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类多变量输出误差系统的耦合多新息辨识方法

摘要

辅助模型辨识思想、多新息辨识理论、耦合辨识概念是研究复杂多变量系统辨识的新理念和原理.将它们结合起来研究类多变量输出误差系统的辨识问题,提出了多元辅助模型辨识方法、多元辅助模型多新息辨识方法、变递推间隔多元辅助模型多新息辨识方法.为减小算法的计算量和提高参数估计精度,将系统模型分解为一些子辨识模型,应用辅助模型辨识思想、多新息辨识理论、耦合辨识概念,研究和推导了部分耦合辅助模型辨识方法、部分耦合辅助模型多新息辨识方法.讨论了几个典型辨识算法的计算量,给出了参数估计的计算步骤和计算流程图.

关键词

参数估计;递推辨识;梯度搜索;最小二乘;辅助模型辨识思想;多新息辨识理论;递阶辨识原理;耦合辨识概念;类多变量系统

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0 引言

系统辨识和模型参数估计是控制问题的基础.工业过程系统本质上都是多变量系统.多变量系统具有结构复杂、变量多、存在非线性、干扰不确定性、维数高等特点,这些因素会导致辨识精度低和辨识算法计算量大.如何提高多变量系统的辨识效率和参数估计精度,已经成为多变量系统辨识的一个重要研究课题.

为了解决存在不可测内部变量系统的辨识问题,提高辨识算法的参数估计精度,解决复杂结构系统的辨识问题和参数耦合多变量系统的辨识问题,近年来,一些新的辨识方法不断问世[1-5],例如基于辅助模型辨识思想的辅助模型辨识方法[6]、基于梯度搜索和最小二乘搜索或牛顿搜索的迭代辨识方法[7]、基于多新息辨识理论的多新息辨识方法[8]、基于递阶辨识原理的递阶辨识方法[9]、基于耦合辨识概念的耦合辨识方法[10]等.

由于多变量系统维数高、输入输出变量多、参数数目多,使得辨识算法的计算量或计算效率成为突出问题.为此,文献[11]讨论了线性回归系统、多元线性回归系统、多变量系统的随机梯度辨识算法、(递推)最小二乘辨识算法的计算量.文献[12]针对伪线性回归系统、多元伪线性回归系统、多变量伪线性回归系统,讨论了最小二乘迭代辨识算法、基于块矩阵求逆的最小二乘迭代辨识算法及其计算效率,主要通过块矩阵求逆方法来减小最小二乘迭代算法求逆的计算量.文献[13]研究了信息向量耦合型多变量系统的子系统递推最小二乘辨识方法,给出了计算量小的联合递推最小二乘辨识算法;研究了部分信息向量耦合型多变量系统的子系统最小二乘辨识算法,提出了计算量小的基于块矩阵求逆的最小二乘辨识算法;给出了部分信息向量耦合型多变量系统的子系统递推最小二乘辨识算法,提出和推导了基于辨识模型分解的递推最小二乘辨识算法,讨论了算法的计算效率.

文献[14]将多元线性回归系统分解成多个子系统,研究了耦合随机梯度辨识算法和耦合多新息随机梯度辨识算法.文献[15]讨论了多元伪线性滑动平均系统的多元增广随机梯度算法,为减小算法的计算量,给出了子系统增广随机梯度算法,利用耦合辨识概念和多新息辨识理论,推导了部分耦合增广随机梯度算法和部分耦合多新息增广随机梯度算法,进一步提出了多元伪线性自回归滑动平均系统

的部分耦合广义增广随机梯度算法和部分耦合多新息广义增广随机梯度算法.文献[16-17]首次提出了多变量系统的耦合辨识概念,研究了非均匀采样数据系统的部分参数耦合随机梯度辨识方法和多元线性回归系统的全耦合最小二乘辨识方法,并进行了收敛性分析.

文献[18-21]提出了双率采样数据系统的辅助模型递推最小二乘辨识方法和辅助模型随机梯度辨识方法、损失数据系统的辅助模型递推最小二乘方法和稀少量测数据系统的辅助模型多新息随机梯度辨识方法.文献[22-24]提出了线性回归系统的多新息随机梯度辨识方法、多新息投影辨识方法、多新息最小二乘辨识方法、变递推间隔多新息随机梯度辨识方法和变递推间隔多新息最小二乘辨识方法.

针对类多变量输出误差系统,本文利用辅助模型辨识思想、多新息辨识理论、耦合辨识概念,提出了多元辅助模型辨识方法、多元辅助模型多新息辨识方法、变递推间隔多元辅助模型多新息辨识方法.进一步将系统模型分解为一些子辨识模型,提出了部分耦合辅助模型辨识方法、部分耦合辅助模型多新息辨识方法,并讨论了几个典型辨识算法的计算量,给出了计算步骤和计算流程图.

1 多元辅助模型辨识方法

考虑类多变量输出误差系统

$$\boldsymbol{y}(t)=\frac{\boldsymbol{Q}(z)}{\alpha(z)}\boldsymbol{u}(t)+\boldsymbol{v}(t), \tag{1}$$

其中 $\boldsymbol{y}(t):=[y_1(t),y_2(t),\cdots,y_m(t)]^{\mathrm{T}}\in\mathbf{R}^m$ 为 m 维观测输出向量,$\boldsymbol{u}(t)\in\mathbf{R}^r$ 为 r 维观测输入向量,$\boldsymbol{v}(t):=[v_1(t),v_2(t),\cdots,v_m(t)]^{\mathrm{T}}\in\mathbf{R}^m$ 为 m 维白噪声向量,$\alpha(z)\in\mathbf{R}$ 为单位后移算子 z^{-1} 的特征多项式,$\boldsymbol{Q}(z)\in\mathbf{R}^{m\times r}$ 为单位后移算子 z^{-1} 的矩阵多项式,它们可以表示为

$$\alpha(z):=1+\alpha_1z^{-1}+\alpha_2z^{-2}+\cdots+\alpha_nz^{-n},\quad \alpha_i\in\mathbf{R}, \tag{2}$$

$$\boldsymbol{Q}(z):=\boldsymbol{Q}_1z^{-1}+\boldsymbol{Q}_2z^{-2}+\cdots+\boldsymbol{Q}_nz^{-n},\quad \boldsymbol{Q}_i\in\mathbf{R}^{m\times r}. \tag{3}$$

下面推导系统(1)的辨识模型.令

$$\boldsymbol{x}(t):=\frac{\boldsymbol{Q}(z)}{\alpha(z)}\boldsymbol{u}(t)\in\mathbf{R}^m. \tag{4}$$

将式(2)和式(3)代入式(4)得到

$$(1+\alpha_1z^{-1}+\alpha_2z^{-2}+\cdots+\alpha_nz^{-n})\boldsymbol{x}(t)=(\boldsymbol{Q}_1z^{-1}+\boldsymbol{Q}_2z^{-2}+\cdots+\boldsymbol{Q}_nz^{-n})\boldsymbol{u}(t).$$

移项可得

$$\boldsymbol{x}(t)=-\alpha_1\boldsymbol{x}(t-1)-\alpha_2\boldsymbol{x}(t-2)-\cdots-\alpha_n\boldsymbol{x}(t-n)+\boldsymbol{Q}_1\boldsymbol{u}(t-1)+\boldsymbol{Q}_2\boldsymbol{u}(t-2)+\cdots+\boldsymbol{Q}_n\boldsymbol{u}(t-n).$$

将上式代入式(1)得到

$$\boldsymbol{y}(t)=-\alpha_1\boldsymbol{x}(t-1)-\alpha_2\boldsymbol{x}(t-2)-\cdots-\alpha_n\boldsymbol{x}(t-n)+\boldsymbol{Q}_1\boldsymbol{u}(t-1)+\boldsymbol{Q}_2\boldsymbol{u}(t-2)+\cdots+\boldsymbol{Q}_n\boldsymbol{u}(t-n)+\boldsymbol{v}(t).$$

定义模型参数向量 $\boldsymbol{\alpha}$、参数矩阵 $\boldsymbol{\theta}$、信息向量 $\boldsymbol{\varphi}(t)$ 和信息矩阵 $\boldsymbol{\phi}(t)$ 如下:

$$\boldsymbol{\alpha}:=[\alpha_1,\alpha_2,\cdots,\alpha_n]^{\mathrm{T}}\in\mathbf{R}^n,$$

$$\boldsymbol{\theta}^{\mathrm{T}}:=[\boldsymbol{Q}_1,\boldsymbol{Q}_2,\cdots,\boldsymbol{Q}_n]\in\mathbf{R}^{m\times(nr)},$$

$$\boldsymbol{\varphi}(t):=[\boldsymbol{u}^{\mathrm{T}}(t-1),\boldsymbol{u}^{\mathrm{T}}(t-2),\cdots,\boldsymbol{u}^{\mathrm{T}}(t-n)]^{\mathrm{T}}\in\mathbf{R}^{nr},$$

$$\boldsymbol{\phi}(t):=[-\boldsymbol{x}(t-1),-\boldsymbol{x}(t-2),\cdots,-\boldsymbol{x}(t-n)]\in\mathbf{R}^{m\times n}.$$

于是,可以得到类多变量输出误差系统(1)的递阶辨识模型:

$$\boldsymbol{y}(t)=\boldsymbol{\phi}(t)\boldsymbol{\alpha}+\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\varphi}(t)+\boldsymbol{v}(t). \tag{5}$$

类多变量输出误差系统的递阶辨识模型(5)既包含参数向量 $\boldsymbol{\alpha}$,又包含参数矩阵 $\boldsymbol{\theta}$,使得辨识算法比较复杂.

为了便于辨识,使用 Kronecker 积,将模型(5)中的信息向量 $\boldsymbol{\varphi}(t)$ 和信息矩阵 $\boldsymbol{\phi}(t)$ 化为一个大信息矩阵 $\boldsymbol{\Phi}(t)$;将参数向量 $\boldsymbol{\alpha}$ 和参数矩阵 $\boldsymbol{\theta}$ 化为一个大参数向量 $\boldsymbol{\vartheta}$,定义信息矩阵 $\boldsymbol{\Phi}(t)$ 和参数向量 $\boldsymbol{\vartheta}$ 如下:

$$\boldsymbol{\Phi}(t):=[\boldsymbol{\phi}(t),\boldsymbol{\varphi}^{\mathrm{T}}(t)\otimes\boldsymbol{I}_m]\in\mathbf{R}^{m\times n_0},\quad n_0:=n+mnr,$$

$$\boldsymbol{\vartheta}:=\begin{bmatrix}\boldsymbol{\alpha}\\ \mathrm{col}[\boldsymbol{\theta}^{\mathrm{T}}]\end{bmatrix}\in\mathbf{R}^{n_0}.$$

则递阶辨识模型(5)可以写为下列多元伪线性回归模型(multivariate pseudo-linear regressive model):

$$\boldsymbol{y}(t)=\boldsymbol{\Phi}(t)\boldsymbol{\vartheta}+\boldsymbol{v}(t). \tag{6}$$

假设阶次 m,n 和 r 已知,且当 $t\leqslant 0$ 时,$\boldsymbol{y}(t)=0$,$\boldsymbol{u}(t)=0$,$\boldsymbol{v}(t)=0$,辨识的目标就是基于辅助模型辨识思想和多新息辨识理论,利用观测数据$\{\boldsymbol{y}(t),\boldsymbol{\Phi}(t):t=1,2,3,\cdots\}$,研究和提出辅助模型多新息辨识方法,估计系统参数向量 $\boldsymbol{\vartheta}$.

1.1 多元辅助模型辨识方法

1.1.1 多元辅助模型随机梯度辨识算法

定义和极小化梯度准则函数(gradient criterion function):

$$J_1(\boldsymbol{\vartheta}):=\|\boldsymbol{y}(t)-\boldsymbol{\Phi}(t)\boldsymbol{\vartheta}\|^2,$$

令 $\hat{\boldsymbol{\vartheta}}(t)$ 为参数向量 $\boldsymbol{\vartheta}$ 在时刻 t 的估计. $\|\boldsymbol{X}\|^2=\mathrm{tr}[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}]$定义为矩阵 $\boldsymbol{X}$ 的范数.参考文献[14],可以得到估计参数向量 $\boldsymbol{\vartheta}$ 的多元随机梯度算法(Multivariate Stochastic Gradient algorithm,M-SG 算法):

$$\hat{\boldsymbol{\vartheta}}(t)=\hat{\boldsymbol{\vartheta}}(t-1)+\frac{\boldsymbol{\Phi}^{\mathrm{T}}(t)}{r(t)}[\boldsymbol{y}(t)-\boldsymbol{\Phi}(t)\hat{\boldsymbol{\vartheta}}(t-1)], \tag{7}$$

$$r(t)=r(t-1)+\|\boldsymbol{\Phi}(t)\|^2,\quad r(0)=1. \tag{8}$$

与多元线性系统相比,这里多元伪线性回归系统辨识的困难在于:$\boldsymbol{\Phi}(t)$中包含了未知回归项$\boldsymbol{x}(t-i)$.将$\boldsymbol{\Phi}(t)$中$\boldsymbol{x}(t-i)$用其估计值(即辅助模型的输出)$\hat{\boldsymbol{x}}(t-i)$代替,代替后的信息矩阵$\boldsymbol{\Phi}(t)$记作$\boldsymbol{\Psi}(t)$,即:

$$\boldsymbol{\Psi}(t):=[\hat{\boldsymbol{\phi}}(t),\boldsymbol{\varphi}^{\mathrm{T}}(t)\otimes\boldsymbol{I}_m]\in\mathbf{R}^{m\times n_0},$$
$$\hat{\boldsymbol{\phi}}(t):=[-\hat{\boldsymbol{x}}(t-1),-\hat{\boldsymbol{x}}(t-2),\cdots,-\hat{\boldsymbol{x}}(t-n)]\in\mathbf{R}^{m\times n},$$
$$\hat{\boldsymbol{x}}(t):=\boldsymbol{\Psi}(t)\hat{\boldsymbol{\vartheta}}(t)\in\mathbf{R}^m. \tag{9}$$

式(9)称为估算$\boldsymbol{x}(t)$的辅助模型,$\boldsymbol{x}(t)$为辅助模型的输出向量或输出估计.

于是,用$\boldsymbol{\Psi}(t)$代替式(7)—(8)中$\boldsymbol{\Phi}(t)$,能够获得估计多元伪线性回归辨识模型(6)参数向量$\boldsymbol{\vartheta}$的多元辅助模型随机梯度算法(Multivariate Auxiliary Model based Stochastic Gradient algorithm, M-AM-SG算法)[2]:

$$\hat{\boldsymbol{\vartheta}}(t)=\hat{\boldsymbol{\vartheta}}(t-1)+\frac{\boldsymbol{\Psi}^{\mathrm{T}}(t)}{r(t)}[\boldsymbol{y}(t)-\boldsymbol{\Psi}(t)\hat{\boldsymbol{\vartheta}}(t-1)],$$
$$\hat{\boldsymbol{\vartheta}}(0)=\mathbf{1}_{n_0}/p_0, \tag{10}$$
$$r(t)=r(t-1)+\|\boldsymbol{\Psi}(t)\|^2,\quad r(0)=1, \tag{11}$$
$$\boldsymbol{\Psi}(t)=[\hat{\boldsymbol{\phi}}(t),\boldsymbol{\varphi}^{\mathrm{T}}(t)\otimes\boldsymbol{I}_m], \tag{12}$$
$$\boldsymbol{\varphi}(t)=[\boldsymbol{u}^{\mathrm{T}}(t-1),\boldsymbol{u}^{\mathrm{T}}(t-2),\cdots,\boldsymbol{u}^{\mathrm{T}}(t-n)]^{\mathrm{T}}, \tag{13}$$
$$\hat{\boldsymbol{\phi}}(t)=[-\hat{\boldsymbol{x}}(t-1),-\hat{\boldsymbol{x}}(t-2),\cdots,-\hat{\boldsymbol{x}}(t-n)], \tag{14}$$
$$\hat{\boldsymbol{x}}(t)=\boldsymbol{\Psi}(t)\hat{\boldsymbol{\vartheta}}(t),\hat{\boldsymbol{x}}(-i)=\mathbf{1}_m/p_0,i=0,1,\cdots,n-1. \tag{15}$$

注意到式(11)中$r(t)$需要计算信息矩阵$\boldsymbol{\Psi}(t)$每一项的平方和,为提高收敛速度,可以取信息矩阵的最大特征值,即将式(11)修改为

$$r(t)=r(t-1)+\lambda_{\max}[\boldsymbol{\Psi}(t)\boldsymbol{\Psi}^{\mathrm{T}}(t)],\quad r(0)=1. \tag{16}$$

表1列出了M-AM-SG算法(10)—(15)中各变量的维数.表2列出了M-AM-SG算法每一步递推计算中的乘法次数、加法次数和flop数.为了提高M-AM-SG算法(10)—(15)的收敛速度和暂态性能,参照文献[14],可以在算法中引入遗忘因子λ或收敛指数ε,得到相应的遗忘因子M-AM-SG算法和修正M-AM-SG算法[14].

