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分布参数高阶随机时滞 Hopfield 神经网络的指数稳定性

摘要

利用 Lyapunov 稳定性理论,积分不等式和 Halanay 不等式,研究了具分布参数的高阶随机时滞 Hopfield 神经网络的均方指数稳定性,得到了保证系统指数稳定且与扩散项相关的充分性条件,并给出了指数收敛率,同时放松了现有文献中对变时滞的要求,因而在一定程度上了改进了现有文献的结果.最后给出了数值算例验证所得结果的有效性.

关键词

指数稳定; Hopfield 神经网络; Halanay 不等式;分布参数;变时滞

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0 引言

美国物理学家 Hopfield 于 1982 年[1]和 1984 年[2]在美国科学院院刊上发表了 2 篇关于人工神经网络研究的论文,引起了巨大的反响.人们重新认识到神经网络的威力以及付诸应用的现实性.随即,一大批学者和研究人员围绕着 Hopfield 提出的方法展开了进一步的工作,形成了 20 世纪 80 年代中期以来人工神经网络的研究热潮. Hopfield 神经网络的稳定性有深远的理论意义与广泛的应用背景,特别是应用电子电路来实现某些新的优化智能计算问题,该研究领域已有一批研究结果[3-11].实际中,由于神经元和放大器的有限切换速度,在生物学和人工神经网络中必然存在时滞.众所周知,时滞对系统的动态性质有很大的影响,例如,时滞常常导致系统失稳.研究时滞神经网络的动态性具有重要意义,有关时滞 Hopfield 神经网络稳定性的研究结果可参见文献[5-16].由于高阶神经网络模型在网络的逼近能力、收敛速度、存储水平和容错能力等方面较之一阶神经网络模型具有更强的功能,因此,对高阶神经网络的研究愈来愈受到国内外科学工作者的重视[5-8,13-22].由于神经网络是通过电子电路实现的,其电热效应不可避免,故用随机方程描述更切实际;又由于电磁场的密度一般来说是不均匀的,电子在不均匀的电磁场运行过程中,势必涉及扩散问题,因此,自然就用扩散方程来描述.研究具分布参数的高阶随机神经网络应更具有一般性.本文利用 Lyapunov 稳定性理论,积分不等式和 Halanay 不等式,研究了具反应扩散的高阶随机时滞 Hopfield 神经网络的均方指数稳定性,得到了保证系统指数稳定的充分性条件并给出了指数收敛率,同时放松了现有文献中对变时滞的要求,在一定程度上了改进了现有文献的结果.本文最后给出数值算例验证了所得结果的有效性.

1 模型描述与基本假设

考虑如下具反应扩散的高阶随机时滞 Hopfield 神经网络系统:

$$\mathrm{d}u_i(t,\boldsymbol{x}) = \Big[\sum_{k=1}^{r}\frac{\partial}{\partial \boldsymbol{x}_k}\left(D_{ik}\frac{\partial u_i(t,\boldsymbol{x})}{\partial \boldsymbol{x}_k}\right) - c_i u_i(t,\boldsymbol{x}) +$$

$$\sum_{j=1}^{n} a_{ij} g_j(u_j(t,\boldsymbol{x})) + \sum_{j=1}^{n} b_{ij} g_j(u_j(t-\tau_j(t),\boldsymbol{x})) + \sum_{j=1}^{n}\sum_{l=1}^{n} T_{ijl}\, g_j(u_j(t-\tau_j(t),\boldsymbol{x})) \times g_l(u_l(t-\tau_l(t),\boldsymbol{x})) + J_i \Big] \mathrm{d}t + \sum_{j=1}^{n} \sigma_{ij}(t,u_j(t,\boldsymbol{x}),u_j(t-\tau_j(t),\boldsymbol{x}))\mathrm{d}w_j(t), \quad (1)$$

