boldsymbol{T}\boldsymbol{p}(\boldsymbol{x},t)=v(\boldsymbol{x},t)$,其中 $\boldsymbol{T}=\begin{bmatrix}\boldsymbol{I}_{n-r} & \boldsymbol{B}_1\boldsymbol{B}_2^{-1}\\ \boldsymbol{0} & \boldsymbol{I}_r\end{bmatrix}$.于是,系统(1)可转化为

$$\mathrm{d}\boldsymbol{p}(x,t)=[\tilde{d}\Delta\boldsymbol{p}(\boldsymbol{x},t)+\tilde{\boldsymbol{A}}\boldsymbol{p}(\boldsymbol{x},t)+\tilde{\boldsymbol{E}}\boldsymbol{p}(\boldsymbol{x},t-\tau)+\tilde{\boldsymbol{B}}\boldsymbol{u}(\boldsymbol{x},t)]\mathrm{d}t+\sum_{i=1}^{m}\sigma_i\tilde{\boldsymbol{F}}_i\boldsymbol{v}(\boldsymbol{x},t)\mathrm{d}W_i(t),\qquad(2)$$

$\tilde{d}=\boldsymbol{T}^{-1}d\boldsymbol{T}=d\boldsymbol{T}^{-1}\boldsymbol{T}=d,\tilde{\boldsymbol{B}}=\boldsymbol{T}^{-1}\boldsymbol{B}=[\boldsymbol{0},\boldsymbol{B}_2]^{\mathrm{T}},\tilde{\boldsymbol{A}}=\boldsymbol{T}^{-1}\boldsymbol{A}\boldsymbol{T}$,$\tilde{\boldsymbol{E}}=\boldsymbol{T}^{-1}\boldsymbol{E}\boldsymbol{T},\tilde{\boldsymbol{F}}_i=\boldsymbol{T}^{-1}\boldsymbol{F}_i\boldsymbol{T}$.

记 $\tilde{\boldsymbol{A}}=\begin{bmatrix}\boldsymbol{A}_{11} & \boldsymbol{A}_{12}\\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22}\end{bmatrix}$,$\tilde{\boldsymbol{E}}=\begin{bmatrix}\boldsymbol{E}_{11} & \boldsymbol{E}_{12}\\ \boldsymbol{E}_{21} & \boldsymbol{E}_{22}\end{bmatrix}$,$\tilde{\boldsymbol{F}_i}=\begin{bmatrix}\boldsymbol{F}_{i11} & \boldsymbol{F}_{i12}\\ \boldsymbol{F}_{i21} & \boldsymbol{F}_{i22}\end{bmatrix}$,其中矩阵 $\boldsymbol{A}_{22},\boldsymbol{E}_{22},\boldsymbol{F}_{i22}\in\mathbf{R}^{r\times r},\boldsymbol{A}_{11},\boldsymbol{E}_{11},\boldsymbol{F}_{i11}\in\mathbf{R}^{(n-r)\times(n-r)},\boldsymbol{A}_{12},\boldsymbol{E}_{12},\boldsymbol{F}_{i12}\in\mathbf{R}^{(n-r)\times r},\boldsymbol{A}_{21},\boldsymbol{E}_{21},\boldsymbol{F}_{i21}\in\mathbf{R}^{r\times(n-r)}$.于是,系统(2)又可改写为

$$\begin{cases}\mathrm{d}\boldsymbol{p}_1(\boldsymbol{x},t)=[d\Delta\boldsymbol{p}_1(\boldsymbol{x},t)+\boldsymbol{A}_{11}\boldsymbol{p}_1(\boldsymbol{x},t)+\\ \quad\boldsymbol{A}_2\boldsymbol{p}_2(\boldsymbol{x},t)+\boldsymbol{E}_{11}\boldsymbol{p}_1(\boldsymbol{x},t-\tau)+\\ \quad\boldsymbol{E}_{12}\boldsymbol{p}_2(\boldsymbol{x},t-\tau)]\mathrm{d}t+\sum_{i=1}^{m}\sigma_i(\boldsymbol{F}_{i11}\boldsymbol{p}_1+\boldsymbol{F}_{i12}\boldsymbol{p}_2)\mathrm{d}W_i(t),\\ \mathrm{d}\boldsymbol{p}_2(\boldsymbol{x},t)=[d\Delta\boldsymbol{p}_2(\boldsymbol{x},t)+\boldsymbol{A}_{21}\boldsymbol{p}_1(\boldsymbol{x},t)+\\ \quad\boldsymbol{A}_{22}\boldsymbol{p}_2(\boldsymbol{x},t)+\boldsymbol{E}_{21}\boldsymbol{p}_1(\boldsymbol{x},t-\tau)+\\ \quad\boldsymbol{E}_{22}\boldsymbol{p}_2(\boldsymbol{x},t-\tau)+\boldsymbol{B}_2\boldsymbol{u}(\boldsymbol{x},t)]\mathrm{d}t+\\ \quad\sum_{i=1}^{m}\sigma_i(\boldsymbol{F}_{i21}\boldsymbol{p}_1+\boldsymbol{F}_{i22}\boldsymbol{p}_2)\mathrm{d}W_i(t).\end{cases}\qquad(5)$$