表1 M-AM-SG算法中各变量维数

Table 1 The dimensions of the variables in the M-AM-SG algorithm

变量名称	维数
输出向量	$\boldsymbol{y}(t)\in\mathbf{R}^m$
输入向量	$\boldsymbol{u}(t)\in\mathbf{R}^r$
参数估计向量	$\hat{\boldsymbol{\vartheta}}(t)\in\mathbf{R}^{n_0}$
估计的信息矩阵	$\boldsymbol{\Psi}(t)\in\mathbf{R}^{m\times n_0}$
输入信息向量	$\boldsymbol{\varphi}(t)\in\mathbf{R}^{nr}$
估计的信息矩阵	$\hat{\boldsymbol{\phi}}(t)\in\mathbf{R}^{m\times n}$
辅助模型的输出向量	$\hat{\boldsymbol{x}}(t)\in\mathbf{R}^m$

1.1.2 多元辅助模型递推最小二乘辨识算法

定义和极小化最小二乘准则函数(least squares criterion function)

$$J_2(\boldsymbol{\vartheta}):=\sum_{j=1}^{t}\|\boldsymbol{y}(j)-\boldsymbol{\Phi}(j)\boldsymbol{\vartheta}\|^2,$$

参考文献[5,10],可以得到计算系统(6)参数估计向量$\hat{\boldsymbol{\vartheta}}(t)$的递推最小二乘算法:

$$\hat{\boldsymbol{\vartheta}}(t)=\hat{\boldsymbol{\vartheta}}(t-1)+\boldsymbol{P}(t)\boldsymbol{\Phi}^{\mathrm{T}}(t)[\boldsymbol{y}(t)-\boldsymbol{\Phi}(t)\hat{\boldsymbol{\vartheta}}(t-1)], \tag{17}$$
$$\boldsymbol{P}^{-1}(t)=\boldsymbol{P}^{-1}(t-1)+\boldsymbol{\Phi}^{\mathrm{T}}(t)\boldsymbol{\Phi}(t). \tag{18}$$

为避免计算式(18)大矩阵$\boldsymbol{P}(t)\in\mathbf{R}^{n_0\times n_0}$的逆,应用矩阵求逆公式

$$(\boldsymbol{A}+\boldsymbol{B}\boldsymbol{C})^{-1}=\boldsymbol{A}^{-1}-\boldsymbol{A}^{-1}\boldsymbol{B}(\boldsymbol{I}+\boldsymbol{C}\boldsymbol{A}^{-1}\boldsymbol{B})^{-1}\boldsymbol{C}\boldsymbol{A}^{-1}$$

于式(18),引入增益矩阵$\boldsymbol{L}(t):=\boldsymbol{P}(t)\boldsymbol{\Phi}^{\mathrm{T}}(t)\in\mathbf{R}^{n_0\times m}$,就能够得到与式(17)—(18)等价的递推最小二乘算法:

$$\hat{\boldsymbol{\vartheta}}(t)=\hat{\boldsymbol{\vartheta}}(t-1)+\boldsymbol{L}(t)[\boldsymbol{y}(t)-\boldsymbol{\Phi}(t)\hat{\boldsymbol{\vartheta}}(t-1)], \tag{19}$$
$$\boldsymbol{L}(t)=\boldsymbol{P}(t-1)\boldsymbol{\Phi}^{\mathrm{T}}(t)[\boldsymbol{I}_m+\boldsymbol{\Phi}(t)\boldsymbol{P}(t-1)\boldsymbol{\Phi}^{\mathrm{T}}(t)]^{-1}, \tag{20}$$
$$\boldsymbol{P}(t)=\boldsymbol{P}(t-1)-\boldsymbol{L}(t)\boldsymbol{\Phi}(t)\boldsymbol{P}(t-1). \tag{21}$$

表2 多元辅助模型随机梯度算法的计算量

Table 2 The computational efficiency of the M-AM-SG algorithm

变量	表达式	乘法次数	加法次数
$\hat{\boldsymbol{\vartheta}}(t)$	$\hat{\boldsymbol{\vartheta}}(t)=\hat{\boldsymbol{\vartheta}}(t-1)+\boldsymbol{\Psi}^{\mathrm{T}}(t)[\boldsymbol{e}(t)/r(t)]\in\mathbf{R}^{n_0}$	$m+mn_0$	mn_0
	$\boldsymbol{e}(t):=\boldsymbol{y}(t)-\boldsymbol{\Psi}(t)\hat{\boldsymbol{\vartheta}}(t-1)\in\mathbf{R}^m$	mn_0	mn_0
$r(t)$	$r(t)=r(t-1)+\|\boldsymbol{\Psi}(t)\|^2\in\mathbf{R}$	mn_0	mn_0
$\hat{\boldsymbol{x}}(t)$	$\hat{\boldsymbol{x}}(t)=\boldsymbol{\Psi}(t)\hat{\boldsymbol{\vartheta}}(t)\in\mathbf{R}^m$	mn_0	mn_0-m
	总数	$4mn_0+m$	$4mn_0-m$
	总flop数	$8mn_0$	

为了能够计算参数估计,与多元随机梯度辨识算法一样,将信息矩阵 $\boldsymbol{\Phi}(t)$ 中未知回归项 $\boldsymbol{x}(t-i)$ 用其估计值 $\hat{\boldsymbol{x}}(t-i)$ 代替,代替后的 $\boldsymbol{\Phi}(t)$ 记作 $\boldsymbol{\Psi}(t)$,则得到可实现的估计参数向量 $\boldsymbol{\vartheta}$ 的多元辅助模型递推最小二乘算法(Multivariate Auxiliary Model based Recursive Least Squares algorithm,M-AM-RLS 算法)[2]:

$$\hat{\boldsymbol{\vartheta}}(t)=\hat{\boldsymbol{\vartheta}}(t-1)+\boldsymbol{L}(t)[\boldsymbol{y}(t)-\boldsymbol{\Psi}(t)\hat{\boldsymbol{\vartheta}}(t-1)],$$

$$\hat{\boldsymbol{\vartheta}}(0)=\mathbf{1}_{n_0}/p_0, \tag{22}$$

$$\boldsymbol{L}(t)=\boldsymbol{P}(t-1)\boldsymbol{\Psi}^{\mathrm{T}}(t)[\boldsymbol{I}+\boldsymbol{\Psi}(t)\boldsymbol{P}(t-1)\boldsymbol{\Psi}^{\mathrm{T}}(t)]^{-1}, \tag{23}$$

$$\boldsymbol{P}(t)=\boldsymbol{P}(t-1)-\boldsymbol{L}(t)\boldsymbol{\Psi}(t)\boldsymbol{P}(t-1),$$

$$\boldsymbol{P}(0)=p_0\boldsymbol{I}_{n_0}, \tag{24}$$

$$\boldsymbol{\Psi}(t)=[\hat{\boldsymbol{\phi}}(t),\boldsymbol{\varphi}^{\mathrm{T}}(t)\otimes\boldsymbol{I}_m], \tag{25}$$

$$\boldsymbol{\varphi}(t)=[\boldsymbol{u}^{\mathrm{T}}(t-1),\boldsymbol{u}^{\mathrm{T}}(t-2),\cdots,\boldsymbol{u}^{\mathrm{T}}(t-n)]^{\mathrm{T}}, \tag{26}$$

$$\hat{\boldsymbol{\phi}}(t)=[-\hat{\boldsymbol{x}}(t-1),-\hat{\boldsymbol{x}}(t-2),\cdots,-\hat{\boldsymbol{x}}(t-n)], \tag{27}$$

$$\hat{\boldsymbol{x}}(t)=\boldsymbol{\Psi}(t)\hat{\boldsymbol{\vartheta}}(t),\quad \hat{\boldsymbol{x}}(-i)=\mathbf{1}_m/p_0,\quad i=0,1,\cdots,n-1. \tag{28}$$

表 3 列出了 M-AM-RLS 算法(22)—(28)中各变量的维数. M-AM-RLS 算法的计算量如表 4 所示[11-13].

表 3 M-AM-RLS 算法中各变量维数

Table 3 The dimensions of the variables in the M-AM-RLS algorithm

变量名称	维数
输出向量	$\boldsymbol{y}(t)\in\mathbf{R}^m$
输入向量	$\boldsymbol{u}(t)\in\mathbf{R}^r$
参数估计向量	$\hat{\boldsymbol{\vartheta}}(t)\in\mathbf{R}^{n_0}$
估计的信息矩阵	$\boldsymbol{\Psi}(t)\in\mathbf{R}^{m\times n_0}$
增益矩阵	$\boldsymbol{L}(t)\in\mathbf{R}^{n_0\times m}$
协方差矩阵	$\boldsymbol{P}(t)\in\mathbf{R}^{n_0\times n_0}$
输入信息向量	$\boldsymbol{\varphi}(t)\in\mathbf{R}^{nr}$
估计的信息矩阵	$\hat{\boldsymbol{\phi}}(t)\in\mathbf{R}^{m\times n}$
辅助模型的输出向量	$\hat{\boldsymbol{x}}(t)\in\mathbf{R}^m$

1.2 多元辅助模型多新息辨识方法

与多元辅助模型递推最小二乘算法相比,多元辅助模型随机梯度算法的计算量小、收敛速度慢.为了改进多元辅助模型辨识算法的收敛速度,本文引入新息长度,将新息向量扩展为一个大新息向量,得到多元辅助模型多新息辨识方法.

定义堆积信息矩阵 $\boldsymbol{\Gamma}(p,t)$ 和堆积输出向量 $\boldsymbol{Y}(p,t)$ 如下:

$$\boldsymbol{\Gamma}(p,t):=[\boldsymbol{\Psi}^{\mathrm{T}}(t),\boldsymbol{\Psi}^{\mathrm{T}}(t-1),\cdots,\boldsymbol{\Psi}^{\mathrm{T}}(t-p+1)]\in\mathbf{R}^{n_0\times(mp)},$$

$$\boldsymbol{Y}(p,t):=\begin{bmatrix}\boldsymbol{y}(t)\\ \boldsymbol{y}(t-1)\\ \vdots\\ \boldsymbol{y}(t-p+1)\end{bmatrix}\in\mathbf{R}^{mp}.$$

令新息长度 $p\geqslant 1$.应用多新息辨识理论,可以将多元辅助模型辨识算法(10)中新息向量 $\boldsymbol{e}(t):=\boldsymbol{y}(t)-\boldsymbol{\Psi}(t)\hat{\boldsymbol{\vartheta}}(t-1)\in\mathbf{R}^m$ 扩展为一个大新息向量

$$\boldsymbol{E}(p,t):=\begin{bmatrix}\boldsymbol{y}(t)-\boldsymbol{\Psi}(t)\hat{\boldsymbol{\vartheta}}(t-1)\\ \boldsymbol{y}(t-1)-\boldsymbol{\Psi}(t-1)\hat{\boldsymbol{\vartheta}}(t-1)\\ \vdots\\ \boldsymbol{y}(t-p+1)-\boldsymbol{\Psi}(t-p+1)\hat{\boldsymbol{\vartheta}}(t-1)\end{bmatrix}=$$

$$\boldsymbol{Y}(p,t)-\boldsymbol{\Gamma}^{\mathrm{T}}(p,t)\hat{\boldsymbol{\vartheta}}(t-1)\in\mathbf{R}^{mp}. \tag{29}$$

1.2.1 多元辅助模型多新息随机梯度辨识算法

借助于多新息辨识理论[1,21-23],推广多元辅助模型随机梯度辨识算法(10)—(15),可以得到新息长度为 p 的多元辅助模型多新息随机梯度算法(Multivariate Auxiliary Model based Multi-Innovation Stochastic Gradient algorithm,M-AM-MISG 算法)[2]:

$$\hat{\boldsymbol{\vartheta}}(t)=\hat{\boldsymbol{\vartheta}}(t-1)+\frac{\boldsymbol{\Gamma}(p,t)}{r(t)}\boldsymbol{E}(p,t),\quad \hat{\boldsymbol{\vartheta}}(0)=\mathbf{1}_{n_0}/p_0, \tag{30}$$

$$\boldsymbol{E}(p,t)=\boldsymbol{Y}(p,t)-\boldsymbol{\Gamma}^{\mathrm{T}}(p,t)\hat{\boldsymbol{\vartheta}}(t-1), \tag{31}$$

$$r(t)=r(t-1)+\|\boldsymbol{\Psi}(t)\|^2,\quad r(0)=1, \tag{32}$$

表 4 多元辅助模型递推最小二乘算法的计算量

Table 4 The computational efficiency of the M-AM-RLS algorithm

变量	表达式	乘法次数	加法次数
$\hat{\boldsymbol{\vartheta}}(t)$	$\hat{\boldsymbol{\vartheta}}(t)=\hat{\boldsymbol{\vartheta}}(t-1)+\boldsymbol{L}(t)\boldsymbol{e}(t)\in\mathbf{R}^{n_0}$	mn_0	mn_0
	$\boldsymbol{e}(t):=\boldsymbol{y}(t)-\boldsymbol{\Psi}(t)\hat{\boldsymbol{\vartheta}}(t-1)\in\mathbf{R}^m$	mn_0	mn_0
$\boldsymbol{L}(t)$	$\boldsymbol{L}(t):=\boldsymbol{R}(t)\boldsymbol{\Lambda}'(t)\in\mathbf{R}^{n_0\times m}$	m^2n_0	$m^2n_0-mn_0$
	$\boldsymbol{R}(t):=\boldsymbol{P}(t-1)\boldsymbol{\Psi}^{\mathrm{T}}(t)\in\mathbf{R}^{n_0\times m}$	mn_0^2	$mn_0^2-mn_0$
	$\boldsymbol{\Lambda}(t):=\boldsymbol{I}_m+\boldsymbol{\Psi}(t)\boldsymbol{R}(t)\in\mathbf{R}^{m\times m}$	m^2n_0	m^2n_0
	$\boldsymbol{\Lambda}'(t):=\boldsymbol{\Lambda}^{-1}(t)\in\mathbf{R}^{m\times m}$	m^3	m^3-n^2
$\boldsymbol{P}(t)$	$\boldsymbol{P}(t)=\boldsymbol{P}(t-1)-\boldsymbol{L}(t)\boldsymbol{R}^{\mathrm{T}}(t)\in\mathbf{R}^{n_0\times n_0}$	mn_0^2	mn_0^2
$\hat{\boldsymbol{x}}(t)$	$\hat{\boldsymbol{x}}(t)=\boldsymbol{\Psi}(t)\hat{\boldsymbol{\vartheta}}(t)\in\mathbf{R}^m$	mn_0	mn_0-m
总数		$2mn_0^2+2m^2n_0+3mn_0+m^3$	$2mn_0^2+2m^2n_0+mn_0+m^3-m^2-m$
总 flop 数		$4mn_0^2+4m^2n_0+4mn_0+2m^3-m^2-m$	

$$\boldsymbol{Y}(p,t)=[\boldsymbol{y}^{\mathrm{T}}(t),\boldsymbol{y}^{\mathrm{T}}(t-1),\cdots,\boldsymbol{y}^{\mathrm{T}}(t-p+1)]^{\mathrm{T}}, \quad (33)$$
$$\boldsymbol{\Gamma}(p,t)=[\boldsymbol{\Psi}^{\mathrm{T}}(t),\boldsymbol{\Psi}^{\mathrm{T}}(t-1),\cdots,\boldsymbol{\Psi}^{\mathrm{T}}(t-p+1)], \quad (34)$$
$$\boldsymbol{\Psi}(t)=[\hat{\boldsymbol{\phi}}(t),\boldsymbol{\varphi}^{\mathrm{T}}(t)\otimes\boldsymbol{I}_m], \quad (35)$$
$$\boldsymbol{\varphi}(t)=[\boldsymbol{u}^{\mathrm{T}}(t-1),\boldsymbol{u}^{\mathrm{T}}(t-2),\cdots,\boldsymbol{u}^{\mathrm{T}}(t-n)]^{\mathrm{T}}, \quad (36)$$
$$\hat{\boldsymbol{\phi}}(t)=[-\hat{\boldsymbol{x}}(t-1),-\hat{\boldsymbol{x}}(t-2),\cdots,-\hat{\boldsymbol{x}}(t-n)], \quad (37)$$
$$\hat{\boldsymbol{x}}(t)=\boldsymbol{\Psi}(t)\hat{\boldsymbol{\vartheta}}(t),\quad \hat{\boldsymbol{x}}(-i)=\mathbf{1}_m/p_0,\quad i=0,1,\cdots,n-1. \quad (38)$$

当新息长度 $p=1$ 时,多元辅助模型多新息随机梯度算法退化为多元辅助模型随机梯度算法.

1) 遗忘因子多元辅助模型多新息随机梯度辨识算法.引入遗忘因子 λ,根据多元辅助模型多新息随机梯度辨识算法(30)—(38),能够得到估计伪线性回归辨识模型(6)参数向量 $\boldsymbol{\vartheta}$ 的遗忘因子多元辅助模型多新息随机梯度算法[2]:

$$\hat{\boldsymbol{\vartheta}}(t)=\hat{\boldsymbol{\vartheta}}(t-1)+\frac{\boldsymbol{\Gamma}(p,t)}{r(t)}\boldsymbol{E}(p,t),\quad \hat{\boldsymbol{\vartheta}}(0)=\mathbf{1}_{n_0}/p_0, \quad (39)$$
$$\boldsymbol{E}(p,t)=\boldsymbol{Y}(p,t)-\boldsymbol{\Gamma}^{\mathrm{T}}(p,t)\hat{\boldsymbol{\vartheta}}(t-1), \quad (40)$$
$$r(t)=\lambda r(t-1)+\|\boldsymbol{\Psi}(t)\|^2,0\leqslant\lambda\leqslant 1,\quad r(0)=1, \quad (41)$$
$$\boldsymbol{Y}(p,t)=[\boldsymbol{y}^{\mathrm{T}}(t),\boldsymbol{y}^{\mathrm{T}}(t-1),\cdots,\boldsymbol{y}^{\mathrm{T}}(t-p+1)]^{\mathrm{T}}, \quad (42)$$
$$\boldsymbol{\Gamma}(p,t)=[\boldsymbol{\Psi}^{\mathrm{T}}(t),\boldsymbol{\Psi}^{\mathrm{T}}(t-1),\cdots,\boldsymbol{\Psi}^{\mathrm{T}}(t-p+1)], \quad (43)$$
$$\boldsymbol{\Psi}(t)=[\hat{\boldsymbol{\phi}}(t),\boldsymbol{\varphi}^{\mathrm{T}}(t)\otimes\boldsymbol{I}_m], \quad (44)$$
$$\boldsymbol{\varphi}(t)=[\boldsymbol{u}^{\mathrm{T}}(t-1),\boldsymbol{u}^{\mathrm{T}}(t-2),\cdots,\boldsymbol{u}^{\mathrm{T}}(t-n)]^{\mathrm{T}}, \quad (45)$$
$$\hat{\boldsymbol{\phi}}(t)=[-\hat{\boldsymbol{x}}(t-1),-\hat{\boldsymbol{x}}(t-2),\cdots,-\hat{\boldsymbol{x}}(t-n)], \quad (46)$$
$$\hat{\boldsymbol{x}}(t)=\boldsymbol{\Psi}(t)\hat{\boldsymbol{\vartheta}}(t),\quad \hat{\boldsymbol{x}}(-i)=\mathbf{1}_m/p_0,\quad i=0,1,\cdots,n-1. \quad (47)$$

2) 修正多元辅助模型多新息随机梯度辨识算法.引入收敛指数 ε,从多元辅助模型多新息随机梯度辨识算法(30)—(38),可以得到估计伪线性回归辨识模型(6)参数向量 $\boldsymbol{\vartheta}$ 的修正多元辅助模型多新息随机梯度算法(Modified Multivariate Auxiliary Model based Multi-Innovation Stochastic Gradient algorithm,M-M-AM-MISG 算法)[2]:

$$\hat{\boldsymbol{\vartheta}}(t)=\hat{\boldsymbol{\vartheta}}(t-1)+\frac{\boldsymbol{\Gamma}(p,t)}{r^{\varepsilon}(t)}\boldsymbol{E}(p,t),\quad \frac{1}{2}<\varepsilon\leqslant 1,$$
$$\hat{\boldsymbol{\vartheta}}(0)=\mathbf{1}_{n_0}/p_0, \quad (48)$$
$$\boldsymbol{E}(p,t)=\boldsymbol{Y}(p,t)-\boldsymbol{\Gamma}^{\mathrm{T}}(p,t)\hat{\boldsymbol{\vartheta}}(t-1), \quad (49)$$
$$r(t)=r(t-1)+\|\boldsymbol{\Psi}(t)\|^2,\quad r(0)=1, \quad (50)$$
$$\boldsymbol{Y}(p,t)=[\boldsymbol{y}^{\mathrm{T}}(t),\boldsymbol{y}^{\mathrm{T}}(t-1),\cdots,\boldsymbol{y}^{\mathrm{T}}(t-p+1)]^{\mathrm{T}}, \quad (51)$$
$$\boldsymbol{\Gamma}(p,t)=[\boldsymbol{\Psi}^{\mathrm{T}}(t),\boldsymbol{\Psi}^{\mathrm{T}}(t-1),\cdots,\boldsymbol{\Psi}^{\mathrm{T}}(t-p+1)], \quad (52)$$
$$\boldsymbol{\Psi}(t)=[\hat{\boldsymbol{\phi}}(t),\boldsymbol{\varphi}^{\mathrm{T}}(t)\otimes\boldsymbol{I}_m], \quad (53)$$
$$\boldsymbol{\varphi}(t)=[\boldsymbol{u}^{\mathrm{T}}(t-1),\boldsymbol{u}^{\mathrm{T}}(t-2),\cdots,\boldsymbol{u}^{\mathrm{T}}(t-n)]^{\mathrm{T}}, \quad (54)$$
$$\hat{\boldsymbol{\phi}}(t)=[-\hat{\boldsymbol{x}}(t-1),-\hat{\boldsymbol{x}}(t-2),\cdots,-\hat{\boldsymbol{x}}(t-n)], \quad (55)$$
$$\hat{\boldsymbol{x}}(t)=\boldsymbol{\Psi}(t)\hat{\boldsymbol{\vartheta}}(t),\quad \hat{\boldsymbol{x}}(-i)=\mathbf{1}_m/p_0,\quad i=0,1,\cdots,n-1. \quad (56)$$