其中 $i=1,2,\cdots,n,n\geqslant 2$ 表示网络中神经元的个数，$\boldsymbol{x}=(x_1,x_2,\cdots,x_r)^{\mathrm{T}}\in\mathscr{O}\subset\mathbf{R}^r$，$\mathscr{O}$ 是具有光滑边界 $\partial\mathscr{O}$ 的有界紧集且满足 $\mathrm{mes}\mathscr{O}>0$. $u_i(t,\boldsymbol{x})$ 是第 i 个神经元在时间 t 和空间 x 处的状态变量，$D_{ik}=D_{ik}(t,\boldsymbol{x},\boldsymbol{u})\geqslant 0$ 是其相应的转移扩散算子，记 $\gamma_i=\min_{1\leqslant k\leqslant m}\{\sup_{t>0,\boldsymbol{x}\in\mathscr{O}} D_{ik}(t,\boldsymbol{x},u_i(t,\boldsymbol{x}))\}$. $c_i>0$ 表示在与神经网络不连通并且无外部附加电压的情况下第 i 个神经元恢复孤立静息状态的速率；a_{ij}，b_{ij} 代表神经元之间的一阶连接权；T_{ijl} 代表神经元之间的二阶连接权；g_j 为神经元在时间 t 和空间 x 处的信号传输函数或激活函数；J_i 代表第 i 个神经元的外部输入；$\tau_j(t)$ 代表传递时变时滞，这里假设 $\tau=\sup_{1\leqslant j\leqslant n,t\in\mathbf{R}}\{\tau_j(t)\}$. σ_{il} $(i=1,2,\cdots,n)$ 为随机干扰的权重函数，$\boldsymbol{w}(t)=[w_1(t),w_2(t),\cdots,w_n(t)]^{\mathrm{T}}$ 是定义在完备概率空间 $(\Omega,\mathscr{F},P)$ 上具有自然流 $\{\mathscr{F}_t\}_{t\geqslant 0}$ 的 n-维 Brown 运动.

系统(1)的初边值条件分别为

$$u_i(\boldsymbol{x},s)=\boldsymbol{\phi}_i(\boldsymbol{x},s),\quad s\in[-\tau,0], \quad (2)$$

$$u_i(\boldsymbol{x},t)=0,(\boldsymbol{x},t)\in\partial\mathscr{O}\times[-\tau,\infty). \quad (3)$$

或者

$$\frac{\partial u_i(\boldsymbol{x},t)}{\partial\boldsymbol{v}}+\boldsymbol{N}u_i(\boldsymbol{x},t)=0,\quad (\boldsymbol{x},t)\in\partial\mathscr{O}\times[-\tau,\infty), \quad (4)$$

其中 $\boldsymbol{v}$ 是 $\partial\mathscr{O}$ 上的单位外法向量，$\boldsymbol{N}$ 是对角线元素为正的对角矩阵. 记 $L^2(\mathscr{O})$ 是 $\mathscr{O}$ 上的 Lebergue 平方可积函数空间，标量实值函数 $h(\boldsymbol{x})\in L^2(\mathscr{O})$ 的范数定义为

$$\|h\|^2=\int_{\mathscr{O}} h^2(\boldsymbol{x})\mathrm{d}\boldsymbol{x},$$

若 $h(\boldsymbol{x})$ 是函数向量，即，$h(\boldsymbol{x})=(h_1(\boldsymbol{x}),h_2(\boldsymbol{x}),\cdots,h_n(\boldsymbol{x}))^{\mathrm{T}}$，则其范数定义为

$$\|h\|^2=\int_{\mathscr{O}} h^{\mathrm{T}}(\boldsymbol{x})h(\boldsymbol{x})\mathrm{d}\boldsymbol{x},$$

$\mathscr{L}_2(\mathscr{O}\times[0,\infty);\mathbf{R}^{n_f})$ 是满足条件

$$E\int_0^t\int_{\mathscr{O}} f(\boldsymbol{x},s)^{\mathrm{T}} f(\boldsymbol{x},s)\mathrm{d}\boldsymbol{x}\mathrm{d}t:=$$

$$\int_{\Omega}\int_0^t\int_{\mathscr{O}} f(\boldsymbol{x},s,\Omega)^{\mathrm{T}} f(\boldsymbol{x},s,\Omega)\mathrm{d}\boldsymbol{x}\mathrm{d}tP\mathrm{d}\Omega<\infty$$