设切换函数为

$$\boldsymbol{S}(\boldsymbol{x},t)=\boldsymbol{C}_1\boldsymbol{p}_1(\boldsymbol{x},t)+\boldsymbol{C}_2\boldsymbol{p}_2(\boldsymbol{x},t),\qquad(6)$$

其中 $\boldsymbol{C}_1\in\mathbf{R}^{r\times(n-r)},\boldsymbol{C}_2\in\mathbf{R}^{r\times r}$ 皆为待定矩阵.于是,切换面为 $\boldsymbol{S}_0=\{(\boldsymbol{p}_1,\boldsymbol{p}_2)^{\mathrm{T}}\mid\boldsymbol{C}_1\boldsymbol{p}_1+\boldsymbol{C}_2\boldsymbol{p}_2=\boldsymbol{0}\}$.

现取 $\boldsymbol{C}_2$ 可逆,则在切换面 $\boldsymbol{S}_0$ 上有 $\boldsymbol{p}_2=-\boldsymbol{C}_2^{-1}\boldsymbol{C}_1\boldsymbol{p}_1$,代入式(5)的第一式,得到滑动模运动方程

$$\mathrm{d}\boldsymbol{p}_1(\boldsymbol{x},t)=[d\Delta\boldsymbol{p}_1(\boldsymbol{x},t)+\boldsymbol{G}\boldsymbol{p}_1(\boldsymbol{x},t)+\boldsymbol{H}\boldsymbol{p}_1(\boldsymbol{x},t-\tau)]\mathrm{d}t+\sum_{i=1}^{m}\sigma_i\boldsymbol{H}_i\boldsymbol{p}_1\mathrm{d}W_i(t),\qquad(7)$$

这里,$\boldsymbol{G}=\boldsymbol{A}_{11}-\boldsymbol{A}_{12}\boldsymbol{C}_2^{-1}\boldsymbol{C}_1,\boldsymbol{H}=\boldsymbol{E}_{11}-\boldsymbol{E}_{12}\boldsymbol{C}_2^{-1}\boldsymbol{C}_1,\boldsymbol{H}_i=\boldsymbol{F}_{i11}-\boldsymbol{F}_{i12}\boldsymbol{C}_2^{-1}\boldsymbol{C}_1$.

设计变结构控制器 $\boldsymbol{u}(x,t)$ 为

$$\begin{aligned}\boldsymbol{u}(\boldsymbol{x},t)=&(\boldsymbol{C}_2\boldsymbol{B}_2)^{-1}[-k\boldsymbol{S}(\boldsymbol{x},t)-\\&(\boldsymbol{C}_1\boldsymbol{A}_{11}+\boldsymbol{C}_2\boldsymbol{A}_{21})\boldsymbol{p}_1(\boldsymbol{x},t)-(\boldsymbol{C}_1\boldsymbol{A}_{12}+\boldsymbol{C}_2\boldsymbol{A}_{22})\boldsymbol{p}_2(\boldsymbol{x},t)-\\&(\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{x},t))^{-1}\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{x},t-\tau)\boldsymbol{K}\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{x},t-\tau)],\end{aligned}\qquad(8)$$

其中,$k>d$ 为常数,$\boldsymbol{K}=\mathrm{diag}(k_1,\cdots,k_n)$ 为待定矩阵.系统(1)在变结构控制器(8)作用下的闭环系统是

$$\mathrm{d}\boldsymbol{p}(\boldsymbol{x},t)=[\tilde{d}\Delta\boldsymbol{p}(\boldsymbol{x},t)+\boldsymbol{f}(\boldsymbol{p}(\boldsymbol{x},t),\boldsymbol{p}(\boldsymbol{x},t-\tau))]\mathrm{d}t+\sum_{i=1}^{m}\sigma_i\tilde{\boldsymbol{F}}_i\boldsymbol{v}(\boldsymbol{x},t)\mathrm{d}W_i(t),\qquad(9)$$

其中,$\boldsymbol{f}(\cdot,\cdot)\in\mathbf{R}^n$.