1.2.2 多元辅助模型多新息递推最小二乘辨识算法

借助于多新息辨识理论[1,23-24],根据多元辅助模型递推最小二乘辨识算法(22)—(28),可以得到新息长度为 p 的多元辅助模型多新息递推最小二乘算法(Multivariate Auxiliary Model based Multi-Innovation Recursive Least Squares algorithm,M-AM-MI-RLS 算法)[1]:

$$\hat{\boldsymbol{\vartheta}}(t)=\hat{\boldsymbol{\vartheta}}(t-1)+\boldsymbol{L}(t)\boldsymbol{E}(p,t),\quad \hat{\boldsymbol{\vartheta}}(0)=\mathbf{1}_{n_0}/p_0, \quad (57)$$
$$\boldsymbol{E}(p,t)=\boldsymbol{Y}(p,t)-\boldsymbol{\Gamma}^{\mathrm{T}}(p,t)\hat{\boldsymbol{\vartheta}}(t-1), \quad (58)$$
$$\boldsymbol{L}(t)=\boldsymbol{P}(t-1)\boldsymbol{\Gamma}(p,t)[\boldsymbol{I}+\boldsymbol{\Gamma}^{\mathrm{T}}(p,t)\boldsymbol{P}(t-1)\boldsymbol{\Gamma}(p,t)]^{-1}, \quad (59)$$
$$\boldsymbol{P}(t)=\boldsymbol{P}(t-1)-\boldsymbol{L}(t)\boldsymbol{\Gamma}^{\mathrm{T}}(p,t)\boldsymbol{P}(t-1),$$
$$\boldsymbol{P}(0)=p_0\boldsymbol{I}_{n_0}, \quad (60)$$
$$\boldsymbol{Y}(p,t)=[\boldsymbol{y}^{\mathrm{T}}(t),\boldsymbol{y}^{\mathrm{T}}(t-1),\cdots,\boldsymbol{y}^{\mathrm{T}}(t-p+1)]^{\mathrm{T}}, \quad (61)$$
$$\boldsymbol{\Gamma}(p,t)=[\boldsymbol{\Psi}^{\mathrm{T}}(t),\boldsymbol{\Psi}^{\mathrm{T}}(t-1),\cdots,\boldsymbol{\Psi}^{\mathrm{T}}(t-p+1)], \quad (62)$$
$$\boldsymbol{\Psi}(t)=[\hat{\boldsymbol{\phi}}(t),\boldsymbol{\varphi}^{\mathrm{T}}(t)\otimes\boldsymbol{I}_m], \quad (63)$$
$$\boldsymbol{\varphi}(t)=[\boldsymbol{u}^{\mathrm{T}}(t-1),\boldsymbol{u}^{\mathrm{T}}(t-2),\cdots,\boldsymbol{u}^{\mathrm{T}}(t-n)]^{\mathrm{T}}, \quad (64)$$
$$\hat{\boldsymbol{\phi}}(t)=[-\hat{\boldsymbol{x}}(t-1),-\hat{\boldsymbol{x}}(t-2),\cdots,-\hat{\boldsymbol{x}}(t-n)], \quad (65)$$
$$\hat{\boldsymbol{x}}(t)=\boldsymbol{\Psi}(t)\hat{\boldsymbol{\vartheta}}(t),\quad \hat{\boldsymbol{x}}(-i)=\mathbf{1}_m/p_0,\quad i=0,1,\cdots,n-1. \quad (66)$$

1.3 变递推间隔多元辅助模型多新息辨识方法

为了处理数据丢失的情况,定义一个整数序列(integer sequence)$\{t_s,s=0,1,2,\cdots\}$满足

$$0=t_0<t_1<t_2<t_3<\cdots<t_{s-1}<t_s<\cdots,$$

且 $t_s^*:=t_s-t_{s-1}\geqslant 1$,使得 $t=t_s(s=1,2,3,\cdots)$时,$\boldsymbol{y}(t)$可得到,即对任意 $s=1,2,3,\cdots,\boldsymbol{y}(t_s)$都可得到.

1.3.1 变递推间隔多元辅助模型多新息投影算法

参考文献[1,23],定义和使用梯度搜索极小化准则函数(criterion function)

$$J_3(\boldsymbol{\vartheta}):=\|\boldsymbol{Y}(p,t_s)-\boldsymbol{\Gamma}^{\mathrm{T}}(p,t_s)\boldsymbol{\vartheta}\|^2, \quad (67)$$

能够得到变递推间隔多元辅助模型多新息投影算法(interval-Varying Multivariate Auxiliary Model based Multi-Innovation Projection algorithm, V-M-AM-MI-Proj 算法)[1,23]:

$$\hat{\boldsymbol{\vartheta}}(t_s)=\hat{\boldsymbol{\vartheta}}(t_{s-1})+\mu(t_s)\boldsymbol{\Gamma}(p,t_s)\boldsymbol{E}(p,t_s),$$
$$s=1,2,3,\cdots \quad (68)$$
$$\hat{\boldsymbol{\vartheta}}(t)=\hat{\boldsymbol{\vartheta}}(t_s),\quad t\in T_s:=\{t_s,t_s+1,\cdots,t_{s+1}-1\}, \quad (69)$$
$$\mu(t_s)=\frac{\|\boldsymbol{\Gamma}(p,t_s)\boldsymbol{E}(p,t_s)\|^2}{\|\boldsymbol{\Gamma}^{\mathrm{T}}(p,t_s)\boldsymbol{\Gamma}(p,t_s)\boldsymbol{E}(p,t_s)\|^2}, \quad (70)$$
$$\boldsymbol{E}(p,t_s)=\boldsymbol{Y}(p,t_s)-\boldsymbol{\Gamma}^{\mathrm{T}}(p,t_s)\hat{\boldsymbol{\vartheta}}(t_{s-1}), \quad (71)$$
$$\boldsymbol{\Gamma}(p,t_s)=[\boldsymbol{\Psi}^{\mathrm{T}}(t_s),\boldsymbol{\Psi}^{\mathrm{T}}(t_s-1),\cdots,\boldsymbol{\Psi}^{\mathrm{T}}(t_s-p+1)], \quad (72)$$
$$\boldsymbol{Y}(p,t_s)=[\boldsymbol{y}^{\mathrm{T}}(t_s),\boldsymbol{y}^{\mathrm{T}}(t_s-1),\cdots,\boldsymbol{y}^{\mathrm{T}}(t_s-p+1)]^{\mathrm{T}}, \quad (73)$$
$$\boldsymbol{\Psi}(t_s)=[\hat{\boldsymbol{\phi}}(t_s),\boldsymbol{\varphi}^{\mathrm{T}}(t_s)\otimes\boldsymbol{I}_m], \quad (74)$$
$$\boldsymbol{\varphi}(t_s)=[\boldsymbol{u}^{\mathrm{T}}(t_s-1),\boldsymbol{u}^{\mathrm{T}}(t_s-2),\cdots,\boldsymbol{u}^{\mathrm{T}}(t_s-n)]^{\mathrm{T}}, \quad (75)$$
$$\hat{\boldsymbol{\phi}}(t_s)=[-\hat{\boldsymbol{x}}(t_s-1),-\hat{\boldsymbol{x}}(t_s-2),\cdots,-\hat{\boldsymbol{x}}(t_s-n)], \quad (76)$$
$$\hat{\boldsymbol{x}}(t_s)=\boldsymbol{\Psi}(t_s)\hat{\boldsymbol{\vartheta}}(t_s),\quad \hat{\boldsymbol{x}}(t_1-i)=\mathbf{1}_m/p_0,\quad i=1,2,\cdots,n. \quad (77)$$

当 $t_s^*=p=1$ 时，多元辅助模型多新息算法就是常规的单新息多元辅助模型投影算法.式(69)表示在数据丢失的区间内，参数估计保持不变.这个算法计算收敛因子(步长)$\mu(t_s)$比较复杂，下面给出简化的变递推间隔多元辅助模型多新息投影算法(V-M-AM-MI-Proj 算法)[1,23]：

$$\hat{\boldsymbol{\vartheta}}(t_s)=\hat{\boldsymbol{\vartheta}}(t_{s-1})+\frac{\boldsymbol{\Gamma}(p,t_s)}{\|\boldsymbol{\Gamma}(p,t_s)\|^2}\boldsymbol{E}(p,t_s),\quad s=1,2,3,\cdots \tag{78}$$

$$\hat{\boldsymbol{\vartheta}}(t)=\hat{\boldsymbol{\vartheta}}(t_s),\quad t\in T_s:=\{t_s,t_s+1,\cdots,t_{s+1}-1\}, \tag{79}$$

$$\boldsymbol{E}(p,t_s)=\boldsymbol{Y}(p,t_s)-\boldsymbol{\Gamma}^{\mathrm{T}}(p,t_s)\hat{\boldsymbol{\vartheta}}(t_{s-1}), \tag{80}$$

$$\boldsymbol{\Gamma}(p,t_s)=[\boldsymbol{\Psi}^{\mathrm{T}}(t_s),\boldsymbol{\Psi}^{\mathrm{T}}(t_s-1),\cdots,\boldsymbol{\Psi}^{\mathrm{T}}(t_s-p+1)], \tag{81}$$

$$\boldsymbol{Y}(p,t_s)=[\boldsymbol{y}^{\mathrm{T}}(t_s),\boldsymbol{y}^{\mathrm{T}}(t_s-1),\cdots,\boldsymbol{y}^{\mathrm{T}}(t_s-p+1)]^{\mathrm{T}}, \tag{82}$$

$$\boldsymbol{\Psi}(t_s)=[\hat{\boldsymbol{\phi}}(t_s),\boldsymbol{\varphi}(t_s)\otimes\boldsymbol{I}_m], \tag{83}$$

$$\boldsymbol{\varphi}(t_s)=[\boldsymbol{u}^{\mathrm{T}}(t_s-1),\boldsymbol{u}^{\mathrm{T}}(t_s-2),\cdots,\boldsymbol{u}^{\mathrm{T}}(t_s-n)]^{\mathrm{T}}, \tag{84}$$

$$\hat{\boldsymbol{\phi}}(t_s)=[-\hat{\boldsymbol{x}}(t_s-1),-\hat{\boldsymbol{x}}(t_s-2),\cdots,-\hat{\boldsymbol{x}}(t_s-n)], \tag{85}$$

$$\hat{\boldsymbol{x}}(t_s)=\boldsymbol{\Psi}(t_s)\hat{\boldsymbol{\vartheta}}(t_s),\quad \hat{\boldsymbol{x}}(t_1-i)=\mathbf{1}_m/p_0,\quad i=1,2,\cdots,n. \tag{86}$$

为了防止式(78)右边第二项分母为零，解决的方法之一是，当 $\|\boldsymbol{\Gamma}(p,t_s)\|^2=0$ 时，令 $\hat{\boldsymbol{\vartheta}}(t_s)=\hat{\boldsymbol{\vartheta}}(t_{s-1})$，或者在式(78)右边第二项分母上加上一个正常数，将式(78)修改为

$$\hat{\boldsymbol{\vartheta}}(t_s)=\hat{\boldsymbol{\vartheta}}(t_{s-1})+\frac{\boldsymbol{\Gamma}(p,t_s)}{1+\|\boldsymbol{\Gamma}(p,t_s)\|^2}[\boldsymbol{Y}(p,t_s)-\boldsymbol{\Gamma}^{\mathrm{T}}(p,t_s)\hat{\boldsymbol{\vartheta}}(t_{s-1})].$$

变递推间隔多元辅助模型多新息投影辨识算法对噪声敏感，下面是变递推间隔辅助模型多新息广义投影辨识算法.

1.3.2 变递推间隔多元辅助模型多新息广义投影算法

变递推间隔多元辅助模型多新息投影辨识算法(78)—(86)可以推广为变递推间隔多元辅助模型多新息广义投影算法(interval-Varying Multivariate Auxiliary Model based Multi-Innovation Generalized Projection algorithm, V-M-AM-MI-GP 算法)[1,21,25]：

$$\hat{\boldsymbol{\vartheta}}(t_s)=\hat{\boldsymbol{\vartheta}}(t_{s-1})+\frac{\boldsymbol{\Gamma}(p,t_s)}{r(q,t_s)}[\boldsymbol{Y}(p,t_s)-\boldsymbol{\Gamma}^{\mathrm{T}}(p,t_s)\hat{\boldsymbol{\vartheta}}(t_{s-1})],\quad s=1,2,3,\cdots \tag{87}$$

$$\hat{\boldsymbol{\vartheta}}(t)=\hat{\boldsymbol{\vartheta}}(t_s),\quad t\in T_s=\{t_s,t_s+1,\cdots,t_{s+1}-1\}, \tag{88}$$

$$r(q,t_s)=\mathrm{tr}[\boldsymbol{\Gamma}(q,t_s)\boldsymbol{\Gamma}^{\mathrm{T}}(q,t_s)],\quad q\geqslant p, \tag{89}$$

$$\boldsymbol{\Gamma}(p,t_s)=[\boldsymbol{\Psi}^{\mathrm{T}}(t_s),\boldsymbol{\Psi}^{\mathrm{T}}(t_s-1),\cdots,\boldsymbol{\Psi}^{\mathrm{T}}(t_s-p+1)], \tag{90}$$

$$\boldsymbol{Y}(p,t_s)=[\boldsymbol{y}^{\mathrm{T}}(t_s),\boldsymbol{y}^{\mathrm{T}}(t_s-1),\cdots,\boldsymbol{y}^{\mathrm{T}}(t_s-p+1)]^{\mathrm{T}}, \tag{91}$$

$$\boldsymbol{\Psi}(t_s)=[\hat{\boldsymbol{\phi}}(t_s),\boldsymbol{\varphi}^{\mathrm{T}}(t_s)\otimes\boldsymbol{I}_m], \tag{92}$$

$$\boldsymbol{\varphi}(t_s)=[\boldsymbol{u}^{\mathrm{T}}(t_s-1),\boldsymbol{u}^{\mathrm{T}}(t_s-2),\cdots,\boldsymbol{u}^{\mathrm{T}}(t_s-n)]^{\mathrm{T}}, \tag{93}$$

$$\hat{\boldsymbol{\phi}}(t_s)=[-\hat{\boldsymbol{x}}(t_s-1),-\hat{\boldsymbol{x}}(t_s-2),\cdots,-\hat{\boldsymbol{x}}(t_s-n)], \tag{94}$$

$$\hat{\boldsymbol{x}}(t_s)=\boldsymbol{\Psi}(t_s)\hat{\boldsymbol{\vartheta}}(t_s),\quad \hat{\boldsymbol{x}}(t_1-i)=\mathbf{1}_m/p_0,\quad i=1,2,\cdots,n. \tag{95}$$

变递推间隔多元辅助模型多新息广义投影辨识算法可以通过加大 q 来克服噪声敏感性，适当选择 q 可以减小时变多元系统参数估计误差上界.

1.3.3 变递推间隔多元辅助模型多新息随机梯度算法

进一步可推广为变递推间隔多元辅助模型多新息随机梯度算法(interval-Varying Multivariate Auxiliary Model based Multi-Innovation Stochastic Gradient algorithm, V-M-AM-MISG 算法)[1,21,23,25]：

$$\hat{\boldsymbol{\vartheta}}(t_s)=\hat{\boldsymbol{\vartheta}}(t_{s-1})+\frac{\boldsymbol{\Gamma}(p,t_s)}{r(t_s)}[\boldsymbol{Y}(p,t_s)-\boldsymbol{\Gamma}^{\mathrm{T}}(p,t_s)\hat{\boldsymbol{\vartheta}}(t_{s-1})],\quad s=1,2,3,\cdots \tag{96}$$

$$\hat{\boldsymbol{\vartheta}}(t)=\hat{\boldsymbol{\vartheta}}(t_s),\quad t\in T_s=\{t_s,t_s+1,\cdots,t_{s+1}-1\}, \tag{97}$$

$$r(t_s)=\|\boldsymbol{\Gamma}(s,t_s)\|^2=\sum_{i=0}^{s-1}\|\boldsymbol{\Psi}(t_s-i)\|^2, \tag{98}$$

$$\boldsymbol{\Gamma}(p,t_s)=[\boldsymbol{\Psi}^{\mathrm{T}}(t_s),\boldsymbol{\Psi}^{\mathrm{T}}(t_s-1),\cdots,\boldsymbol{\Psi}^{\mathrm{T}}(t_s-p+1)], \tag{99}$$

$$\boldsymbol{Y}(p,t_s)=[\boldsymbol{y}^{\mathrm{T}}(t_s),\boldsymbol{y}^{\mathrm{T}}(t_s-1),\cdots,\boldsymbol{y}^{\mathrm{T}}(t_s-p+1)]^{\mathrm{T}}, \tag{100}$$

$$\boldsymbol{\Psi}(t_s)=[\hat{\boldsymbol{\phi}}(t_s),\boldsymbol{\varphi}^{\mathrm{T}}(t_s)\otimes\boldsymbol{I}_m], \tag{101}$$

$$\boldsymbol{\varphi}(t_s)=[\boldsymbol{u}^{\mathrm{T}}(t_s-1),\boldsymbol{u}^{\mathrm{T}}(t_s-2),\cdots,\boldsymbol{u}^{\mathrm{T}}(t_s-n)]^{\mathrm{T}}, \tag{102}$$

$$\hat{\boldsymbol{\phi}}(t_s)=[-\hat{\boldsymbol{x}}(t_s-1),-\hat{\boldsymbol{x}}(t_s-2),\cdots,-\hat{\boldsymbol{x}}(t_s-n)], \tag{103}$$

$$\hat{\boldsymbol{x}}(t_s)=\boldsymbol{\Psi}(t_s)\hat{\boldsymbol{\vartheta}}(t_s),\quad \hat{\boldsymbol{x}}(t_1-i)=\mathbf{1}_m/p_0,\quad i=1,2,\cdots,n. \tag{104}$$

当递推间隔 $t_s^*\equiv 1$ 时，V-M-AM-MISG 算法退化为 M-AM-MISG 算法.随着递推步数 s 的增加，$r(t_s)\to\infty$，算法增益矩阵 $\boldsymbol{\Gamma}(p,t_s)/r(t_s)$ 趋于零，故变递推间隔多元辅助模型多新息随机梯度辨识算法没有跟踪时变参数的能力.当然，也可以推导变递推间隔修正多元辅助模型多新息随机梯度辨识算法和变递推间隔遗忘因子多元辅助模型多新息随机梯度辨识算法.