的可测函数集，其中 E 表示数学期望，$f(\boldsymbol{x},t):\mathscr{O}\times[0,T]\to\mathbf{R}^{n_f}$，是空间点 $\boldsymbol{x}\in\mathscr{O}$ 处的随机过程. 记 $\boldsymbol{\phi}(s,\boldsymbol{x})=((\boldsymbol{\phi}_1(s,\boldsymbol{x}),\boldsymbol{\phi}_2(s,\boldsymbol{x}),\cdots,\boldsymbol{\phi}_n(s,\boldsymbol{x}))^{\mathrm{T}}:-\tau<s\leqslant 0\}$ 为 $\mathbf{R}^n$-值 $\mathscr{F}_0$-可测随机过程且满足

$$E\|\boldsymbol{\phi}\|^2=E\{\sup_{-\tau\leqslant s\leqslant 0}\|\boldsymbol{\phi}(\cdot,s)\|^2\}<\infty.$$

给出以下基本假设：

假设 1 假设激励函数 g_i 连续可微且存在非负常数 χ_i，L_i，使得

$$|g_i(u_i)|\leqslant\chi_i,\quad 0<\frac{g_i(u_i)-g_i(v_i)}{u_i-v_i}\leqslant L_i,$$

$$\forall u_i\neq v_i,\quad u_i,v_i\in\mathbf{R},\quad i=1,2,\cdots,n.$$

假设 2 随机干扰函数 σ_{ij} 是全局 Lipschtiz 的且存在非负常数 μ_{ij}，υ_{ij}，使得

$$\sigma_{ij}^{\ 2}(t,u,v)\leqslant\mu_{ij}u^2+\upsilon_{ij}v^2.$$

假设系统(1)满足初始条件(2)的解存在，令 u^* 是系统(1)的平衡点，$\boldsymbol{u}(t,\boldsymbol{x})=(u_1(t,\boldsymbol{x}),\cdots,u_n(t,\boldsymbol{x}))^{\mathrm{T}}$ 是系统(1)的任意解. 这里 $\sigma_{ij}(t,u_j^{\ *},u_j^{\ *})=0$，$i=1,2,\cdots,n$. 记 $v_i(t,\boldsymbol{x})=u_i(t,\boldsymbol{x})-u_i^{\ *}$，则有

$$\mathrm{d}v_i(t,\boldsymbol{x})=\Big[\sum_{k=1}^{r}\frac{\partial}{\partial x_k}\Big(D_{ik}\frac{\partial v_i(t,\boldsymbol{x})}{\partial x_k}\Big)-c_iv_i(t,\boldsymbol{x})+\sum_{j=1}^{n}a_{ij}f_j(v_j(t,\boldsymbol{x}))+\sum_{j=1}^{n}\Big(b_{ij}+\sum_{l=1}^{n}(T_{ijl}+T_{ilj})\zeta_l\Big)\times f_j(v_j(t-\tau_j(t),\boldsymbol{x}))\Big]\mathrm{d}t+\sum_{j=1}^{n}\sigma_{ij}(t,v_j(t,\boldsymbol{x}),v_j(t-\tau_j(t),\boldsymbol{x}))\mathrm{d}w_j(t) \quad (5)$$

其中 $i=1,2,\cdots,n$，$f_j(v_j(t,\boldsymbol{x}))=g_j(u_j(t,\boldsymbol{x}))-g_j(u_j^{\ *})$，$f_j(v_j(t-\tau_j(t),\boldsymbol{x}))=g_j(u_j(t-\tau_j(t),\boldsymbol{x}))-g_j(u_j^{\ *})$，$\sigma_{ij}(t,v_j(t,\boldsymbol{x}),v_j(t-\tau_{ij}(t),\boldsymbol{x}))=\sigma_{ij}(t,u_j(t,\boldsymbol{x}),u_j(t-\tau_j(t),\boldsymbol{x}))$，$\zeta_l=T_{ijl}/(T_{ijl}+T_{ilj})g_l(u_l(t-\tau_l(t),\boldsymbol{x}))+T_{ilj}/(T_{ijl}+T_{ilj})g_l(u_l^*)$，当 $T_{ijl}+T_{ilj}\neq 0$，时，其位于 $g_l(u_l(t-\tau_l(t),\boldsymbol{x}))$ 和 $g_l(u_l^*)$ 之间，否则 $\zeta_l=0$. 则对任意的 $i=1,2,\cdots,n$，直接计算可得

$$|f_j(z)|\leqslant L_j|z|,zf_j(z)\geqslant 0,\quad\forall z\in\mathbf{R}.$$

为了得到主要结果，给出下述定义和引理：

定义 1 系统(5)的平衡点是均方指数稳定的，如果存在常数 $\lambda>0$ 和 $M>0$，使得

$$E\|v(\cdot,t)\|^2\leqslant M\mathrm{e}^{-\delta t}E\|\boldsymbol{\phi}\|^2,\quad t\geqslant 0$$