3 滑动模运动的可达性

对于随机系统,滑动曲面仍为 $\boldsymbol{S}_0=0$,只是闭环系统的运动达到滑动模上的程度须用概率、数学期望等数字特征来刻画.因此,对随机系统来说,滑动模的可达性的证明比较困难,为此,参见文献[12],给出如下定义.

定义 2 若对系统(9)从任意位置$(t_0,\boldsymbol{p}_0)$出发的运动 $\boldsymbol{p}(\boldsymbol{x},t,t_0,\boldsymbol{p}_0)$,存在有限时间 $T>0$,使当 $t-t_0>T$ 时,

$$\mathbf{E}\|\boldsymbol{S}(\boldsymbol{x},t)\|=0,\quad\mathbf{E}\|\boldsymbol{S}(\boldsymbol{x},t)\|^2=0,\qquad(10)$$

则称滑动模具有可达性;若存在有限时间 $T>0$,使当 $t-t_0>T$ 时,

$$\mathbf{E}\|\boldsymbol{S}(\boldsymbol{x},t)\|=0,\quad\mathbf{E}\|\boldsymbol{S}(\boldsymbol{x},t)\|^2\to0\ (t\to+\infty),\qquad(11)$$

则称滑动模具有次可达性.

先给出一个引理.

引理 1[7] 对任意的 $\boldsymbol{x}\in\Omega$,有

$$\bar{\boldsymbol{Q}}=\bar{\boldsymbol{Q}}(\boldsymbol{x},t)=\boldsymbol{Q}^{\mathrm{T}}(\boldsymbol{x},t)\Delta\boldsymbol{Q}(\boldsymbol{x},t)-\boldsymbol{Q}^{\mathrm{T}}(\boldsymbol{x},t)\boldsymbol{Q}(\boldsymbol{x},t)\leqslant0.\qquad(12)$$

证明 记 $\Omega_1=\{\boldsymbol{x}\mid\bar{\boldsymbol{Q}}(\boldsymbol{x},t)>0,\boldsymbol{x}\in\Omega\}$,$\Omega_2=\{\boldsymbol{x}\mid\bar{Q}(\boldsymbol{x},t)\leqslant0,\boldsymbol{x}\in\Omega\}$,并假设 Ω_1 非空,显然有 $\partial\Omega_1=[\partial\Omega\cap\partial\Omega_1]\cup[\Omega\cap\partial\Omega_1]$,且 $\bar{\boldsymbol{Q}}(\boldsymbol{x},t)=0,\boldsymbol{x}\in\partial\Omega_1$.

于是

$$0 < \int_{\Omega_1} \bar{\boldsymbol{Q}}(\boldsymbol{x},t)\mathrm{d}\boldsymbol{x} =$$

$$\int_{\Omega_1} \boldsymbol{Q}^{\mathrm{T}}(\boldsymbol{x},t)\Delta\boldsymbol{Q}(\boldsymbol{x},t)\mathrm{d}\boldsymbol{x} - \int_{\Omega_1}\boldsymbol{Q}^{\mathrm{T}}(\boldsymbol{x},t)\boldsymbol{Q}(\boldsymbol{x},t)\mathrm{d}\boldsymbol{x} =$$

$$\boldsymbol{Q}^{\mathrm{T}}(\boldsymbol{x},t)\nabla\boldsymbol{Q}(\boldsymbol{x},t)\Big|_{\partial\Omega_1} - \int_{\Omega_1}(\nabla\boldsymbol{Q}(\boldsymbol{x},t))^{\mathrm{T}}\nabla\boldsymbol{Q}(\boldsymbol{x},t)\mathrm{d}\boldsymbol{x} -$$

$$\int_{\Omega_1}\boldsymbol{Q}^{\mathrm{T}}(\boldsymbol{x},t)\boldsymbol{Q}(\boldsymbol{x},t)\mathrm{d}\boldsymbol{x}.$$