1.3.4 变递推间隔多元辅助模型多新息最小二乘算法

借助于最小二乘原理，将 V-M-AM-MI-GP 算法进一步推广，能够得到变递推间隔多元辅助模型多新息递推最小二乘算法(interval-Varying Multivariate Auxiliary Model based Multi-Innovation Recursive Least Squares algorithm, V-M-AM-MI-RLS 算法)[1,21,23]：

$$\hat{\boldsymbol{\vartheta}}(t_s)=\hat{\boldsymbol{\vartheta}}(t_{s-1})+\boldsymbol{L}(t_s)[\boldsymbol{Y}(p,t_s)-\boldsymbol{\Gamma}^{\mathrm{T}}(p,t_s)\hat{\boldsymbol{\vartheta}}(t_{s-1})],\quad s=1,2,3,\cdots \tag{105}$$

$$\hat{\boldsymbol{\vartheta}}(t)=\hat{\boldsymbol{\vartheta}}(t_s),\quad t\in T_s=\{t_s,t_s+1,\cdots,t_{s+1}-1\}, \tag{106}$$

$$\boldsymbol{L}(t_s)=\boldsymbol{P}(t_s)\boldsymbol{\Gamma}(p,t_s)=\boldsymbol{P}(t_{s-1})\boldsymbol{\Gamma}(p,t_s)[\boldsymbol{I}_{mp}+\boldsymbol{\Gamma}^{\mathrm{T}}(p,t_s)\boldsymbol{P}(t_{s-1})\boldsymbol{\Gamma}(p,t_s)]^{-1},\qquad(107)$$

$$\boldsymbol{P}(t_s)=\boldsymbol{P}(t_{s-1})-\boldsymbol{L}(t_s)\boldsymbol{\Gamma}^{\mathrm{T}}(p,t_s)\boldsymbol{P}(t_{s-1}),\quad \boldsymbol{P}(0)=p_0\boldsymbol{I}_{n_0},\qquad(108)$$

$$\boldsymbol{\Gamma}(p,t_s)=[\boldsymbol{\Psi}^{\mathrm{T}}(t_s),\boldsymbol{\Psi}^{\mathrm{T}}(t_s-1),\cdots,\boldsymbol{\Psi}^{\mathrm{T}}(t_s-p+1)],\qquad(109)$$

$$\boldsymbol{Y}(p,t_s)=[\boldsymbol{y}^{\mathrm{T}}(t_s),\boldsymbol{y}^{\mathrm{T}}(t_s-1),\cdots,\boldsymbol{y}^{\mathrm{T}}(t_s-p+1)]^{\mathrm{T}},\qquad(110)$$

$$\boldsymbol{\Psi}(t_s)=[\hat{\boldsymbol{\phi}}(t_s),\boldsymbol{\varphi}^{\mathrm{T}}(t_s)\otimes\boldsymbol{I}_m],\qquad(111)$$

$$\boldsymbol{\varphi}(t_s)=[\boldsymbol{u}^{\mathrm{T}}(t_s-1),\boldsymbol{u}^{\mathrm{T}}(t_s-2),\cdots,\boldsymbol{u}^{\mathrm{T}}(t_s-n)]^{\mathrm{T}},\qquad(112)$$

$$\hat{\boldsymbol{\phi}}(t_s)=[-\hat{\boldsymbol{x}}(t_s-1),-\hat{\boldsymbol{x}}(t_s-2),\cdots,-\hat{\boldsymbol{x}}(t_s-n)],\qquad(113)$$

$$\hat{\boldsymbol{x}}(t_s)=\boldsymbol{\Psi}(t_s)\hat{\boldsymbol{\vartheta}}(t_s),\quad \hat{\boldsymbol{x}}(t_1-i)=\boldsymbol{1}_m/p_0,\quad i=1,2,\cdots,n.\qquad(114)$$

在这些变递推间隔的辨识算法中，递推间隔 t_s 不必取连续的自然数，可以是变化的.当遇到坏数据或不可信数据时，可以跳过这部分数据，因而具有克服损失数据的能力.

2 部分耦合辅助模型辨识方法

将类多变量输出误差系统(1)的递阶辨识模型(5)重写如下：

$$\boldsymbol{y}(t)=\boldsymbol{\phi}(t)\boldsymbol{\alpha}+\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\varphi}(t)+\boldsymbol{v}(t).\qquad(115)$$

辨识模型(115)中包含了一个参数向量 $\boldsymbol{\alpha}$ 和一个参数矩阵 $\boldsymbol{\theta}$，辨识的目标是基于耦合辨识概念和多新息辨识理论，利用系统的观测数据$\{\boldsymbol{u}(t),\boldsymbol{y}(t):t=1,2,3,\cdots\}$，提出耦合多新息类辨识方法，估计系统模型参数向量 $\boldsymbol{\alpha}$ 和参数矩阵 $\boldsymbol{\theta}$.

令 $\boldsymbol{\theta}_i$ 为参数矩阵 $\boldsymbol{\theta}$ 的第 i 列，$\boldsymbol{\phi}_i^{\mathrm{T}}(t)$ 为信息矩阵 $\boldsymbol{\phi}(t)$ 的第 i 行，即

$$\boldsymbol{\theta}:=[\boldsymbol{\theta}_1,\boldsymbol{\theta}_2,\cdots,\boldsymbol{\theta}_m]\in\mathbf{R}^{(nr)\times m},$$

$$\boldsymbol{\phi}(t):=[\boldsymbol{\phi}_1(t),\boldsymbol{\phi}_2(t),\cdots,\boldsymbol{\phi}_m(t)]^{\mathrm{T}}\in\mathbf{R}^{m\times n}.$$

将辨识模型(115)进一步写为

$$\begin{bmatrix}y_1(t)\\y_2(t)\\\vdots\\y_m(t)\end{bmatrix}=\begin{bmatrix}\boldsymbol{\phi}_1^{\mathrm{T}}(t)\\\boldsymbol{\phi}_2^{\mathrm{T}}(t)\\\vdots\\\boldsymbol{\phi}_m^{\mathrm{T}}(t)\end{bmatrix}\boldsymbol{\alpha}+\begin{bmatrix}\boldsymbol{\theta}_1^{\mathrm{T}}\\\boldsymbol{\theta}_2^{\mathrm{T}}\\\vdots\\\boldsymbol{\theta}_m^{\mathrm{T}}\end{bmatrix}\boldsymbol{\varphi}(t)+\begin{bmatrix}v_1(t)\\v_2(t)\\\vdots\\v_m(t)\end{bmatrix}.$$

上式可以分解为 m 个子辨识模型

$$y_i(t)=\boldsymbol{\phi}_i^{\mathrm{T}}(t)\boldsymbol{\alpha}+\boldsymbol{\theta}_i^{\mathrm{T}}\boldsymbol{\varphi}(t)+v_i(t)=\boldsymbol{\phi}_i^{\mathrm{T}}(t)\boldsymbol{\alpha}+\boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{\theta}_i+v_i(t)=[\boldsymbol{\phi}_i^{\mathrm{T}}(t),\boldsymbol{\varphi}^{\mathrm{T}}(t)]\begin{bmatrix}\boldsymbol{\alpha}\\\boldsymbol{\theta}_i\end{bmatrix}+v_i(t),\quad i=1,2,\cdots,m.\qquad(116)$$

定义子系统信息向量为 $\boldsymbol{\psi}_i(t):=\begin{bmatrix}\boldsymbol{\phi}_i(t)\\\boldsymbol{\varphi}(t)\end{bmatrix}\in\mathbf{R}^{n+nr}$，则子辨识模型(116)可以写成

$$y_i(t)=\boldsymbol{\psi}_i^{\mathrm{T}}(t)\begin{bmatrix}\boldsymbol{\alpha}\\\boldsymbol{\theta}_i\end{bmatrix}+v_i(t),\quad i=1,2,\cdots,m.\qquad(117)$$

在式(116)的 m 个子辨识模型中，每个子辨识模型有一个共同的子参数向量 $\boldsymbol{\alpha}$ 和一个共同的子信息向量 $\boldsymbol{\varphi}(t)$，每个子辨识模型还包含了一个不同的子参数向量 $\boldsymbol{\theta}_i$ 和一个不同的子信息向量 $\boldsymbol{\phi}_i(t)$.下面研究部分耦合辅助模型随机梯度辨识方法和部分耦合辅助模型最小二乘辨识方法.

2.1 部分耦合子系统辅助模型辨识算法

2.2.1 子系统辅助模型随机梯度辨识算法

使用负梯度搜索，根据式(117)，定义和极小化梯度准则函数

$$J_3(\boldsymbol{\alpha},\boldsymbol{\theta}_i):=\sum_{j=1}^{t}\left\{y_i(j)-\boldsymbol{\psi}_i^{\mathrm{T}}(j)\begin{bmatrix}\boldsymbol{\alpha}\\\boldsymbol{\theta}_i\end{bmatrix}\right\}^2,i=1,2,\cdots,m,$$

可以得到估计参数向量 $\boldsymbol{\alpha}$ 和 $\boldsymbol{\theta}_i$ 的子系统随机梯度辨识算法：

$$\begin{bmatrix}\hat{\boldsymbol{\alpha}}(t)\\\hat{\boldsymbol{\theta}}_i(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}(t-1)\\\hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}+\frac{\boldsymbol{\psi}_i(t)}{r_i(t)}\left\{y_i(t)-\boldsymbol{\psi}_i^{\mathrm{T}}(t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}(t-1)\\\hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}\right\},\qquad(118)$$

$$r_i(t)=r_i(t-1)+\|\boldsymbol{\psi}_i(t)\|^2,\quad r_i(0)=1.\qquad(119)$$

由于 $\boldsymbol{\psi}_i(t)$ 中包含的 $\boldsymbol{x}(t-i)$ 是未知的，上述算法中参数向量的估计 $\hat{\boldsymbol{\alpha}}(t)$ 和 $\hat{\boldsymbol{\theta}}_i(t)$ 无法实现.借助于辅助模型辨识思想，将信息向量 $\boldsymbol{\psi}_i(t)$ 中的 $\boldsymbol{x}(t-i)$ 用其估计 $\hat{\boldsymbol{x}}(t-i)$ 代替，定义

$$\hat{\boldsymbol{\psi}}_i(t):=\begin{bmatrix}\hat{\boldsymbol{\phi}}_i(t)\\\boldsymbol{\varphi}(t)\end{bmatrix}\in\mathbf{R}^{n+nr},\quad i=1,2,\cdots,m,$$

$$\hat{\boldsymbol{\phi}}(t):=[\hat{\boldsymbol{\phi}}_1(t),\hat{\boldsymbol{\phi}}_2(t),\cdots,\hat{\boldsymbol{\phi}}_m(t)]^{\mathrm{T}}\in\mathbf{R}^{m\times n},$$

$$\hat{\boldsymbol{\phi}}(t):=[-\hat{\boldsymbol{x}}(t-1),-\hat{\boldsymbol{x}}(t-2),\cdots,-\hat{\boldsymbol{x}}(t-n)]\in\mathbf{R}^{m\times n}.$$

用 $\hat{\boldsymbol{\psi}}_i(t)$ 代替式(118)—(119)中 $\boldsymbol{\psi}_i(t)$，这种代替后的辨识算法是可实现的.为区分每个子系统中的 $\hat{\boldsymbol{\alpha}}(t)$，加下标 i，记作 $\hat{\boldsymbol{\alpha}}_i(t)$，便得到子系统辅助模型随机梯度算法(Subsystem Auxiliary Model based Stochastic Gradient algorithm，S-AM-SG 算法)[1]：

$$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_i(t)\\\hat{\boldsymbol{\theta}}_i(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}_i(t-1)\\\hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}+\frac{\hat{\boldsymbol{\psi}}_i(t)}{r_i(t)}\left\{y_i(t)-\hat{\boldsymbol{\psi}}_i^{\mathrm{T}}(t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}_i(t-1)\\\hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}\right\},$$

$$\hat{\boldsymbol{\alpha}}_i(0)=\boldsymbol{1}_n/p_0,\quad \hat{\boldsymbol{\theta}}_i(0)=\boldsymbol{1}_{nr}/p_0,\qquad(120)$$

$$r_i(t)=r_i(t-1)+\|\hat{\boldsymbol{\psi}}_i(t)\|^2,\quad r_i(0)=1,\qquad(121)$$

$$\hat{\boldsymbol{\psi}}_i(t)=\begin{bmatrix}\hat{\boldsymbol{\phi}}_i(t)\\\boldsymbol{\varphi}(t)\end{bmatrix},\qquad(122)$$

$$\boldsymbol{\varphi}(t)=[\boldsymbol{u}^{\mathrm{T}}(t-1),\boldsymbol{u}^{\mathrm{T}}(t-2),\cdots,\boldsymbol{u}^{\mathrm{T}}(t-n)]^{\mathrm{T}},\qquad(123)$$

$$\hat{\boldsymbol{\phi}}(t)=[-\hat{\boldsymbol{x}}(t-1),-\hat{\boldsymbol{x}}(t-2),\cdots,-\hat{\boldsymbol{x}}(t-n)]\qquad(124)$$

$$=[\hat{\boldsymbol{\phi}}_1(t),\hat{\boldsymbol{\phi}}_2(t),\cdots,\hat{\boldsymbol{\phi}}_m(t)]^{\mathrm{T}},\qquad(125)$$

$$\hat{\boldsymbol{x}}(t)=[\hat{x}_1(t),\hat{x}_2(t),\cdots,\hat{x}_m(t)]^{\mathrm{T}},\qquad(126)$$

$$\hat{x}_i(t)=\hat{\boldsymbol{\phi}}_i^{\mathrm{T}}(t)\hat{\boldsymbol{\alpha}}_i(t)+\boldsymbol{\varphi}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}_i(t).\qquad(127)$$

式(127)为估算未知内部变量 $x_i(t)$ 的辅助模型.表 5 列出了 S-AM-SG 算法(120)—(127)中各变量的维数.

表 5　S-AM-SG 算法中各变量维数

Table 5　The dimensions of the variables in the S-AM-SG algorithm

变量名称	维数
输出向量	$y_i(t)\in\mathbf{R}$
输入向量	$\boldsymbol{u}(t)\in\mathbf{R}^r$
参数估计向量	$\hat{\boldsymbol{\theta}}_i(t)\in\mathbf{R}^{nr}$
参数估计向量	$\hat{\boldsymbol{\alpha}}_i(t)\in\mathbf{R}^n$
估计的信息矩阵	$\hat{\boldsymbol{\phi}}(t)\in\mathbf{R}^{m\times n}$
估计的信息向量	$\hat{\boldsymbol{\psi}}_i(t)\in\mathbf{R}^{n+nr}$
输入信息向量	$\boldsymbol{\varphi}(t)\in\mathbf{R}^{nr}$
估计的信息向量	$\hat{\boldsymbol{\phi}}_i(t)\in\mathbf{R}^n$
辅助模型的输出向量	$\hat{\boldsymbol{x}}(t)\in\mathbf{R}^m$

2.1.2　子系统辅助模型递推最小二乘辨识算法

对于子辨识模型(117),定义和极小化最小二乘准则函数

$$J_4(\boldsymbol{\alpha},\boldsymbol{\theta}_i):=\sum_{j=1}^{t}\left\{y_i(j)-\boldsymbol{\psi}_i^{\mathrm{T}}(j)\begin{bmatrix}\boldsymbol{\alpha}\\\boldsymbol{\theta}_i\end{bmatrix}\right\}^2,\quad i=1,2,\cdots,m,$$

参照文献[1],可以得到估计参数向量 $\boldsymbol{\alpha}$ 和 $\boldsymbol{\theta}_i$ 的子系统递推最小二乘算法:

$$\begin{bmatrix}\hat{\boldsymbol{\alpha}}(t)\\\hat{\boldsymbol{\theta}}_i(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}(t-1)\\\hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}+\boldsymbol{P}_i(t)\boldsymbol{\psi}_i(t)\left\{y_i(t)-\boldsymbol{\psi}_i^{\mathrm{T}}(t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}(t-1)\\\hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}\right\},\tag{128}$$

$$\boldsymbol{P}_i^{-1}(t)=\boldsymbol{P}_i^{-1}(t-1)+\boldsymbol{\psi}_i(t)\boldsymbol{\psi}_i^{\mathrm{T}}(t).\tag{129}$$

与子系统随机梯度辨识算法一样,$\boldsymbol{\psi}_i(t)$ 中包含未知的 $\boldsymbol{x}(t-i)$ 同样用其估计 $\hat{\boldsymbol{x}}(t-i)$ 代替,并给每个子系统中 $\boldsymbol{\alpha}$ 的估计加下标 i,便得到下列子系统辅助模型递推最小二乘算法(Subsystem Auxiliary Model based Recursive Least Squares algorithm,S-AM-RLS 算法)[1]:

$$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_i(t)\\\hat{\boldsymbol{\theta}}_i(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}_i(t-1)\\\hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}+\boldsymbol{P}_i(t)\hat{\boldsymbol{\psi}}_i(t)\left\{y_i(t)-\hat{\boldsymbol{\psi}}_i^{\mathrm{T}}(t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}_i(t-1)\\\hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}\right\},\tag{130}$$

$$\boldsymbol{P}_i^{-1}(t)=\boldsymbol{P}_i^{-1}(t-1)+\hat{\boldsymbol{\psi}}_i(t)\hat{\boldsymbol{\psi}}_i^{\mathrm{T}}(t),\tag{131}$$

$$\hat{\boldsymbol{\psi}}_i(t)=\begin{bmatrix}\hat{\boldsymbol{\phi}}_i(t)\\\boldsymbol{\varphi}(t)\end{bmatrix},\tag{132}$$

$$\boldsymbol{\varphi}(t)=[\boldsymbol{u}^{\mathrm{T}}(t-1),\boldsymbol{u}^{\mathrm{T}}(t-2),\cdots,\boldsymbol{u}^{\mathrm{T}}(t-n)]^{\mathrm{T}},\tag{133}$$

$$\hat{\boldsymbol{\phi}}(t)=[-\hat{\boldsymbol{x}}(t-1),-\hat{\boldsymbol{x}}(t-2),\cdots,-\hat{\boldsymbol{x}}(t-n)]\tag{134}$$

$$=[\hat{\boldsymbol{\phi}}_1(t),\hat{\boldsymbol{\phi}}_2(t),\cdots,\hat{\boldsymbol{\phi}}_m(t)]^{\mathrm{T}},\tag{135}$$

$$\hat{\boldsymbol{x}}(t)=[\hat{x}_1(t),\hat{x}_2(t),\cdots,\hat{x}_m(t)]^{\mathrm{T}},\tag{136}$$

$$\hat{x}_i(t)=\hat{\boldsymbol{\phi}}_i^{\mathrm{T}}(t)\hat{\boldsymbol{\alpha}}_i(t)+\boldsymbol{\varphi}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}_i(t).\tag{137}$$

为避免参数估计误差协方差阵 $\boldsymbol{P}_i(t)\in\mathbf{R}^{(n+nr)\times(n+nr)}$ 的求逆运算,在式(131)中应用矩阵求逆公式

$$(\boldsymbol{A}+\boldsymbol{BC})^{-1}=\boldsymbol{A}^{-1}-\boldsymbol{A}^{-1}\boldsymbol{B}(\boldsymbol{I}+\boldsymbol{CA}^{-1}\boldsymbol{B})^{-1}\boldsymbol{CA}^{-1},$$

并引入增益向量 $\boldsymbol{L}_i(t):=\boldsymbol{P}_i(t)\hat{\boldsymbol{\psi}}_i(t)\in\mathbf{R}^{n+nr}$,则 S-AM-RLS 算法(130)—(137)可以等价表示为

$$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_i(t)\\\hat{\boldsymbol{\theta}}_i(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}_i(t-1)\\\hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}+\boldsymbol{L}_i(t)\left\{y_i(t)-\hat{\boldsymbol{\psi}}_i^{\mathrm{T}}(t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}_i(t-1)\\\hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}\right\},$$

$$\hat{\boldsymbol{\alpha}}_i(0)=\mathbf{1}_n/p_0,\quad\hat{\boldsymbol{\theta}}_i(0)=\mathbf{1}_{nr}/p_0,\tag{138}$$

$$\boldsymbol{L}_i(t)=\boldsymbol{P}_i(t)\hat{\boldsymbol{\psi}}_i(t)=\boldsymbol{P}_i(t-1)\hat{\boldsymbol{\psi}}_i(t)[1+\hat{\boldsymbol{\psi}}_i^{\mathrm{T}}(t)\boldsymbol{P}_i(t-1)\hat{\boldsymbol{\psi}}_i(t)]^{-1},\tag{139}$$

$$\boldsymbol{P}_i(t)=[\boldsymbol{I}_{n+nr}-\boldsymbol{L}_i(t)\hat{\boldsymbol{\psi}}_i^{\mathrm{T}}(t)]\boldsymbol{P}_i(t-1),\quad\boldsymbol{P}_i(0)=p_0\boldsymbol{I}_{n+nr},\tag{140}$$

$$\hat{\boldsymbol{\psi}}_i(t)=\begin{bmatrix}\hat{\boldsymbol{\phi}}_i(t)\\\boldsymbol{\varphi}(t)\end{bmatrix},\tag{141}$$

$$\boldsymbol{\varphi}(t)=[\boldsymbol{u}^{\mathrm{T}}(t-1),\boldsymbol{u}^{\mathrm{T}}(t-2),\cdots,\boldsymbol{u}^{\mathrm{T}}(t-n)]^{\mathrm{T}},\tag{142}$$

$$\hat{\boldsymbol{\phi}}(t)=[-\hat{\boldsymbol{x}}(t-1),-\hat{\boldsymbol{x}}(t-2),\cdots,-\hat{\boldsymbol{x}}(t-n)]\tag{143}$$

$$=[\hat{\boldsymbol{\phi}}_1(t),\hat{\boldsymbol{\phi}}_2(t),\cdots,\hat{\boldsymbol{\phi}}_m(t)]^{\mathrm{T}},\tag{144}$$

$$\hat{\boldsymbol{x}}(t)=[\hat{x}_1(t),\hat{x}_2(t),\cdots,\hat{x}_m(t)]^{\mathrm{T}},\tag{145}$$

$$\hat{x}_i(t)=\hat{\boldsymbol{\phi}}_i^{\mathrm{T}}(t)\hat{\boldsymbol{\alpha}}_i(t)+\boldsymbol{\varphi}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}_i(t).\tag{146}$$

为了方便起见,表 6 列出了 S-AM-RLS 算法(138)—(146)中各变量的维数.

2.1.3　部分耦合子系统辅助模型随机梯度辨识算法

在 S-AM-SG 算法(120)—(127)中,对参数向量 $\boldsymbol{\alpha}$ 进行了 m 次估计,得到 m 个 $\hat{\boldsymbol{\alpha}}_i(t)$,而实际只需要一个估计,取每个子系统的估计 $\hat{\boldsymbol{\alpha}}_i(t)$ 的平均值作为参数向量 $\boldsymbol{\alpha}$ 的估计,即

$$\hat{\boldsymbol{\alpha}}(t)=\frac{\hat{\boldsymbol{\alpha}}_1(t)+\hat{\boldsymbol{\alpha}}_2(t)+\cdots+\hat{\boldsymbol{\alpha}}_m(t)}{m}\in\mathbf{R}^n.$$

用平均值 $\hat{\boldsymbol{\alpha}}(t-1)$ 代替 S-AM-SG 算法中的 $\hat{\boldsymbol{\alpha}}_i(t-1)$,就得到了一个简单的部分耦合子系统辅助模型随机梯度算法(Partially Coupled Subsystem Auxiliary Model based Stochastic Gradient algorithm,PC-S-AM-SG 算法):

表 6　S-AM-RLS 算法中各变量维数

Table 6　The dimensions of the variables in the S-AM-RLS algorithm

变量名称	维数
输出向量	$y_i(t)\in\mathbf{R}$
输入向量	$\boldsymbol{u}(t)\in\mathbf{R}^r$
参数估计向量	$\hat{\boldsymbol{\alpha}}_i(t)\in\mathbf{R}^n$
参数估计向量	$\hat{\boldsymbol{\theta}}_i(t)\in\mathbf{R}^{nr}$
增益向量	$\boldsymbol{L}_i(t)\in\mathbf{R}^{n+nr}$
协方差矩阵	$\boldsymbol{P}_i(t)\in\mathbf{R}^{(n+nr)\times(n+nr)}$
估计的信息矩阵	$\hat{\boldsymbol{\phi}}(t)\in\mathbf{R}^{m\times n}$
估计的信息向量	$\hat{\boldsymbol{\phi}}_i(t)\in\mathbf{R}^n$
估计的信息向量	$\hat{\boldsymbol{\psi}}_i(t)\in\mathbf{R}^{n+nr}$
输入信息向量	$\boldsymbol{\varphi}(t)\in\mathbf{R}^{nr}$
辅助模型的输出向量	$\hat{\boldsymbol{x}}(t)\in\mathbf{R}^m$

$$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_i(t)\\ \hat{\boldsymbol{\theta}}_i(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}(t-1)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}+\frac{\hat{\boldsymbol{\psi}}_i(t)}{r_i(t)}\left\{y_i(t)-\hat{\boldsymbol{\psi}}_i^{\mathrm{T}}(t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}(t-1)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}\right\},$$

$$\hat{\boldsymbol{\alpha}}(0)=\mathbf{1}_n/p_0,\quad \hat{\boldsymbol{\theta}}_i(0)=\mathbf{1}_{nr}/p_0,\tag{147}$$

$$r_i(t)=r_i(t-1)+\|\hat{\boldsymbol{\psi}}_i(t)\|^2,\quad r_i(0)=1,\tag{148}$$

$$\hat{\boldsymbol{\psi}}_i(t)=\begin{bmatrix}\hat{\boldsymbol{\phi}}_i(t)\\ \boldsymbol{\varphi}(t)\end{bmatrix},\tag{149}$$

$$\boldsymbol{\varphi}(t)=[\boldsymbol{u}^{\mathrm{T}}(t-1),\boldsymbol{u}^{\mathrm{T}}(t-2),\cdots,\boldsymbol{u}^{\mathrm{T}}(t-n)]^{\mathrm{T}},\tag{150}$$

$$\hat{\boldsymbol{\phi}}(t)=[-\hat{\boldsymbol{x}}(t-1),-\hat{\boldsymbol{x}}(t-2),\cdots,-\hat{\boldsymbol{x}}(t-n)]\tag{151}$$

$$=[\hat{\boldsymbol{\phi}}_1(t),\hat{\boldsymbol{\phi}}_2(t),\cdots,\hat{\boldsymbol{\phi}}_m(t)]^{\mathrm{T}},\tag{152}$$

$$\hat{\boldsymbol{x}}(t)=\hat{\boldsymbol{\phi}}(t)\hat{\boldsymbol{\alpha}}(t)+\hat{\boldsymbol{\theta}}^{\mathrm{T}}(t)\boldsymbol{\varphi}(t),\tag{153}$$

$$\hat{\boldsymbol{\alpha}}(t)=\frac{\hat{\boldsymbol{\alpha}}_1(t)+\hat{\boldsymbol{\alpha}}_2(t)+\cdots+\hat{\boldsymbol{\alpha}}_m(t)}{m},\tag{154}$$

$$\hat{\boldsymbol{\theta}}(t)=[\hat{\boldsymbol{\theta}}_1(t),\hat{\boldsymbol{\theta}}_2(t),\cdots,\hat{\boldsymbol{\theta}}_m(t)].\tag{155}$$

式(153)为估算未知内部变量 $\boldsymbol{x}(t)$ 的辅助模型.

为了提高部分耦合子系统辅助模型随机梯度算法的暂态性能和收敛速度,在式(147)中引入收敛指数(convergence index)ε,或在式(148)中引入遗忘因子(Forgetting Factor,FF)λ,则式(147)和式(148)可以改写为

$$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_i(t)\\ \hat{\boldsymbol{\theta}}_i(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}(t-1)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}+\frac{\hat{\boldsymbol{\psi}}_i(t)}{r_i^{\varepsilon}(t)}\left\{y_i(t)-\hat{\boldsymbol{\psi}}_i^{\mathrm{T}}(t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}(t-1)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}\right\},$$

$$\frac{1}{2}<\varepsilon\leqslant 1,$$

$$r_i(t)=\lambda r_i(t-1)+\|\hat{\boldsymbol{\psi}}_i(t)\|^2,\quad 0\leqslant\lambda\leqslant 1,\quad r_i(0)=1.$$

2.1.4　部分耦合子系统辅助模型递推最小二乘辨识算法

与 S-AM-SG 算法的处理方法相同,取每个子系统的估计 $\hat{\boldsymbol{\alpha}}_i(t)$ 的平均值作为参数向量 $\boldsymbol{\alpha}$ 的估计,用平均值 $\hat{\boldsymbol{\alpha}}(t-1)$ 代替 $\hat{\boldsymbol{\alpha}}_i(t-1)$,根据 S-AM-RLS 算法(138)—(146),能够获得部分耦合子系统辅助模型递推最小二乘算法(Partially Coupled Subsystem Auxiliary Model based Recursive Least Squares algorithm,PC-S-AM-RLS 算法)[1]:

$$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_i(t)\\ \hat{\boldsymbol{\theta}}_i(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}(t-1)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}+\boldsymbol{L}_i(t)\left\{y_i(t)-\hat{\boldsymbol{\psi}}_i^{\mathrm{T}}(t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}(t-1)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}\right\},$$

$$\hat{\boldsymbol{\alpha}}(0)=\mathbf{1}_n/p_0,\quad \hat{\boldsymbol{\theta}}_i(0)=\mathbf{1}_{nr}/p_0,\tag{156}$$

$$\boldsymbol{L}_i(t)=\boldsymbol{P}_i(t)\hat{\boldsymbol{\psi}}_i(t)=$$

$$\boldsymbol{P}_i(t-1)\hat{\boldsymbol{\psi}}_i(t)[1+\hat{\boldsymbol{\psi}}_i^{\mathrm{T}}(t)\boldsymbol{P}_i(t-1)\hat{\boldsymbol{\psi}}_i(t)]^{-1},\tag{157}$$

$$\boldsymbol{P}_i(t)=[\boldsymbol{I}_{n+nr}-\boldsymbol{L}_i(t)\hat{\boldsymbol{\psi}}_i^{\mathrm{T}}(t)]\boldsymbol{P}_i(t-1),$$

$$\boldsymbol{P}_i(0)=p_0\boldsymbol{I}_{n+nr},\tag{158}$$

$$\hat{\boldsymbol{\psi}}_i(t)=\begin{bmatrix}\hat{\boldsymbol{\phi}}_i(t)\\ \boldsymbol{\varphi}(t)\end{bmatrix},\tag{159}$$

$$\boldsymbol{\varphi}(t)=[\boldsymbol{u}^{\mathrm{T}}(t-1),\boldsymbol{u}^{\mathrm{T}}(t-2),\cdots,\boldsymbol{u}^{\mathrm{T}}(t-n)]^{\mathrm{T}},\tag{160}$$

$$\hat{\boldsymbol{\phi}}(t)=[-\hat{\boldsymbol{x}}(t-1),-\hat{\boldsymbol{x}}(t-2),\cdots,-\hat{\boldsymbol{x}}(t-n)]\tag{161}$$

$$=[\hat{\boldsymbol{\phi}}_1(t),\hat{\boldsymbol{\phi}}_2(t),\cdots,\hat{\boldsymbol{\phi}}_m(t)]^{\mathrm{T}},\tag{162}$$

$$\hat{\boldsymbol{x}}(t)=\hat{\boldsymbol{\phi}}(t)\hat{\boldsymbol{\alpha}}(t)+\hat{\boldsymbol{\theta}}^{\mathrm{T}}(t)\boldsymbol{\varphi}(t),\tag{163}$$

$$\hat{\boldsymbol{\alpha}}(t)=\frac{\hat{\boldsymbol{\alpha}}_1(t)+\hat{\boldsymbol{\alpha}}_2(t)+\cdots+\hat{\boldsymbol{\alpha}}_m(t)}{m},\tag{164}$$

$$\hat{\boldsymbol{\theta}}(t)=[\hat{\boldsymbol{\theta}}_1(t),\hat{\boldsymbol{\theta}}_2(t),\cdots,\hat{\boldsymbol{\theta}}_m(t)].\tag{165}$$

为了避免子系统辨识算法的冗余计算,下面基于部分耦合子系统辅助模型随机梯度算法和部分耦合子系统辅助模型递推最小二乘算法,推导部分耦合辅助模型随机梯度算法和部分耦合辅助模型递推最小二乘算法.

2.2　部分耦合辅助模型辨识算法

2.2.1　部分耦合辅助模型随机梯度辨识算法

对于递推参数估计算法,参数估计随着数据长度 t 的增大而收敛于真参数,故可以认为第 $i-1$ 个子系统在时刻 t 的参数估计 $\hat{\boldsymbol{\alpha}}_{i-1}(t)$ 比第 i 个子系统在时刻 $t-1$ 的参数估计 $\hat{\boldsymbol{\alpha}}_i(t-1)$ 更接近真参数 $\boldsymbol{\alpha}$.参考文献[1,16]中的部分耦合随机梯度辨识方法,用 $\hat{\boldsymbol{\alpha}}_{i-1}(t)$ 代替式(147)右边的 $\hat{\boldsymbol{\alpha}}(t-1)$,用 $\hat{\boldsymbol{\alpha}}_m(t-1)$ 代替式(147)中 $i=1$ 时的 $\hat{\boldsymbol{\alpha}}(t-1)$,对式(148)中的 $r_i(t)$ 也进行同样的耦合代替处理,则得到部分耦合辅助模型随机梯度算法(Partially Coupled Auxiliary Model based Stochastic Gradient algorithm,PC-AM-SG 算法):

$$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_1(t)\\ \hat{\boldsymbol{\theta}}_1(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}_m(t-1)\\ \hat{\boldsymbol{\theta}}_1(t-1)\end{bmatrix}+\frac{\hat{\boldsymbol{\psi}}_1(t)}{r_1(t)}\left\{y_1(t)-\hat{\boldsymbol{\psi}}_1^{\mathrm{T}}(t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}_m(t-1)\\ \hat{\boldsymbol{\theta}}_1(t-1)\end{bmatrix}\right\},$$

$$\hat{\boldsymbol{\alpha}}_m(0)=\mathbf{1}_n/p_0,\quad \hat{\boldsymbol{\theta}}_1(0)=\mathbf{1}_{nr}/p_0, \tag{166}$$

$$r_1(t)=r_m(t-1)+\|\hat{\boldsymbol{\phi}}_1(t)\|^2+\|\boldsymbol{\varphi}(t)\|^2,\quad r_m(0)=1, \tag{167}$$

$$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_i(t)\\ \hat{\boldsymbol{\theta}}_i(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}_{i-1}(t)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}+\frac{\hat{\boldsymbol{\psi}}_i(t)}{r_i(t)}\left\{y_i(t)-\hat{\boldsymbol{\psi}}_i^{\mathrm{T}}(t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}_{i-1}(t)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}\right\},$$

$$\hat{\boldsymbol{\theta}}_i(0)=\mathbf{1}_{nr}/p_0,\quad i=2,3,\cdots,m, \tag{168}$$

$$r_i(t)=r_{i-1}(t)+\|\hat{\boldsymbol{\phi}}_i(t)\|^2+\|\boldsymbol{\varphi}(t)\|^2,\quad r_i(0)=1, \tag{169}$$

$$\hat{\boldsymbol{\psi}}_i(t)=\begin{bmatrix}\hat{\boldsymbol{\phi}}_i(t)\\ \boldsymbol{\varphi}(t)\end{bmatrix},\quad i=1,2,\cdots,m, \tag{170}$$

$$\boldsymbol{\varphi}(t)=[\boldsymbol{u}^{\mathrm{T}}(t-1),\boldsymbol{u}^{\mathrm{T}}(t-2),\cdots,\boldsymbol{u}^{\mathrm{T}}(t-n)]^{\mathrm{T}}, \tag{171}$$

$$\hat{\boldsymbol{\phi}}(t)=[-\hat{\boldsymbol{x}}(t-1),-\hat{\boldsymbol{x}}(t-2),\cdots,-\hat{\boldsymbol{x}}(t-n)] \tag{172}$$

$$=[\hat{\boldsymbol{\phi}}_1(t),\hat{\boldsymbol{\phi}}_2(t),\cdots,\hat{\boldsymbol{\phi}}_m(t)]^{\mathrm{T}}, \tag{173}$$

$$\hat{\boldsymbol{x}}(t)=[\hat{x}_1(t),\hat{x}_2(t),\cdots,\hat{x}_m(t)]^{\mathrm{T}}, \tag{174}$$

$$\hat{x}_i(t)=\hat{\boldsymbol{\phi}}_i^{\mathrm{T}}(t)\hat{\boldsymbol{\alpha}}_m(t)+\boldsymbol{\varphi}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}_i(t). \tag{175}$$

上述算法中,$\hat{\boldsymbol{\alpha}}_i(t)\in\mathbf{R}^n$ 和 $\hat{\boldsymbol{\theta}}_i(t)\in\mathbf{R}^{nr}$ 为第 i 个子系统在时刻 t 的参数估计,并且各个子系统辨识算法间的参数估计 $\hat{\boldsymbol{\alpha}}_i(t)$ 是耦合计算的,去掉子系统辨识算法间 $r_i(t)$ 的耦合,便得到一个新的部分耦合辅助模型随机梯度算法[1]:

$$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_1(t)\\ \hat{\boldsymbol{\theta}}_1(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}_m(t-1)\\ \hat{\boldsymbol{\theta}}_1(t-1)\end{bmatrix}+\frac{\hat{\boldsymbol{\psi}}_1(t)}{r_1(t)}\left\{y_1(t)-\hat{\boldsymbol{\psi}}_1^{\mathrm{T}}(t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}_m(t-1)\\ \hat{\boldsymbol{\theta}}_1(t-1)\end{bmatrix}\right\}, \tag{176}$$

$$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_i(t)\\ \hat{\boldsymbol{\theta}}_i(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}_{i-1}(t)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}+\frac{\hat{\boldsymbol{\psi}}_i(t)}{r_i(t)}\left\{y_i(t)-\hat{\boldsymbol{\psi}}_i^{\mathrm{T}}(t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}_{i-1}(t)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}\right\},\quad i=2,3,\cdots,m, \tag{177}$$

$$r_i(t)=r_i(t-1)+\|\hat{\boldsymbol{\phi}}_i(t)\|^2+\|\boldsymbol{\varphi}(t)\|^2,\quad i=1,2,\cdots,m, \tag{178}$$

$$\hat{\boldsymbol{\psi}}_i(t)=\begin{bmatrix}\hat{\boldsymbol{\phi}}_i(t)\\ \boldsymbol{\varphi}(t)\end{bmatrix}, \tag{179}$$

$$\boldsymbol{\varphi}(t)=[\boldsymbol{u}^{\mathrm{T}}(t-1),\boldsymbol{u}^{\mathrm{T}}(t-2),\cdots,\boldsymbol{u}^{\mathrm{T}}(t-n)]^{\mathrm{T}}, \tag{180}$$

$$\hat{\boldsymbol{\phi}}(t)=[-\hat{\boldsymbol{x}}(t-1),-\hat{\boldsymbol{x}}(t-2),\cdots,-\hat{\boldsymbol{x}}(t-n)] \tag{181}$$

$$=[\hat{\boldsymbol{\phi}}_1(t),\hat{\boldsymbol{\phi}}_2(t),\cdots,\hat{\boldsymbol{\phi}}_m(t)]^{\mathrm{T}}, \tag{182}$$

$$\hat{\boldsymbol{x}}(t)=[\hat{x}_1(t),\hat{x}_2(t),\cdots,\hat{x}_m(t)]^{\mathrm{T}}, \tag{183}$$

$$\hat{x}_i(t)=\hat{\boldsymbol{\phi}}_i^{\mathrm{T}}(t)\hat{\boldsymbol{\alpha}}_m(t)+\boldsymbol{\varphi}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}_i(t). \tag{184}$$

PC-AM-SG 算法(176)—(184)的计算量如表 7 所示.