若 $\boldsymbol{x}\in\partial\Omega\cap\partial\Omega_1$,则 $\boldsymbol{x}\in\Omega$.于是,利用边值条件(3)可得 $\boldsymbol{Q}^{\mathrm{T}}(\boldsymbol{x},t)\nabla\boldsymbol{Q}(\boldsymbol{x},t)\Big|_{\partial\Omega_1}=0$.这与式(12)矛盾.若 $\boldsymbol{x}\in\Omega\cap\partial\Omega_1$,则 $\boldsymbol{x}\in\partial\Omega$.由于 $\bar{\boldsymbol{Q}}(\boldsymbol{x},t)=0(\boldsymbol{x}\in\partial\Omega_1)$,则从 $\bar{\boldsymbol{Q}}(\boldsymbol{x},t)$ 的定义可知

$$\Delta\boldsymbol{Q}(\boldsymbol{x},t)=\boldsymbol{Q}(\boldsymbol{x},t),\boldsymbol{x}\in\partial\Omega_1.$$

引理 1 证毕.

记 $\boldsymbol{U}\triangleq(\boldsymbol{C}_1\boldsymbol{E}_{12}+\boldsymbol{C}_2\boldsymbol{E}_{22})\boldsymbol{C}_2^{-1}$.

定理 1 假设下列条件成立:

(Ⅰ) $\boldsymbol{p}(\boldsymbol{x},t)$ 在 $\Omega\times[0,+\infty)$ 上有界;

(Ⅱ) 存在 $\boldsymbol{C}_1,\boldsymbol{C}_2$,使得 $\boldsymbol{C}_1\boldsymbol{E}_{11}+\boldsymbol{C}_2\boldsymbol{E}_{21}=(\boldsymbol{C}_1\boldsymbol{E}_{12}+\boldsymbol{C}_2\boldsymbol{E}_{22})\boldsymbol{C}_2^{-1}\boldsymbol{C}_1$,同时,存在 $k_1,\cdots,k_n>0$,使得矩阵

$$\widehat{\boldsymbol{U}}=\begin{pmatrix}-2(k-d+\eta)\boldsymbol{I} & \boldsymbol{U}\\ \boldsymbol{U}^{\mathrm{T}} & -\eta\boldsymbol{I}\end{pmatrix}$$

负定,并设其最大特征值为 $-\lambda$;

(Ⅲ) 存在 $\beta>0$,使得 $\lambda-\beta+\tau\mathrm{e}^{\beta\tau}>0$,则在变结构控制器(8)的作用下,整个切换面 S_0 均为滑动模态区,变结构控制系统(9)的滑动模具有次可达性.

证明 因为

$$\mathrm{d}\boldsymbol{S}(\boldsymbol{x},t)=\boldsymbol{C}_1\mathrm{d}\boldsymbol{p}_1(\boldsymbol{x},t)+\boldsymbol{C}_2\mathrm{d}\boldsymbol{p}_2(\boldsymbol{x},t)=$$
$$[d\Delta\boldsymbol{S}-k\boldsymbol{S}+(\boldsymbol{C}_1\boldsymbol{E}_{11}+\boldsymbol{C}_2\boldsymbol{E}_{21})\boldsymbol{p}_1(\boldsymbol{x},t-\tau)+$$
$$(\boldsymbol{C}_1\boldsymbol{E}_{12}+\boldsymbol{C}_2\boldsymbol{E}_{22})\boldsymbol{p}_2(\boldsymbol{x},t-\tau)]\mathrm{d}t+\sum_{i=1}^{m}\sigma_i\hat{\boldsymbol{F}}_i\mathrm{d}W_i(t),$$

其中,$\hat{\boldsymbol{F}}_i=\boldsymbol{C}_1(\boldsymbol{F}_{i11}\boldsymbol{p}_1+\boldsymbol{F}_{i12}\boldsymbol{p}_2)+\boldsymbol{C}_2(\boldsymbol{F}_{i21}\boldsymbol{p}_1+\boldsymbol{F}_{i22}\boldsymbol{p}_2)$.

考虑辅助泛函

$$\mathscr{G}(\boldsymbol{x},t)=\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{x},t)\boldsymbol{S}(\boldsymbol{x},t)+\eta\int_{-\tau}^{0}\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{x},t+\zeta)\boldsymbol{S}(\boldsymbol{x},t+\zeta)\mathrm{d}\zeta,\tag{13}$$

并注意引理 1 有

$$\mathrm{d}\mathscr{G}(\boldsymbol{x},t)=2\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{x},t)\mathrm{d}\boldsymbol{S}(\boldsymbol{x},t)+$$
$$[\eta\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{x},t)\boldsymbol{S}(\boldsymbol{x},t)-\eta\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{x},t-\tau)\boldsymbol{S}(\boldsymbol{x},t-\tau)]\mathrm{d}t=$$
$$[2\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{x},t)d\Delta\boldsymbol{S}(\boldsymbol{x},t)-2k\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{x},t)\boldsymbol{S}(\boldsymbol{x},t)+$$
$$2\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{x},t)\boldsymbol{U}\boldsymbol{S}(\boldsymbol{x},t-\tau)+\eta\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{x},t)\boldsymbol{S}(\boldsymbol{x},t)-$$
$$\eta\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{x},t-\tau)\boldsymbol{S}(\boldsymbol{x},t-\tau)]\mathrm{d}t+\sum_{i=1}^{m}\sigma_i\hat{\boldsymbol{F}}_i\mathrm{d}W_i(t)=$$
$$[2d\bar{\boldsymbol{S}}(\boldsymbol{x},t)-2(k-d+\eta)\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{x},t)\boldsymbol{S}(x,t)+$$
$$2\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{x},t)\boldsymbol{U}\boldsymbol{S}(\boldsymbol{x},t-\tau)-\eta\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{x},t-\tau)\boldsymbol{S}(\boldsymbol{x},t-\tau)]\mathrm{d}t+$$
$$\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{x},t)\sum_{i=1}^{m}\sigma_i\hat{\boldsymbol{F}}_i\mathrm{d}W_i(t)\leqslant$$
$$(\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{x},t),\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{x},t-\tau))\widehat{\boldsymbol{U}}(\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{x},t),\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{x},t-\tau))^{\mathrm{T}}\mathrm{d}t+$$
$$\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{x},t)\sum_{i=1}^{m}\sigma_i\hat{\boldsymbol{F}}_i\mathrm{d}W_i(t).\tag{14}$$

取 $\eta\in(0,\lambda-\beta+\tau\mathrm{e}^{\beta\tau})$,考虑辅助函数

$$V(t,\boldsymbol{Z})=\mathrm{e}^{\beta t}\boldsymbol{Z}(t)=\mathrm{e}^{\beta t}\int_{\Omega}\mathscr{G}(\boldsymbol{x},t)\mathrm{d}\boldsymbol{x},$$

由 Itô 公式,并注意式(13),(14),有

$$\mathrm{d}(\mathrm{e}^{\beta t}\boldsymbol{Z}(t))=\mathrm{e}^{\beta t}\Big[\beta\|\boldsymbol{S}(\boldsymbol{x},t)\|^2+$$
$$\beta\eta\int_{\Omega}\int_{-\tau}^{0}\boldsymbol{S}^2(\boldsymbol{x},t+\zeta)\mathrm{d}\zeta\mathrm{d}\boldsymbol{x}+\eta\int_{\Omega}(\boldsymbol{S}^2(\boldsymbol{x},t)-$$
$$\boldsymbol{S}^2(\boldsymbol{x},t-\tau))\mathrm{d}\boldsymbol{x}\Big]\mathrm{d}t+\mathrm{e}^{\beta t}d\|\boldsymbol{S}(\boldsymbol{x},t)\|^2\leqslant$$
$$\mathrm{e}^{\beta t}\Big\{\Big[-(\lambda-\beta-\eta)\|\boldsymbol{S}(\boldsymbol{x},t)\|^2-(\lambda+\eta)\|\boldsymbol{S}(\boldsymbol{x},t-\tau)\|^2+$$
$$\beta\eta\int_{\Omega}\int_{-\tau}^{0}\boldsymbol{S}^2(\boldsymbol{x},t+\zeta)\mathrm{d}\zeta\mathrm{d}\boldsymbol{x}\Big]\mathrm{d}t+$$
$$\int_{\Omega}\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{x},t)\sum_{i=1}^{m}\sigma_i\hat{\boldsymbol{F}}_i\mathrm{d}W_i(t)\mathrm{d}\boldsymbol{x}\Big\}.$$

从 0 到任意的 $T>0$ 积分上面不等式两边,且取数学期望便有

$$\mathbf{E}(\mathrm{e}^{\beta T}\boldsymbol{Z})\leqslant\rho_1+\mathbf{E}\Big[-(\lambda-\beta-\eta)\int_0^{\mathrm{T}}\mathrm{e}^{\beta t}\|\boldsymbol{S}(\boldsymbol{x},t)\|^2\mathrm{d}t-$$
$$(\lambda+\eta)\int_0^{T}\mathrm{e}^{\beta t}\|\boldsymbol{S}(\boldsymbol{x},t-\tau)\|^2\mathrm{d}t+$$
$$\beta\int_{\Omega}\int_0^{T}\mathrm{e}^{\beta t}\int_{-\tau}^{0}\boldsymbol{S}^2(\boldsymbol{x},t+\zeta)\mathrm{d}\zeta\mathrm{d}t\mathrm{d}\boldsymbol{x}\Big],\tag{15}$$