PC-AM-SG 算法(176)—(184)计算参数估计向量 $\hat{\boldsymbol{\alpha}}_m(t)$ 和 $\hat{\boldsymbol{\theta}}_i(t)$ 的步骤如下:

① 置初值:令 $t=1$,$\hat{\boldsymbol{\alpha}}_m(0)=\mathbf{1}_n/p_0$,$\hat{\boldsymbol{\theta}}_i(0)=\mathbf{1}_{nr}/p_0$,$r_i(0)=1$,$i=1,2,\cdots,m$,$\hat{\boldsymbol{x}}(-k)=\mathbf{1}_m/p_0$,$k\geqslant 0$,$p_0=10^6$.

② 收集观测数据 $\boldsymbol{u}(t)$ 和 $\boldsymbol{y}(t)$,用式(180)构成信息向量 $\boldsymbol{\varphi}(t)$,根据式(181)构成信息矩阵 $\hat{\boldsymbol{\phi}}(t)$,并从 $\hat{\boldsymbol{\phi}}(t)$ 中读出向量 $\hat{\boldsymbol{\phi}}_i(t)$,以构成式(179)中的信息向量 $\hat{\boldsymbol{\psi}}_i(t)$.

③ 用式(178)计算 $r_i(t)$,用式(176)刷新参数估计 $\hat{\boldsymbol{\alpha}}_1(t)$ 和 $\hat{\boldsymbol{\theta}}_1(t)$.

④ 依次当 $i=2,3,\cdots,m$ 时,用式(177)刷新参数估计 $\hat{\boldsymbol{\alpha}}_i(t)$ 和 $\hat{\boldsymbol{\theta}}_i(t)$.

⑤ 用式(184)计算估计 $\hat{x}_i(t)$,根据式(183)构成向量 $\hat{\boldsymbol{x}}(t)$.

⑥ t 增 1,转到第②步.

1) 遗忘因子部分耦合辅助模型随机梯度辨识算法. 为了提高 PC-AM-SG 算法的暂态性能,在式(178)中引入遗忘因子(Forgetting Factor, FF)λ,得到

$$r_i(t)=\lambda r_i(t-1)+\|\hat{\boldsymbol{\phi}}_i(t)\|^2+\|\boldsymbol{\varphi}(t)\|^2,\quad 0\leqslant\lambda\leqslant 1. \tag{185}$$

式(176)—(177),式(185)和式(179)—(184)构成了估计参数向量 $\boldsymbol{\alpha}$ 和 $\boldsymbol{\theta}$ 的遗忘因子 PC-AM-SG 算法[22,26-27].

2) 修正部分耦合辅助模型随机梯度辨识算法.

表 7 部分耦合辅助模型随机梯度算法的计算量

Table 7 The computational efficiency of the PC-AM-SG algorithm

表达式	乘法次数	加法次数
$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_i(t)\\ \hat{\boldsymbol{\theta}}_i(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}_{i-1}(t)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}+\frac{\hat{\boldsymbol{\psi}}_i(t)}{r_i(t)}\left\{y_i(t)-\hat{\boldsymbol{\psi}}_i^{\mathrm{T}}(t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}_{i-1}(t)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}\right\}$	$2m(n+nr)+m$	$2m(n+nr)$
$r_i(t)=r_i(t-1)+\|\hat{\boldsymbol{\psi}}_i(t)\|^2$	$m(n+nr)$	$m(n+nr)$
$\hat{x}_i(t)=\hat{\boldsymbol{\phi}}_i^{\mathrm{T}}(t)\hat{\boldsymbol{\alpha}}_m(t)+\boldsymbol{\varphi}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}_i(t)$	$m(n+nr)$	$m(n+nr)-2m$
总数	$4m(n+nr)+m$	$4m(n+nr)-2m$
总 flop 数	$8m(n+nr)-m$	

为了提高 PC-AM-SG 算法的收敛速度,在式(176)和式(177)中引入收敛指数(convergence index)ε,得到

$$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_1(t)\\ \hat{\boldsymbol{\theta}}_1(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}_m(t-1)\\ \hat{\boldsymbol{\theta}}_1(t-1)\end{bmatrix}+\frac{\hat{\boldsymbol{\psi}}_1(t)}{r_1^{\varepsilon}(t)}\left\{y_1(t)-\hat{\boldsymbol{\psi}}_1^{\mathrm{T}}(t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}_m(t-1)\\ \hat{\boldsymbol{\theta}}_1(t-1)\end{bmatrix}\right\},$$

$$\hat{\boldsymbol{\alpha}}_m(0)=\mathbf{1}_n/p_0,\quad \hat{\boldsymbol{\theta}}_1(0)=\mathbf{1}_{nr}/p_0,\quad \frac{1}{2}<\varepsilon\leqslant 1,\tag{186}$$

$$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_i(t)\\ \hat{\boldsymbol{\theta}}_i(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}_{i-1}(t)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}+\frac{\hat{\boldsymbol{\psi}}_i(t)}{r_i^{\varepsilon}(t)}\left\{y_i(t)-\hat{\boldsymbol{\psi}}_i^{\mathrm{T}}(t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}_{i-1}(t)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}\right\},$$

$$\hat{\boldsymbol{\theta}}_i(0)=\mathbf{1}_{nr}/p_0,\quad i=2,3,\cdots,m.\tag{187}$$

式(186)—(187)和式(178)—(184)构成了带收敛指数的 PC-AM-SG 算法,即修正 PC-AM-SG 算法;式(185)—(187)和式(179)—(184)构成了带遗忘因子的修正 PC-AM-SG 算法[28].

2.2.2 部分耦合辅助模型递推最小二乘辨识算法

利用耦合辨识概念[1,16-17],用 $\hat{\boldsymbol{\alpha}}_{i-1}(t)$ 代替式(156)右边的 $\hat{\boldsymbol{\alpha}}(t-1)$,用 $\hat{\boldsymbol{\alpha}}_m(t-1)$ 代替式(156)中 $i=1$ 时的 $\hat{\boldsymbol{\alpha}}(t-1)$,对式(158)中的 $\boldsymbol{P}_i(t)$ 也进行同样的耦合代替处理,则得到部分耦合辅助模型递推最小二乘算法(Partially Coupled Auxiliary Model based Recursive Least Squares algorithm,PC-AM-RLS 算法):

$$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_1(t)\\ \hat{\boldsymbol{\theta}}_1(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}_m(t-1)\\ \hat{\boldsymbol{\theta}}_1(t-1)\end{bmatrix}+\boldsymbol{L}_1(t)\left\{y_1(t)-\hat{\boldsymbol{\psi}}_1^{\mathrm{T}}(t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}_m(t-1)\\ \hat{\boldsymbol{\theta}}_1(t-1)\end{bmatrix}\right\},$$

$$\hat{\boldsymbol{\alpha}}_m(0)=\mathbf{1}_n/p_0,\quad \hat{\boldsymbol{\theta}}_1(0)=\mathbf{1}_{nr}/p_0,\tag{188}$$

$$\boldsymbol{L}_1(t)=\frac{\boldsymbol{P}_m(t-1)\hat{\boldsymbol{\psi}}_1(t)}{1+\hat{\boldsymbol{\psi}}_1^{\mathrm{T}}(t)\boldsymbol{P}_m(t-1)\hat{\boldsymbol{\psi}}_1(t)},\tag{189}$$

$$\boldsymbol{P}_1(t)=[\boldsymbol{I}_{n+nr}-\boldsymbol{L}_1(t)\hat{\boldsymbol{\psi}}_1^{\mathrm{T}}(t)]\boldsymbol{P}_m(t-1),$$
$$\boldsymbol{P}_m(0)=p_0\boldsymbol{I}_{n+nr},\tag{190}$$

$$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_i(t)\\ \hat{\boldsymbol{\theta}}_i(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}_{i-1}(t)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}+\boldsymbol{L}_i(t)\left\{y_i(t)-\hat{\boldsymbol{\psi}}_i^{\mathrm{T}}(t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}_{i-1}(t)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}\right\},$$

$$\hat{\boldsymbol{\theta}}_i(0)=\mathbf{1}_{nr}/p_0,\quad i=2,3,\cdots,m,\tag{191}$$

$$\boldsymbol{L}_i(t)=\frac{\boldsymbol{P}_{i-1}(t)\hat{\boldsymbol{\psi}}_i(t)}{1+\hat{\boldsymbol{\psi}}_i^{\mathrm{T}}(t)\boldsymbol{P}_{i-1}(t)\hat{\boldsymbol{\psi}}_i(t)},\tag{192}$$

$$\boldsymbol{P}_i(t)=[\boldsymbol{I}_{n+nr}-\boldsymbol{L}_i(t)\hat{\boldsymbol{\psi}}_i^{\mathrm{T}}(t)]\boldsymbol{P}_{i-1}(t),\tag{193}$$

$$\hat{\boldsymbol{\psi}}_i(t)=\begin{bmatrix}\hat{\boldsymbol{\phi}}_i(t)\\ \boldsymbol{\varphi}(t)\end{bmatrix},\quad i=1,2,\cdots,m,\tag{194}$$

$$\boldsymbol{\varphi}(t)=[\boldsymbol{u}^{\mathrm{T}}(t-1),\boldsymbol{u}^{\mathrm{T}}(t-2),\cdots,\boldsymbol{u}^{\mathrm{T}}(t-n)]^{\mathrm{T}},\tag{195}$$

$$\hat{\boldsymbol{\phi}}(t)=[-\hat{\boldsymbol{x}}(t-1),-\hat{\boldsymbol{x}}(t-2),\cdots,-\hat{\boldsymbol{x}}(t-n)]\tag{196}$$

$$=[\hat{\boldsymbol{\phi}}_1(t),\hat{\boldsymbol{\phi}}_2(t),\cdots,\hat{\boldsymbol{\phi}}_m(t)]^{\mathrm{T}},\tag{197}$$

$$\hat{\boldsymbol{x}}(t)=[\hat{x}_1(t),\hat{x}_2(t),\cdots,\hat{x}_m(t)]^{\mathrm{T}},\tag{198}$$

$$\hat{x}_i(t)=\hat{\boldsymbol{\phi}}_i^{\mathrm{T}}(t)\hat{\boldsymbol{\alpha}}_m(t)+\boldsymbol{\varphi}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}_i(t).\tag{199}$$

上述算法中,各子系统辨识算法间参数估计 $\hat{\boldsymbol{\alpha}}_i(t)$、$\hat{\boldsymbol{\theta}}_i(t)$ 和协方差阵 $\boldsymbol{P}_i(t)$ 都是耦合的,去掉子系统辨识算法间 $\boldsymbol{P}_i(t)$ 的耦合(去掉耦合更合理),便得到一个较为简单的的部分耦合辅助模型递推最小二乘算法(PC-AM-RLS 算法)[1]:

$$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_1(t)\\ \hat{\boldsymbol{\theta}}_1(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}_m(t-1)\\ \hat{\boldsymbol{\theta}}_1(t-1)\end{bmatrix}+\boldsymbol{L}_1(t)\left\{y_1(t)-\hat{\boldsymbol{\psi}}_1^{\mathrm{T}}(t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}_m(t-1)\\ \hat{\boldsymbol{\theta}}_1(t-1)\end{bmatrix}\right\},\tag{200}$$

$$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_i(t)\\ \hat{\boldsymbol{\theta}}_i(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}_{i-1}(t)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}+\boldsymbol{L}_i(t)\left\{y_i(t)-\hat{\boldsymbol{\psi}}_i^{\mathrm{T}}(t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}_{i-1}(t)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}\right\},$$
$$i=2,3,\cdots,m,\tag{201}$$

$$\boldsymbol{L}_i(t)=\frac{\boldsymbol{P}_i(t-1)\hat{\boldsymbol{\psi}}_i(t)}{1+\hat{\boldsymbol{\psi}}_i^{\mathrm{T}}(t)\boldsymbol{P}_i(t-1)\hat{\boldsymbol{\psi}}_i(t)},\quad i=1,2,\cdots,m,\tag{202}$$

$$\boldsymbol{P}_i(t)=[\boldsymbol{I}_{n+nr}-\boldsymbol{L}_i(t)\hat{\boldsymbol{\psi}}_i^{\mathrm{T}}(t)]\boldsymbol{P}_i(t-1),\tag{203}$$

$$\hat{\boldsymbol{\psi}}_i(t)=\begin{bmatrix}\hat{\boldsymbol{\phi}}_i(t)\\ \boldsymbol{\varphi}(t)\end{bmatrix},\tag{204}$$

$$\boldsymbol{\varphi}(t)=[\boldsymbol{u}^{\mathrm{T}}(t-1),\boldsymbol{u}^{\mathrm{T}}(t-2),\cdots,\boldsymbol{u}^{\mathrm{T}}(t-n)]^{\mathrm{T}},\tag{205}$$

$$\hat{\boldsymbol{\phi}}(t)=[-\hat{\boldsymbol{x}}(t-1),-\hat{\boldsymbol{x}}(t-2),\cdots,-\hat{\boldsymbol{x}}(t-n)]\tag{206}$$

$$=[\hat{\boldsymbol{\phi}}_1(t),\hat{\boldsymbol{\phi}}_2(t),\cdots,\hat{\boldsymbol{\phi}}_m(t)]^{\mathrm{T}},\tag{207}$$

$$\hat{\boldsymbol{x}}(t)=[\hat{x}_1(t),\hat{x}_2(t),\cdots,\hat{x}_m(t)]^{\mathrm{T}},\tag{208}$$

$$\hat{x}_i(t)=\hat{\boldsymbol{\phi}}_i^{\mathrm{T}}(t)\hat{\boldsymbol{\alpha}}_m(t)+\boldsymbol{\varphi}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}_i(t).\tag{209}$$

PC-AM-RLS 算法(200)—(209)计算参数估计向量 $\hat{\boldsymbol{\alpha}}_m(t)$ 和 $\hat{\boldsymbol{\theta}}_i(t)$ 的步骤如下:

① 置初值:令 $t=1$,$\hat{\boldsymbol{\alpha}}_m(0)=\mathbf{1}_n/p_0$,$\hat{\boldsymbol{\theta}}_i(0)=\mathbf{1}_{nr}/p_0$,$\boldsymbol{P}_i(0)=p_0\boldsymbol{I}_{n+nr}$,$i=1,2,\cdots,m$,$\hat{\boldsymbol{x}}(-k)=\mathbf{1}_m/p_0$,$k\geqslant 0$,$p_0=10^6$.

② 收集观测数据 $\boldsymbol{u}(t)$ 和 $\boldsymbol{y}(t)$,用式(205)构成信息向量 $\boldsymbol{\varphi}(t)$,用式(206)构成信息矩阵 $\hat{\boldsymbol{\phi}}(t)$,并从 $\hat{\boldsymbol{\phi}}(t)$ 中读出向量 $\hat{\boldsymbol{\phi}}_i(t)$,以构成式(204)中的信息向量 $\hat{\boldsymbol{\psi}}_i(t)$.

③ 依次当 $i=1,2,\cdots,m$ 时,用式(202)计算增益向量 $\boldsymbol{L}_i(t)$,用式(203)计算协方差阵 $\boldsymbol{P}_i(t)$.用式(200)刷新参数估计 $\hat{\boldsymbol{\alpha}}_1(t)$ 和 $\hat{\boldsymbol{\theta}}_1(t)$.

④ 依次当 $i=2,3,\cdots,m$ 时,用式(201)刷新参数估计向量 $\hat{\boldsymbol{\alpha}}_i(t)$ 和 $\hat{\boldsymbol{\theta}}_i(t)$.

⑤ 用式(209)计算输出估计 $\hat{x}_i(t)$,根据式(208)构成向量 $\hat{\boldsymbol{x}}(t)$.

⑥ t 增 1,转到第②步.

PC-AM-RLS 算法(200)—(209)的计算量如表 8 所示.

3 部分耦合辅助模型多新息辨识方法

本节研究部分耦合辅助模型多新息随机梯度辨识方法和部分耦合辅助模型多新息最小二乘辨识方法.

3.1 部分耦合子系统辅助模型多新息辨识算法

与递推最小二乘算法相比,随机梯度算法的计算量小,但是收敛速度慢,为了改进随机梯度算法的收敛速度,下面引入新息长度,来研究收敛速度快的部分耦合辅助模型多新息随机梯度算法.

3.1.1 部分耦合子系统辅助模型多新息随机梯度辨识算法

定义子系统的信息矩阵 $\boldsymbol{\Gamma}_i(p,t)$ 和堆积输出向量 $\boldsymbol{Y}_i(p,t)$ 如下:

$$\boldsymbol{\Gamma}_i(p,t):=[\hat{\boldsymbol{\psi}}_i(t),\hat{\boldsymbol{\psi}}_i(t-1),\cdots,\hat{\boldsymbol{\psi}}_i(t-p+1)]\in\mathbf{R}^{(n+nr)\times p},$$

$$\boldsymbol{Y}_i(p,t):=\begin{bmatrix} y_i(t)\\ y_i(t-1)\\ \vdots\\ y_i(t-p+1)\end{bmatrix}\in\mathbf{R}^p.$$

令 $p\geqslant 1$ 为新息长度,应用多新息辨识理论,将 PC-S-AM-SG 算法(147)—(155)中的标量新息

$$e_i(t):=y_i(t)-\hat{\boldsymbol{\psi}}_i^{\mathrm{T}}(t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}(t-1)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}=$$

$$y_i(t)-\hat{\boldsymbol{\phi}}_i^{\mathrm{T}}(t)\hat{\boldsymbol{\alpha}}(t-1)-\boldsymbol{\varphi}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}_i(t-1)\in\mathbf{R}$$

扩展为新息向量

$$\boldsymbol{E}_i(p,t):=\begin{bmatrix} y_i(t)-\hat{\boldsymbol{\psi}}_i^{\mathrm{T}}(t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}(t-1)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}\\ y_i(t-1)-\hat{\boldsymbol{\psi}}_i^{\mathrm{T}}(t-1)\begin{bmatrix}\hat{\boldsymbol{\alpha}}(t-1)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}\\ \vdots\\ y_i(t-p+1)-\hat{\boldsymbol{\psi}}_i^{\mathrm{T}}(t-p+1)\begin{bmatrix}\hat{\boldsymbol{\alpha}}(t-1)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}\end{bmatrix}=$$

$$\begin{bmatrix} y_i(t)-\hat{\boldsymbol{\phi}}_i^{\mathrm{T}}(t)\hat{\boldsymbol{\alpha}}(t-1)-\boldsymbol{\varphi}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}_i(t-1)\\ y_i(t-1)-\hat{\boldsymbol{\phi}}_i^{\mathrm{T}}(t-1)\hat{\boldsymbol{\alpha}}(t-1)-\boldsymbol{\varphi}^{\mathrm{T}}(t-1)\hat{\boldsymbol{\theta}}_i(t-1)\\ \vdots\\ y_i(t-p+1)-\hat{\boldsymbol{\phi}}_i^{\mathrm{T}}(t-p+1)\hat{\boldsymbol{\alpha}}(t-1)-\boldsymbol{\varphi}^{\mathrm{T}}(t-p+1)\hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}=$$

$$\boldsymbol{Y}_i(p,t)-\boldsymbol{\Gamma}_i^{\mathrm{T}}(p,t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}(t-1)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}\in\mathbf{R}^p.$$

参照文献[14-15],将部分耦合子系统辅助模型随机梯度算法(147)—(155)加以推广,可以得到部分耦合子系统辅助模型多新息随机梯度算法(Partially Coupled Subsystem Auxiliary Model based Multi-Innovation Stochastic Gradient algorithm,PC-S-AM-MISG 算法):

$$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_i(t)\\ \hat{\boldsymbol{\theta}}_i(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}(t-1)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}+\frac{\boldsymbol{\Gamma}_i(p,t)}{r_i(t)}\boldsymbol{E}_i(p,t),\quad \hat{\boldsymbol{\alpha}}(0)=\mathbf{1}_n/p_0,$$

$$\hat{\boldsymbol{\theta}}_i(0)=\mathbf{1}_{nr}/p_0,\tag{210}$$

$$\boldsymbol{E}_i(p,t)=\boldsymbol{Y}_i(p,t)-\boldsymbol{\Gamma}_i^{\mathrm{T}}(p,t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}(t-1)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix},\tag{211}$$