其中,$\rho_1=\|\boldsymbol{S}(\boldsymbol{x},0)\|^2+\eta\int_{\Omega}\int_{-\tau}^{0}\boldsymbol{S}^2(\boldsymbol{x},\zeta)\mathrm{d}\zeta\mathrm{d}\boldsymbol{x}$.计算

$$\int_{\Omega}\int_0^{T}\mathrm{e}^{\beta t}\int_{-\tau}^{0}\boldsymbol{S}^2(\boldsymbol{x},t+\zeta)\mathrm{d}\zeta\mathrm{d}t\mathrm{d}\boldsymbol{x}=$$
$$\int_{\Omega}\int_{-\tau}^{T}\int_{\zeta\vee0}^{(\zeta+\tau)\wedge T}\mathrm{e}^{\beta t}\mathrm{d}t\boldsymbol{S}^2(\boldsymbol{x},\zeta)\mathrm{d}\zeta\mathrm{d}\boldsymbol{x}\leqslant$$
$$\int_{\Omega}\int_{-\tau}^{T}\tau\mathrm{e}^{\beta(\zeta+\tau)}\boldsymbol{S}^2(\boldsymbol{x},\zeta)\mathrm{d}\zeta\mathrm{d}\boldsymbol{x}\leqslant$$
$$\rho_2+\tau\mathrm{e}^{\beta\tau}\int_{\Omega}\int_0^{T}\mathrm{e}^{\beta\zeta}\boldsymbol{S}^2(\boldsymbol{x},\zeta)\mathrm{d}\zeta\mathrm{d}\boldsymbol{x},$$

其中,$\rho_2=\int_{\Omega}\int_{-\tau}^{0}\tau\mathrm{e}^{\beta(\zeta+\tau)}\boldsymbol{S}^2(\boldsymbol{x},\zeta)\mathrm{d}\zeta\mathrm{d}\boldsymbol{x}$(由条件(Ⅰ)知 $\boldsymbol{S}(\boldsymbol{x},t)$ 在 $\Omega\times[-\tau,0]$ 上有界,从而 ρ_2 为常数).

于是,式(15)化为

$$\mathbf{E}[\mathrm{e}^{\beta T}\boldsymbol{Z}]\leqslant\rho_1+\rho_2+$$
$$\mathbf{E}\Big[-(\lambda-\beta-\eta+\tau\mathrm{e}^{\beta\tau})\int_0^{T}\mathrm{e}^{\beta t}\|\boldsymbol{S}(\boldsymbol{x},t)\|^2\mathrm{d}t-$$

$$(\lambda+\eta)\int_0^T e^{\beta t}\|\boldsymbol{S}(\boldsymbol{x},t-\tau)\|^2 dt\Big].$$

由定理条件(Ⅲ),得到

$$\mathbf{E}\|\boldsymbol{S}(\boldsymbol{x},T)\|^2\leqslant(\rho_1+\rho_2)e^{-\beta T}.$$

由定义(2)可知,滑动模具有次可达性.定理证毕.

4 滑动模运动的稳定性

在本节中分析滑动模方程的渐近稳定性.

定理 2 假设下列条件成立:

(Ⅰ) $\boldsymbol{p}_1(\boldsymbol{x},t)$在$\Omega\times[-\tau,+\infty)$上有界;

(Ⅱ) $\mu_{\max}((d\boldsymbol{I}+\boldsymbol{G})^{\mathrm{T}}+(d\boldsymbol{I}+\boldsymbol{G}))=-\mu<0$,存在常数$\gamma>0$,使得矩阵

$$\hat{\boldsymbol{U}}=\begin{pmatrix}(\alpha-\mu+\gamma)\boldsymbol{I} & \boldsymbol{H}\\ \boldsymbol{H}^{\mathrm{T}} & -\gamma\boldsymbol{I}\end{pmatrix}$$

负定,并设其最大特征值为$-\delta$;

(Ⅲ) 存在$\beta>0$,使得$\lambda-\beta+\tau e^{\beta\tau}>0$,则滑动模运动方程(9)的平凡解是均方指数稳定的.