表 8 部分耦合辅助模型递推最小二乘算法的计算量

Table 8 The computational efficiency of the PC-AM-RLS algorithm

表达式	乘法次数	加法次数
$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_i(t)\\ \hat{\boldsymbol{\theta}}_i(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}_{i-1}(t)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}+\boldsymbol{L}_i(t)e_i(t)\in\mathbf{R}^{n+nr}$	$m(n+nr)$	$m(n+nr)$
$e_i(t):=y_i(t)-\hat{\boldsymbol{\psi}}_i^{\mathrm{T}}(t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}_{i-1}(t)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}\in\mathbf{R}$	$m(n+nr)$	$m(n+nr)$
$\boldsymbol{L}_i(t)=\boldsymbol{\zeta}_i(t)/[1+\hat{\boldsymbol{\psi}}_i^{\mathrm{T}}(t)\boldsymbol{\zeta}_i(t)]\in\mathbf{R}^{n+nr}$	$2m(n+nr)$	$m(n+nr)$
$\boldsymbol{\zeta}_i(t):=\boldsymbol{P}_i(t-1)\hat{\boldsymbol{\psi}}_i(t)\in\mathbf{R}^{n+nr}$	$m(n+nr)^2$	$m(n+nr)^2-m(n+nr)$
$\boldsymbol{P}_i(t)=\boldsymbol{P}_i(t-1)-\boldsymbol{L}_i(t)\boldsymbol{\zeta}_i^{\mathrm{T}}(t)\in\mathbf{R}^{(n+nr)\times(n+nr)}$	$m(n+nr)^2$	$m(n+nr)^2$
$\hat{x}_i(t)=\hat{\boldsymbol{\phi}}_i^{\mathrm{T}}(t)\hat{\boldsymbol{\alpha}}_m(t)+\boldsymbol{\varphi}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}_i(t)\in\mathbf{R}$	$m(n+nr)$	$m(n+nr)-2m$
总数	$2m(n+nr)^2+5m(n+nr)$	$2m(n+nr)^2+3m(n+nr)-2m$
总 flop 数	$4m(n+nr)^2+8m(n+nr)-2m$	

$$r_i(t)=r_i(t-1)+\|\hat{\boldsymbol{\psi}}_i(t)\|^2=r_i(t-1)+\|\hat{\boldsymbol{\phi}}_i(t)\|^2+\|\boldsymbol{\varphi}(t)\|^2,\quad r_i(0)=1,\tag{212}$$

$$\boldsymbol{\Gamma}_i(p,t)=[\hat{\boldsymbol{\psi}}_i(t),\hat{\boldsymbol{\psi}}_i(t-1),\cdots,\hat{\boldsymbol{\psi}}_i(t-p+1)],\tag{213}$$

$$\boldsymbol{Y}_i(p,t)=[y_i(t),y_i(t-1),\cdots,y_i(t-p+1)]^{\mathrm{T}},\tag{214}$$

$$\hat{\boldsymbol{\psi}}_i(t)=\begin{bmatrix}\hat{\boldsymbol{\phi}}_i(t)\\ \boldsymbol{\varphi}(t)\end{bmatrix},\tag{215}$$

$$\boldsymbol{\varphi}(t)=[\boldsymbol{u}^{\mathrm{T}}(t-1),\boldsymbol{u}^{\mathrm{T}}(t-2),\cdots,\boldsymbol{u}^{\mathrm{T}}(t-n)]^{\mathrm{T}},\tag{216}$$

$$\hat{\boldsymbol{\phi}}(t)=[-\hat{\boldsymbol{x}}(t-1),-\hat{\boldsymbol{x}}(t-2),\cdots,-\hat{\boldsymbol{x}}(t-n)]\tag{217}$$

$$=[\hat{\boldsymbol{\phi}}_1(t),\hat{\boldsymbol{\phi}}_2(t),\cdots,\hat{\boldsymbol{\phi}}_m(t)]^{\mathrm{T}},\tag{218}$$

$$\hat{\boldsymbol{x}}(t)=\hat{\boldsymbol{\phi}}(t)\hat{\boldsymbol{\alpha}}(t)+\hat{\boldsymbol{\theta}}^{\mathrm{T}}(t)\boldsymbol{\varphi}(t),\tag{219}$$

$$\hat{\boldsymbol{\alpha}}(t)=\frac{\hat{\boldsymbol{\alpha}}_1(t)+\hat{\boldsymbol{\alpha}}_2(t)+\cdots+\hat{\boldsymbol{\alpha}}_m(t)}{m},\tag{220}$$

$$\hat{\boldsymbol{\theta}}(t)=[\hat{\boldsymbol{\theta}}_1(t),\hat{\boldsymbol{\theta}}_2(t),\cdots,\hat{\boldsymbol{\theta}}_m(t)].\tag{221}$$

为了提高算法的暂态性能和收敛速度,也可以进一步引入遗忘因子 λ 和收敛指数 ε,得到相应的遗忘因子 PC-S-AM-MISG 算法和修正 PC-S-AM-MISG 算法.

1) 遗忘因子部分耦合子系统辅助模型多新息随机梯度算法.

$$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_i(t)\\ \hat{\boldsymbol{\theta}}_i(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}(t-1)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}+\frac{\boldsymbol{\Gamma}_i(p,t)}{r_i(t)}\boldsymbol{E}_i(p,t),$$

$$\hat{\boldsymbol{\alpha}}(0)=\mathbf{1}_n/p_0,\quad \hat{\boldsymbol{\theta}}_i(0)=\mathbf{1}_{nr}/p_0,\tag{222}$$

$$\boldsymbol{E}_i(p,t)=\boldsymbol{Y}_i(p,t)-\boldsymbol{\Gamma}_i^{\mathrm{T}}(p,t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}(t-1)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix},\tag{223}$$

$$r_i(t)=\lambda r_i(t-1)+\|\hat{\boldsymbol{\psi}}_i(t)\|^2=\lambda r_i(t-1)+\|\hat{\boldsymbol{\phi}}_i(t)\|^2+\|\boldsymbol{\varphi}(t)\|^2,$$
$$0\leqslant\lambda\leqslant1,\quad r_i(0)=1,\tag{224}$$

$$\boldsymbol{\Gamma}_i(p,t)=[\hat{\boldsymbol{\psi}}_i(t),\hat{\boldsymbol{\psi}}_i(t-1),\cdots,\hat{\boldsymbol{\psi}}_i(t-p+1)],\tag{225}$$

$$\boldsymbol{Y}_i(p,t)=[y_i(t),y_i(t-1),\cdots,y_i(t-p+1)]^{\mathrm{T}},\tag{226}$$

$$\hat{\boldsymbol{\psi}}_i(t)=\begin{bmatrix}\hat{\boldsymbol{\phi}}_i(t)\\ \boldsymbol{\varphi}(t)\end{bmatrix},\tag{227}$$

$$\boldsymbol{\varphi}(t)=[\boldsymbol{u}^{\mathrm{T}}(t-1),\boldsymbol{u}^{\mathrm{T}}(t-2),\cdots,\boldsymbol{u}^{\mathrm{T}}(t-n)]^{\mathrm{T}},\tag{228}$$

$$\hat{\boldsymbol{\phi}}(t)=[-\hat{\boldsymbol{x}}(t-1),-\hat{\boldsymbol{x}}(t-2),\cdots,-\hat{\boldsymbol{x}}(t-n)]\tag{229}$$

$$=[\hat{\boldsymbol{\phi}}_1(t),\hat{\boldsymbol{\phi}}_2(t),\cdots,\hat{\boldsymbol{\phi}}_m(t)]^{\mathrm{T}},\tag{230}$$

$$\hat{\boldsymbol{x}}(t)=\hat{\boldsymbol{\phi}}(t)\hat{\boldsymbol{\alpha}}(t)+\hat{\boldsymbol{\theta}}^{\mathrm{T}}(t)\boldsymbol{\varphi}(t),\tag{231}$$

$$\hat{\boldsymbol{\alpha}}(t)=\frac{\hat{\boldsymbol{\alpha}}_1(t)+\hat{\boldsymbol{\alpha}}_2(t)+\cdots+\hat{\boldsymbol{\alpha}}_m(t)}{m},\tag{232}$$

$$\hat{\boldsymbol{\theta}}(t)=[\hat{\boldsymbol{\theta}}_1(t),\hat{\boldsymbol{\theta}}_2(t),\cdots,\hat{\boldsymbol{\theta}}_m(t)].\tag{233}$$

2) 修正部分耦合子系统辅助模型多新息随机梯度算法.

$$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_i(t)\\ \hat{\boldsymbol{\theta}}_i(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}(t-1)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}+\frac{\boldsymbol{\Gamma}_i(p,t)}{r_i^{\varepsilon}(t)}\boldsymbol{E}_i(p,t),$$

$$\frac{1}{2}<\varepsilon\leqslant1,\quad \hat{\boldsymbol{\alpha}}(0)=\mathbf{1}_n/p_0,\quad \hat{\boldsymbol{\theta}}_i(0)=\mathbf{1}_{nr}/p_0,\tag{234}$$

$$\boldsymbol{E}_i(p,t)=\boldsymbol{Y}_i(p,t)-\boldsymbol{\Gamma}_i^{\mathrm{T}}(p,t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}(t-1)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix},\tag{235}$$

$$r_i(t)=r_i(t-1)+\|\hat{\boldsymbol{\psi}}_i(t)\|^2=r_i(t-1)+\|\hat{\boldsymbol{\phi}}_i(t)\|^2+\|\boldsymbol{\varphi}(t)\|^2,\quad r_i(0)=1,\tag{236}$$

$$\boldsymbol{\Gamma}_i(p,t)=[\hat{\boldsymbol{\psi}}_i(t),\hat{\boldsymbol{\psi}}_i(t-1),\cdots,\hat{\boldsymbol{\psi}}_i(t-p+1)],\tag{237}$$

$$\boldsymbol{Y}_i(p,t)=[y_i(t),y_i(t-1),\cdots,y_i(t-p+1)]^{\mathrm{T}},\tag{238}$$

$$\hat{\boldsymbol{\psi}}_i(t)=\begin{bmatrix}\hat{\boldsymbol{\phi}}_i(t)\\ \boldsymbol{\varphi}(t)\end{bmatrix},\tag{239}$$

$$\boldsymbol{\varphi}(t)=[\boldsymbol{u}^{\mathrm{T}}(t-1),\boldsymbol{u}^{\mathrm{T}}(t-2),\cdots,\boldsymbol{u}^{\mathrm{T}}(t-n)]^{\mathrm{T}},\tag{240}$$

$$\hat{\boldsymbol{\phi}}(t)=[-\hat{\boldsymbol{x}}(t-1),-\hat{\boldsymbol{x}}(t-2),\cdots,-\hat{\boldsymbol{x}}(t-n)]\tag{241}$$

$$=[\hat{\boldsymbol{\phi}}_1(t),\hat{\boldsymbol{\phi}}_2(t),\cdots,\hat{\boldsymbol{\phi}}_m(t)]^{\mathrm{T}},\tag{242}$$

$$\hat{\boldsymbol{x}}(t)=\hat{\boldsymbol{\phi}}(t)\hat{\boldsymbol{\alpha}}(t)+\hat{\boldsymbol{\theta}}^{\mathrm{T}}(t)\boldsymbol{\varphi}(t),\tag{243}$$

$$\hat{\boldsymbol{\alpha}}(t)=\frac{\hat{\boldsymbol{\alpha}}_1(t)+\hat{\boldsymbol{\alpha}}_2(t)+\cdots+\hat{\boldsymbol{\alpha}}_m(t)}{m},\tag{244}$$

$$\hat{\boldsymbol{\theta}}(t)=[\hat{\boldsymbol{\theta}}_1(t),\hat{\boldsymbol{\theta}}_2(t),\cdots,\hat{\boldsymbol{\theta}}_m(t)].\tag{245}$$

3.1.2 部分耦合子系统辅助模型多新息递推最小二乘辨识算法

子系统信息矩阵 $\boldsymbol{\Gamma}_i(p,t)$、堆积输出向量 $\boldsymbol{Y}_i(p,t)$ 和新息向量 $\boldsymbol{E}_i(p,t)$ 的定义同上,根据部分耦合子系统辅助模型递推最小二乘算法(156)—(165)的结构形式,借助于多新息辨识理论,可以得到部分耦合子系统辅助模型多新息递推最小二乘算法(Partially Coupled Subsystem Auxiliary Model based Multi-Innovation Recursive Least Squares algorithm, PC-S-AM-MI-RLS 算法):

$$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_i(t)\\ \hat{\boldsymbol{\theta}}_i(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}(t-1)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}+\boldsymbol{L}_i(t)\boldsymbol{E}_i(p,t),\ \hat{\boldsymbol{\alpha}}(0)=\mathbf{1}_n/p_0,$$

$$\hat{\boldsymbol{\theta}}_i(0)=\mathbf{1}_{nr}/p_0,\tag{246}$$

$$\boldsymbol{E}_i(p,t)=\boldsymbol{Y}_i(p,t)-\boldsymbol{\Gamma}_i^{\mathrm{T}}(p,t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}(t-1)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix},\tag{247}$$

$$\boldsymbol{L}_i(t)=\boldsymbol{P}_i(t-1)\boldsymbol{\Gamma}_i(p,t)[\boldsymbol{I}_p+\boldsymbol{\Gamma}_i^{\mathrm{T}}(p,t)\boldsymbol{P}_i(t-1)\boldsymbol{\Gamma}_i(p,t)]^{-1},\tag{248}$$

$$\boldsymbol{P}_i(t)=[\boldsymbol{I}_{n+nr}-\boldsymbol{L}_i(t)\boldsymbol{\Gamma}_i^{\mathrm{T}}(p,t)]\boldsymbol{P}_i(t-1),$$
$$\boldsymbol{P}_i(0)=p_0\boldsymbol{I}_{n+nr},\tag{249}$$

$$\boldsymbol{\Gamma}_i(p,t)=[\hat{\boldsymbol{\psi}}_i(t),\hat{\boldsymbol{\psi}}_i(t-1),\cdots,\hat{\boldsymbol{\psi}}_i(t-p+1)],\tag{250}$$

$$\boldsymbol{Y}_i(p,t)=[y_i(t),y_i(t-1),\cdots,y_i(t-p+1)]^{\mathrm{T}},\tag{251}$$

$$\hat{\boldsymbol{\psi}}_i(t)=\begin{bmatrix}\hat{\boldsymbol{\phi}}_i(t)\\ \boldsymbol{\varphi}(t)\end{bmatrix},\tag{252}$$

$$\boldsymbol{\varphi}(t)=[\boldsymbol{u}^{\mathrm{T}}(t-1),\boldsymbol{u}^{\mathrm{T}}(t-2),\cdots,\boldsymbol{u}^{\mathrm{T}}(t-n)]^{\mathrm{T}}, \quad (253)$$

$$\hat{\boldsymbol{\phi}}(t)=[-\hat{\boldsymbol{x}}(t-1),-\hat{\boldsymbol{x}}(t-2),\cdots,-\hat{\boldsymbol{x}}(t-n)] \quad (254)$$

$$=[\hat{\boldsymbol{\phi}}_1(t),\hat{\boldsymbol{\phi}}_2(t),\cdots,\hat{\boldsymbol{\phi}}_m(t)]^{\mathrm{T}}, \quad (255)$$

$$\hat{\boldsymbol{x}}(t)=\hat{\boldsymbol{\phi}}(t)\hat{\boldsymbol{\alpha}}(t)+\hat{\boldsymbol{\theta}}^{\mathrm{T}}(t)\boldsymbol{\varphi}(t), \quad (256)$$

$$\hat{\boldsymbol{\alpha}}(t)=\frac{\hat{\boldsymbol{\alpha}}_1(t)+\hat{\boldsymbol{\alpha}}_2(t)+\cdots+\hat{\boldsymbol{\alpha}}_m(t)}{m}, \quad (257)$$

$$\hat{\boldsymbol{\theta}}(t)=[\hat{\boldsymbol{\theta}}_1(t),\hat{\boldsymbol{\theta}}_2(t),\cdots,\hat{\boldsymbol{\theta}}_m(t)]. \quad (258)$$

3.2 部分耦合辅助模型多新息辨识算法

3.2.1 部分耦合辅助模型多新息随机梯度辨识算法

借助于多新息辨识理论,推广部分耦合辅助模型随机梯度算法(166)—(175),可以得到部分耦合辅助模型多新息随机梯度算法(Partially Coupled Auxiliary Model based Multi-Innovation Stochastic Gradient algorithm,PC-AM-MISG 算法):

$$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_1(t)\\ \hat{\boldsymbol{\theta}}_1(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}_m(t-1)\\ \hat{\boldsymbol{\theta}}_1(t-1)\end{bmatrix}+\frac{\boldsymbol{\Gamma}_1(p,t)}{r_1(t)}\boldsymbol{E}_1(p,t), \quad (259)$$

$$\boldsymbol{E}_1(p,t)=\boldsymbol{Y}_1(p,t)-\boldsymbol{\Gamma}_1^{\mathrm{T}}(p,t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}_m(t-1)\\ \hat{\boldsymbol{\theta}}_1(t-1)\end{bmatrix}, \quad (260)$$

$$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_i(t)\\ \hat{\boldsymbol{\theta}}_i(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}_{i-1}(t)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}+\frac{\boldsymbol{\Gamma}_i(p,t)}{r_i(t)}\boldsymbol{E}_i(p,t),$$
$$i=2,3,\cdots,m, \quad (261)$$

$$\boldsymbol{E}_i(p,t)=\boldsymbol{Y}_i(p,t)-\boldsymbol{\Gamma}_i^{\mathrm{T}}(p,t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}_{i-1}(t)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}, \quad (262)$$

$$r_i(t)=r_i(t-1)+\|\hat{\boldsymbol{\psi}}_i(t)\|^2=$$
$$r_i(t-1)+\|\hat{\boldsymbol{\phi}}_i(t)\|^2+\|\boldsymbol{\varphi}(t)\|^2,$$
$$i=1,2,\cdots,m, \quad (263)$$

$$\boldsymbol{\Gamma}_i(p,t)=[\hat{\boldsymbol{\psi}}_i(t),\hat{\boldsymbol{\psi}}_i(t-1),\cdots,\hat{\boldsymbol{\psi}}_i(t-p+1)], \quad (264)$$

$$\boldsymbol{Y}_i(p,t)=[y_i(t),y_i(t-1),\cdots,y_i(t-p+1)]^{\mathrm{T}}, \quad (265)$$

$$\hat{\boldsymbol{\psi}}_i(t)=\begin{bmatrix}\hat{\boldsymbol{\phi}}_i(t)\\ \boldsymbol{\varphi}(t)\end{bmatrix}, \quad (266)$$

$$\boldsymbol{\varphi}(t)=[\boldsymbol{u}^{\mathrm{T}}(t-1),\boldsymbol{u}^{\mathrm{T}}(t-2),\cdots,\boldsymbol{u}^{\mathrm{T}}(t-n)]^{\mathrm{T}}, \quad (267)$$

$$\hat{\boldsymbol{\phi}}(t)=[-\hat{\boldsymbol{x}}(t-1),-\hat{\boldsymbol{x}}(t-2),\cdots,-\hat{\boldsymbol{x}}(t-n)] \quad (268)$$

$$=[\hat{\boldsymbol{\phi}}_1(t),\hat{\boldsymbol{\phi}}_2(t),\cdots,\hat{\boldsymbol{\phi}}_m(t)]^{\mathrm{T}}, \quad (269)$$

$$\hat{\boldsymbol{x}}(t)=[\hat{x}_1(t),\hat{x}_2(t),\cdots,\hat{x}_m(t)]^{\mathrm{T}}, \quad (270)$$

$$\hat{x}_i(t)=\hat{\boldsymbol{\phi}}_i^{\mathrm{T}}(t)\hat{\boldsymbol{\alpha}}_m(t)+\boldsymbol{\varphi}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}_i(t). \quad (271)$$

PC-AM-MISG 算法(259)—(271)的计算量如表 9 所示.