证明 系统(9)两边同时乘以$\boldsymbol{p}_1^{\mathrm{T}}(\boldsymbol{x},t)$,有

$$\frac{1}{2}d(\boldsymbol{p}_1^{\mathrm{T}}(\boldsymbol{x},t)\boldsymbol{p}_1(\boldsymbol{x},t))=[d\boldsymbol{p}_1^{\mathrm{T}}(\boldsymbol{x},t)\Delta\boldsymbol{p}_1(\boldsymbol{x},t)+\boldsymbol{p}_1^{\mathrm{T}}(\boldsymbol{x},t)\boldsymbol{G}\boldsymbol{p}_1(\boldsymbol{x},t)+\boldsymbol{p}_1^{\mathrm{T}}(\boldsymbol{x},t)\boldsymbol{H}\boldsymbol{p}_1(\boldsymbol{x},t-\tau)]dt+\sum_{i=1}^m\sigma_i\boldsymbol{p}_1^{\mathrm{T}}(\boldsymbol{x},t)\boldsymbol{H}_i\boldsymbol{p}_1(\boldsymbol{x},t)dW_i(t)=$$
$$[\lambda\bar{\boldsymbol{p}}_1(\boldsymbol{x},t)+\boldsymbol{p}_1^{\mathrm{T}}(\boldsymbol{x},t)(d\boldsymbol{I}+\boldsymbol{G})\boldsymbol{p}_1(\boldsymbol{x},t)+\boldsymbol{p}_1^{\mathrm{T}}(\boldsymbol{x},t)\boldsymbol{H}\boldsymbol{p}_1(\boldsymbol{x},t-\tau)]dt+\sum_{i=1}^m\sigma_i\boldsymbol{p}_1^{\mathrm{T}}(\boldsymbol{x},t)\boldsymbol{H}_i\boldsymbol{p}_1(\boldsymbol{x},t)dW_i(t).$$

由引理1知,$\bar{\boldsymbol{p}}_1(\boldsymbol{x},t)\leqslant0$,因此,对上式在$\Omega$上关于$\boldsymbol{x}$积分,得

$$d\int_\Omega\boldsymbol{p}_1^{\mathrm{T}}(\boldsymbol{x},t)\boldsymbol{p}_1(\boldsymbol{x},t)d\boldsymbol{x}\leqslant\Big[2\int_\Omega\boldsymbol{p}_1^{\mathrm{T}}(\boldsymbol{x},t)(d\boldsymbol{I}+\boldsymbol{G})\boldsymbol{p}_1(\boldsymbol{x},t)d\boldsymbol{x}+2\boldsymbol{p}_1^{\mathrm{T}}(\boldsymbol{x},t)\boldsymbol{H}\boldsymbol{p}_1(\boldsymbol{x},t-\tau)\Big]dt+2\int_\Omega\sum_{i=1}^m\sigma_i\boldsymbol{p}_1^{\mathrm{T}}(\boldsymbol{x},t)\boldsymbol{H}_i\boldsymbol{p}_1(\boldsymbol{x},t)dW_i(t)d\boldsymbol{x},\quad(16)$$

取$\alpha\in(0,\lambda)$,考虑辅助函数

$$V(t,\boldsymbol{N})=e^{\alpha t}\boldsymbol{N}(t),$$

其中$\boldsymbol{N}(t)=\|\boldsymbol{p}_1(\boldsymbol{x},t)\|^2+\gamma\int_\Omega\int_{-\tau}^0\boldsymbol{p}_1^2(\boldsymbol{x},t+\xi)d\xi d\boldsymbol{x}$.

由 Itô 公式,并注意到式(16),有

$$d(e^{\alpha t}\boldsymbol{N}(t))=e^{\alpha t}\Big[\alpha\|\boldsymbol{p}_1(\boldsymbol{x},t)\|^2+\alpha\gamma\int_\Omega\int_{-\tau}^0\boldsymbol{p}_1^2(\boldsymbol{x},t+\xi)d\xi d\boldsymbol{x}\Big]dt+e^{\alpha t}d\boldsymbol{N}(t)\leqslant e^{\alpha t}\Big[\alpha\|\boldsymbol{p}_1(\boldsymbol{x},t)\|^2+\alpha\gamma\int_\Omega\int_{-\tau}^0\boldsymbol{p}_1^2(\boldsymbol{x},t+\xi)d\xi d\boldsymbol{x}-\mu\|\boldsymbol{p}_1(\boldsymbol{x},t)\|^2+2\int_\Omega\boldsymbol{p}_1^{\mathrm{T}}(\boldsymbol{x},t)\boldsymbol{H}\boldsymbol{p}_1(\boldsymbol{x},t-\tau)d\boldsymbol{x}+\gamma\|\boldsymbol{p}_1(\boldsymbol{x},t)\|^2-\gamma\int_\Omega\boldsymbol{p}_1^{\mathrm{T}}(\boldsymbol{x},t-\tau)\boldsymbol{p}_1(\boldsymbol{x},t-\tau)d\boldsymbol{x}\Big]dt+$$
$$2e^{\alpha t}\int_\Omega\sum_{i=1}^m\sigma_i\boldsymbol{p}_1^{\mathrm{T}}\boldsymbol{H}_i\boldsymbol{p}_1dW_i(t)d\boldsymbol{x}\leqslant$$
$$e^{\alpha t}\Big[\int_\Omega(\boldsymbol{p}_1^{\mathrm{T}},\boldsymbol{p}_1^{\mathrm{T}}(\boldsymbol{x},t-\tau))\hat{\boldsymbol{U}}(\boldsymbol{p}_1^{\mathrm{T}},\boldsymbol{p}_1^{\mathrm{T}}(\boldsymbol{x},t-\tau))^{\mathrm{T}}d\boldsymbol{x}+\alpha\gamma\int_\Omega\int_{-\tau}^0\boldsymbol{p}_1^2(\boldsymbol{x},t+\xi)d\xi d\boldsymbol{x}\Big]dt+$$
$$2e^{\alpha t}\int_\Omega\sum_{i=1}^m\sigma_i\boldsymbol{p}_1^{\mathrm{T}}\boldsymbol{H}_i\boldsymbol{p}_1dW_i(t)d\boldsymbol{x}\leqslant$$
$$e^{\alpha t}\begin{bmatrix}-\delta(\|\boldsymbol{p}_1(\boldsymbol{x},t)\|^2+\|\boldsymbol{p}_1(\boldsymbol{x},t-\tau)\|^2)+\\ \alpha\gamma\int_\Omega\int_{-\tau}^0\boldsymbol{p}_1^2(\boldsymbol{x},t+\xi)d\xi d\boldsymbol{x}\end{bmatrix}dt+2e^{\alpha t}\int_\Omega\sum_{i=1}^m\sigma_i\boldsymbol{p}_1^{\mathrm{T}}\boldsymbol{H}_i\boldsymbol{p}_1dW_i(t)d\boldsymbol{x}.$$

从0到任意的$T>0$积分上面不等式两边,且取数学期望,类似于定理1相应部分的证明方法可证明$\mathbf{E}\|\boldsymbol{p}_1(\boldsymbol{x},T)\|^2\leqslant\rho_3e^{-\alpha T}$,故滑动模运动方程(9)的平凡解是均方指数稳定的.定理证毕.

如果考虑边界条件(4),则只需要利用下列引理,可以得到与定理1,定理2类似的结论,限于篇幅,不在阐述.

引理 3 如果α_0是特征值问题

$$\begin{cases}\Delta\varphi(\boldsymbol{x})+\alpha\varphi(\boldsymbol{x})=0, & \boldsymbol{x}\in\Omega,\\ \varphi(\boldsymbol{x})=0, & \boldsymbol{x}\in\partial\Omega\end{cases}$$

的最小特征值,则$\alpha_0>0$,且它对应的特征函数$\varphi(\boldsymbol{x})>0,\boldsymbol{x}\in\Omega$,并有界.

5 结论

本文讨论了一类抛物型随机系统的滑动模控制问题,主要是改进了相关文献[5]中部分定理的证明方法,使不等式的估计更加精确,所得结论与系统的扩散系数相关,参见定理2中的条件(Ⅱ).

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Variable structure control for systems with stochastic distributed parameters

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Abstract We study into the sliding mode control of Itô-type stochastic time-delay system with distributed parameters, design the variable structure controller and verify the existence of sliding-mode movement in this paper. At the same time, the invariable property and moving stability are analyzed for sliding-mode control system with uncertain variables on sliding switching surface. Compared with previous studies in this aspect, the conclusion achieved in this paper is correlated with diffusion coefficients and thus has significance in practical application. Conclusions of many similar papers can be improved and extended by using methods proposed in this paper.

Key words stochastic driven; distributed parameter; variable structure; diffusion coefficient