PC-AM-MISG 算法(259)—(271)计算参数估计向量 $\hat{\boldsymbol{\alpha}}_i(t)$ 和 $\hat{\boldsymbol{\theta}}_i(t)$ 的步骤如下:

① 置初值:令 $t=1$,给定新息长度 p,$\hat{\boldsymbol{\alpha}}_m(0)=\mathbf{1}_n/p_0$,$r_i(0)=1$,$\hat{\boldsymbol{\theta}}_i(0)=\mathbf{1}_{nr}/p_0$,$i=1,2,\cdots,m$,$\hat{\boldsymbol{x}}(-k)=\mathbf{1}_m/p_0$,$k\geqslant 0$,$p_0=10^6$.

② 收集数据 $\boldsymbol{u}(t)$ 和 $\boldsymbol{y}(t)$,用式(267)构成 $\boldsymbol{\varphi}(t)$,用式(268)构成矩阵 $\hat{\boldsymbol{\phi}}(t)$,并从 $\hat{\boldsymbol{\phi}}(t)$ 中读出向量 $\hat{\boldsymbol{\phi}}_i(t)$,以构成式(266)中的信息向量 $\hat{\boldsymbol{\psi}}_i(t)$.

③ 依次当 $i=1,2,\cdots,m$ 时,用式(263)计算 $r_i(t)$,用式(264)和(265)构成 $\boldsymbol{\Gamma}_i(p,t)$ 和 $\boldsymbol{Y}_i(p,t)$.

④ 用式(260)计算新息向量 $\boldsymbol{E}_1(p,t)$,用式(259)刷新参数估计 $\hat{\boldsymbol{\alpha}}_1(t)$ 和 $\hat{\boldsymbol{\theta}}_1(t)$.

⑤ 依次当 $i=2,3,\cdots,m$ 时,用式(262)计算新息向量 $\boldsymbol{E}_i(p,t)$,用式(261)刷新参数估计向量 $\hat{\boldsymbol{\alpha}}_i(t)$ 和 $\hat{\boldsymbol{\theta}}_i(t)$.

⑥ 用式(270)计算估计 $\hat{x}_i(t)$,根据式(271)构成向量 $\hat{\boldsymbol{x}}(t)$.t 增 1,转到第②步.

PC-AM-MISG 辨识算法(259)—(271)计算参数估计向量 $\hat{\boldsymbol{\alpha}}_i(t)$ 和 $\hat{\boldsymbol{\theta}}_i(t)$ 的流程如图 1 所示.

当然,也可以进一步引入遗忘因子或收敛指数,得到暂态性能好的遗忘因子 PC-AM-MISG 算法及收

表 9 PC-AM-MISG 算法的计算量

Table 9 The computational efficiency of the PC-AM-MISG algorithm

表达式	乘法次数	加法次数
$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_i(t)\\ \hat{\boldsymbol{\theta}}_i(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}_{i-1}(t)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}+\frac{\boldsymbol{\Gamma}_i(p,t)}{r_i(t)}\boldsymbol{E}_i(p,t)\in\mathbf{R}^{n+nr}$	$m(p+1)(n+nr)$	$mp(n+nr)$
$\boldsymbol{E}_i(p,t)=\boldsymbol{Y}_i(p,t)-\boldsymbol{\Gamma}_i^{\mathrm{T}}(p,t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}_{i-1}(t)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}\in\mathbf{R}^p$	$mp(n+nr)$	$mp(n+nr)$
$r_i(t)=r_i(t-1)+\|\hat{\boldsymbol{\psi}}_i(t)\|^2\in\mathbf{R}$	$m(n+nr)$	$m(n+nr)$
$\hat{x}_i(t)=\hat{\boldsymbol{\phi}}_i^{\mathrm{T}}(t)\hat{\boldsymbol{\alpha}}_m(t)+\boldsymbol{\varphi}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}_i(t)\in\mathbf{R}$	$m(n+nr)$	$m(n+nr)-2m$
总数	$m(2p+3)(n+nr)$	$m(2p+2)(n+nr)-2m$
总 flop 数	$m(4p+5)(n+nr)-2m$	

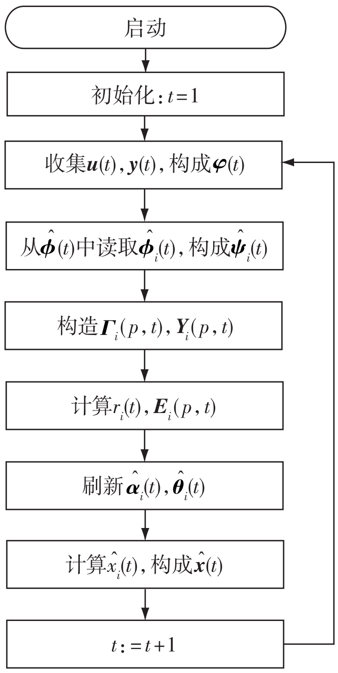

图1　计算 PC-AM-MISG 算法参数估计 $\hat{\boldsymbol{\alpha}}_i(t)$ 和 $\hat{\boldsymbol{\theta}}_i(t)$ 的流程

Fig. 1　The flowchart of computing the parameter estimates $\hat{\boldsymbol{\alpha}}_i(t)$ and $\hat{\boldsymbol{\theta}}_i(t)$ of the PC-AM-MISG algorithm

敛速度快的修正 PC-AM-MISG 算法.

3.2.2　部分耦合辅助模型多新息递推最小二乘辨识算法

借助于多新息辨识理论，推广部分耦合辅助模型递推最小二乘算法(200)—(209)，可以得到部分耦合辅助模型多新息递推最小二乘算法(Partially Coupled Auxiliary Model based Multi-Innovation Recursive Least Squares algorithm，PC-AM-MI-RLS 算法)：

$$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_1(t)\\ \hat{\boldsymbol{\theta}}_1(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}_m(t-1)\\ \hat{\boldsymbol{\theta}}_m(t-1)\end{bmatrix}+\boldsymbol{L}_1(t)\boldsymbol{E}_1(p,t), \tag{272}$$

$$\boldsymbol{E}_1(p,t)=\boldsymbol{Y}_1(p,t)-\boldsymbol{\Gamma}_1^{\mathrm{T}}(p,t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}_m(t-1)\\ \hat{\boldsymbol{\theta}}_1(t-1)\end{bmatrix}, \tag{273}$$

$$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_i(t)\\ \hat{\boldsymbol{\theta}}_i(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}_{i-1}(t)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}+\boldsymbol{L}_i(t)\boldsymbol{E}_i(p,t),\quad i=2,3,\cdots,m, \tag{274}$$

$$\boldsymbol{E}_i(p,t)=\boldsymbol{Y}_i(p,t)-\boldsymbol{\Gamma}_i^{\mathrm{T}}(p,t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}_{i-1}(t)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}, \tag{275}$$

$$\boldsymbol{L}_i(t)=\boldsymbol{P}_i(t-1)\boldsymbol{\Gamma}_i(p,t)[\boldsymbol{I}_p+\boldsymbol{\Gamma}_i^{\mathrm{T}}(p,t)\boldsymbol{P}_i(t-1)\boldsymbol{\Gamma}_i(p,t)]^{-1},\quad i=1,2,\cdots,m, \tag{276}$$

$$\boldsymbol{P}_i(t)=[\boldsymbol{I}_{n+nr}-\boldsymbol{L}_i(t)\boldsymbol{\Gamma}_i^{\mathrm{T}}(p,t)]\boldsymbol{P}_i(t-1), \tag{277}$$

$$\boldsymbol{\Gamma}_i(p,t)=[\hat{\boldsymbol{\psi}}_i(t),\hat{\boldsymbol{\psi}}_i(t-1),\cdots,\hat{\boldsymbol{\psi}}_i(t-p+1)], \tag{278}$$

$$\boldsymbol{Y}_i(p,t)=[y_i(t),y_i(t-1),\cdots,y_i(t-p+1)], \tag{279}$$

$$\hat{\boldsymbol{\psi}}_i(t)=\begin{bmatrix}\hat{\boldsymbol{\phi}}_i(t)\\ \boldsymbol{\varphi}(t)\end{bmatrix}, \tag{280}$$

$$\boldsymbol{\varphi}(t)=[\boldsymbol{u}^{\mathrm{T}}(t-1),\boldsymbol{u}^{\mathrm{T}}(t-2),\cdots,\boldsymbol{u}^{\mathrm{T}}(t-n)]^{\mathrm{T}}, \tag{281}$$

$$\hat{\boldsymbol{\phi}}(t)=[-\hat{\boldsymbol{x}}(t-1),-\hat{\boldsymbol{x}}(t-2),\cdots,-\hat{\boldsymbol{x}}(t-n)] \tag{282}$$

$$=[\hat{\boldsymbol{\phi}}_1(t),\hat{\boldsymbol{\phi}}_2(t),\cdots,\hat{\boldsymbol{\phi}}_m(t)]^{\mathrm{T}}, \tag{283}$$

$$\hat{\boldsymbol{x}}(t)=[\hat{x}_1(t),\hat{x}_2(t),\cdots,\hat{x}_m(t)]^{\mathrm{T}}, \tag{284}$$

$$\hat{x}_i(t)=\hat{\boldsymbol{\phi}}_i^{\mathrm{T}}(t)\hat{\boldsymbol{\alpha}}_m(t)+\boldsymbol{\varphi}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}_i(t). \tag{285}$$

式(284)—(285)可以等价写为

$$\hat{\boldsymbol{x}}(t)=\hat{\boldsymbol{\phi}}(t)\hat{\boldsymbol{\alpha}}_m(t)+\hat{\boldsymbol{\theta}}^{\mathrm{T}}(t)\boldsymbol{\varphi}(t), \tag{286}$$

$$\hat{\boldsymbol{\theta}}(t)=[\hat{\boldsymbol{\theta}}_1(t),\hat{\boldsymbol{\theta}}_2(t),\cdots,\hat{\boldsymbol{\theta}}_m(t)], \tag{287}$$

算法输出的参数估计为 $\hat{\boldsymbol{\alpha}}(t)=\hat{\boldsymbol{\alpha}}_m(t)$ 和 $\hat{\boldsymbol{\theta}}(t)=[\hat{\boldsymbol{\theta}}_1(t),\hat{\boldsymbol{\theta}}_2(t),\cdots,\hat{\boldsymbol{\theta}}_m(t)]$.

PC-AM-MI-RLS 算法(272)—(285)的计算量如表10所示.

PC-AM-MI-RLS 算法(272)—(285)计算参数估计向量 $\hat{\boldsymbol{\alpha}}_i(t)$ 和 $\hat{\boldsymbol{\theta}}_i(t)$ 的步骤如下：

① 置初值：令 $t=1$，$\hat{\boldsymbol{\alpha}}_m(0)=\mathbf{1}_n/p_0$，$\hat{\boldsymbol{\theta}}_i(0)=\mathbf{1}_{nr}/p_0$，$\boldsymbol{P}_i(0)=p_0\boldsymbol{I}_{n+nr}$，$i=1,2,\cdots,m$，$\hat{\boldsymbol{x}}(-k)=\mathbf{1}_m/p_0$，$k\geqslant 0$，$p_0=10^6$.

② 收集数据 $\boldsymbol{u}(t)$ 和 $\boldsymbol{y}(t)$，用式(281)构成信息向量 $\boldsymbol{\varphi}(t)$，用式(282)构成信息矩阵 $\hat{\boldsymbol{\phi}}(t)$，并从 $\hat{\boldsymbol{\phi}}(t)$ 中读出向量 $\hat{\boldsymbol{\phi}}_i(t)$，以构成式(280)中的信息向量 $\hat{\boldsymbol{\psi}}_i(t)$.

③ 依次当 $i=1,2,\cdots,m$ 时，用式(278)和(279)构成 $\boldsymbol{\Gamma}_i(p,t)$ 和 $\boldsymbol{Y}_i(p,t)$，用式(276)计算增益向量 $\boldsymbol{L}_i(t)$，用式(277)计算协方差阵 $\boldsymbol{P}_i(t)$.

④ 用式(273)计算新息向量 $\boldsymbol{E}_1(p,t)$，用式(272)刷新参数估计向量 $\hat{\boldsymbol{\alpha}}_1(t)$ 和 $\hat{\boldsymbol{\theta}}_1(t)$.

⑤ 依次当 $i=2,3,\cdots,m$ 时，用式(275)计算新息向量 $\boldsymbol{E}_i(p,t)$，用式(274)刷新参数估计向量 $\hat{\boldsymbol{\alpha}}_i(t)$ 和 $\hat{\boldsymbol{\theta}}_i(t)$.

⑥ 用式(285)计算估计 $\hat{x}_i(t)$，根据式(284)构成向量 $\hat{\boldsymbol{x}}(t)$. t 增1，转到第②步.

PC-AM-MI-RLS 辨识算法(272)—(285)计算参数估计向量 $\hat{\boldsymbol{\alpha}}_i(t)$ 和 $\hat{\boldsymbol{\theta}}_i(t)$ 的流程如图2所示.

表 10　PC-AM-MI-RLS 算法的计算量

Table 10　The computational efficiency of the PC-AM-MI-RLS algorithm

表达式	乘法次数	加法次数
$\begin{bmatrix}\hat{\boldsymbol{\alpha}}_i(t)\\ \hat{\boldsymbol{\theta}}_i(t)\end{bmatrix}=\begin{bmatrix}\hat{\boldsymbol{\alpha}}_{i-1}(t)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}+\boldsymbol{L}_i(t)\boldsymbol{E}_i(p,t)$	$mp(n+nr)$	$mp(n+nr)$
$\boldsymbol{E}_i(p,t)=\boldsymbol{Y}_i(p,t)-\boldsymbol{\Gamma}_i^{\mathrm{T}}(p,t)\begin{bmatrix}\hat{\boldsymbol{\alpha}}_{i-1}(t)\\ \hat{\boldsymbol{\theta}}_i(t-1)\end{bmatrix}$	$mp(n+nr)$	$mp(n+nr)$
$\boldsymbol{L}_i(t)=\boldsymbol{R}_i(t)\boldsymbol{\Lambda}'_i(t)$	$mp^2(n+nr)$	$mp^2(n+nr)-mp(n+nr)$
$\boldsymbol{R}_i(t):=\boldsymbol{P}_i(t-1)\boldsymbol{\Gamma}_i(p,t)$	$mp(n+nr)^2$	$mp(n+nr)^2-mp(n+nr)$
$\boldsymbol{\Lambda}_i(t):=\boldsymbol{I}_p+\boldsymbol{\Gamma}_i^{\mathrm{T}}(p,t)\boldsymbol{R}_i(t)$	$mp^2(n+nr)$	$mp^2(n+nr)$
$\boldsymbol{\Lambda}'_i(t):=\boldsymbol{\Lambda}_i^{-1}(t)$	mp^3	$m(p^3-p^2)$
$\boldsymbol{P}_i(t)=\boldsymbol{P}_i(t-1)-\boldsymbol{L}_i(t)\boldsymbol{R}_i^{\mathrm{T}}(t)$	$mp(n+nr)^2$	$mp(n+nr)^2$
$\hat{x}_i(t)=\hat{\boldsymbol{\phi}}_i^{\mathrm{T}}(t)\hat{\boldsymbol{\alpha}}_m(t)+\boldsymbol{\varphi}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}_i(t)$	$m(n+nr)$	$m(n+nr)-2m$
总数	$2mp(n+nr)^2+mp^3+m(2p^2+2p+1)(n+nr)$	$2mp(n+nr)^2+m(2p^2+1)(n+nr)+m(p^3-p^2)-2m$
总 flop 数	$4mp(n+nr)^2+m(4p^2+2p+2)(n+nr)+m(2p^3-p^2-2)$	

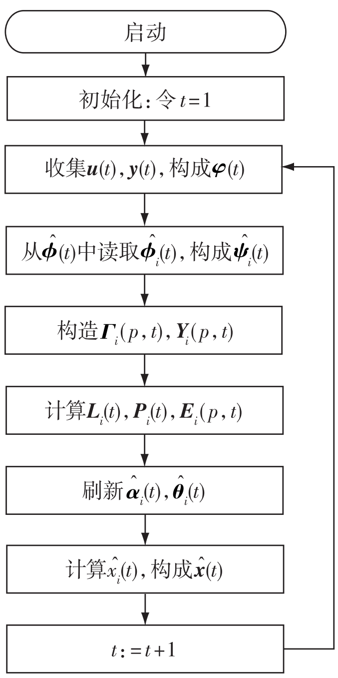

图 2　计算 PC-AM-MI-RLS 算法参数估计 $\hat{\boldsymbol{\alpha}}_i(t)$ 和 $\hat{\boldsymbol{\theta}}_i(t)$ 的流程

Fig. 2　The flowchart of computing the parameter estimates $\hat{\boldsymbol{\alpha}}_i(t)$ and $\hat{\boldsymbol{\theta}}_i(t)$ of the PC-AM-MI-RLS algorithm

4　结语

本文借助于辅助模型辨识思想,将耦合辨识概念与多新息辨识理论相结合,针对类多变量输出误差系统,研究和提出了多元随机梯度辨识方法和多元递推最小二乘辨识方法、部分耦合辅助模型随机梯度辨识方法和部分耦合辅助模型递推最小二乘辨识方法、部分耦合辅助模型多新息随机梯度辨识方法和部分耦合辅助模型多新息递推最小二乘辨识方法.同时给出了几个典型算法的计算量、计算步骤和计算流程图.文中的研究方法可以推广到有色噪声干扰的类多变量方程误差系统

$$\alpha(z)\boldsymbol{y}(t)=\boldsymbol{Q}(z)\boldsymbol{u}(t)+\gamma^{-1}(z)N(z)\boldsymbol{v}(t)$$

和有色噪声干扰的类多变量输出误差系统

$$\boldsymbol{y}(t)=\frac{\boldsymbol{Q}(z)}{\alpha(z)}\boldsymbol{u}(t)+\gamma^{-1}(z)N(z)\boldsymbol{v}(t),$$

其中 $\gamma(z)$ 和 $N(z)$ 为 1,单位阵,首一多项式或首一多项式矩阵.

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Coupled multi-innovation identification methods for multivariable output-error-like systems

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Abstract The auxiliary model identification idea,the multi-innovation identification theory and the coupling identification concept are the new ideas and principles for studying identification problems of complex systems. Combining them,this paper studies the identification methods of multivariable output-error-like systems and presents multivariate auxiliary model identification methods,multivariate auxiliary model based multi-innovation identification methods,interval-varying multivariate auxiliary model based multi-innovation identification methods.In order to reduce the computational complexity of the algorithms,we decompose the system into several sub-identification models and derive the partially coupled auxiliary model based identification methods and the partially coupled auxiliary model based multi-innovation identification methods,using the auxiliary model identification idea,the multi-innovation identification theory and the coupling identification concept.Finally,the computational efficiency,the computational steps and the flowcharts of some typical identification algorithms are discussed.

Key words parameter estimation;recursive identification;gradient search;least squares;auxiliary model identification idea;multi-innovation identification theory;hierarchical identification principle;coupling identification concept;multivariable-like system