定义 2 对任意的连续函数 $V:\mathbf{R}\to\mathbf{R}$，$V(t)$ 的 Dini 导数定义为

$$D^+V(t)=\limsup_{t\to 0^+}\frac{V(t+h)-V(t)}{h},$$

易知 $V(t)$ 是局部 Lipschitz 的，$|D^+V(t)|<\infty$.

引理 1 (Poincarè-Friedrichs 不等式)假设 $\mathscr{O}=$

$\{\boldsymbol{x}|0\leqslant|\boldsymbol{x}|\leqslant l<\infty\}\subset\mathbf{R}^q$ 是一凸集,$y(\boldsymbol{x})\in C^1(\mathscr{O})$ 满足条件 $y(\boldsymbol{x})|_{\partial\mathscr{O}}=0$,则成立不等式

$$\int_{\mathscr{O}}y^2(\boldsymbol{x})\mathrm{d}\boldsymbol{x}\leqslant\frac{l^2}{q}\int_{\mathscr{O}}\sum_{i=1}^{q}\left(\frac{\partial y}{\partial\boldsymbol{x}}\right)^2\mathrm{d}\boldsymbol{x}=\frac{l^2}{q}\int_{\mathscr{O}}|\nabla y|^2\mathrm{d}\boldsymbol{x}.$$

引理 2 (Halanay 不等式)假设 $a>b>0$,$v(t)$ 是定义在 $[t_0-\tau,t_0]$ 上的非负连续函数且满足不等式

$$D^+v(t)\leqslant-av(t)+b\sup_{t-\tau\leqslant s\leqslant t}v(s),\quad t\geqslant t_0,$$

其中 τ 是非负常数,则存在常数 $k,\lambda>0$ 满足

$$v(t)\leqslant k\mathrm{e}^{-\lambda(t-t_0)},\quad t\geqslant t_0,$$

这里 $k=\sup_{t-\tau\leqslant s\leqslant t}v(s)$,$\lambda$ 是方程 $\lambda=a-b\mathrm{e}^{\lambda\tau}$ 的唯一正解.

2 主要结果

定理 1 若假设 1 成立且下列条件满足:

(H_1) $0<\mu<\lambda_m(\hat{\boldsymbol{A}})$;

(H_2) let $\lambda>0$ 使得 $\lambda-\lambda_m(\hat{\boldsymbol{A}})+\mu\mathrm{e}^{\lambda\tau}=0$,

其中

$$\mu=\lambda_M(\boldsymbol{S}),\boldsymbol{C}=\mathrm{diag}(c_1,c_2,\cdots,c_n),$$

$$m_{ij}=|b_{ij}|+\sum_{l=1}^{n}|T_{ijl}+T_{ilj}|\chi_l,\boldsymbol{M}=(m_{ij})_{n\times n},$$

$$\boldsymbol{T}_i=(T_{ijl})_{n\times n},\quad\boldsymbol{\chi}=(\chi_1,\cdots\chi_n)^{\mathrm{T}},$$

$$\boldsymbol{L}=\mathrm{diag}(L_1,\cdots,L_n),\quad a_{jj}^+=\max\{0,a_{jj}\},$$

$$\hat{\boldsymbol{A}}=(\hat{A}_{ij})=\begin{cases}l\dfrac{q}{l^2}\gamma_j+c_j-\dfrac{1}{2}\displaystyle\sum_{k=1}^{n}\mu_kj-L_ja_{jj}^+,j=i,\\-(|a_{ij}|L_j+|a_{ji}|L_i)/2,\quad j\neq i,\end{cases}$$

$$\boldsymbol{S}=\begin{bmatrix}0&\boldsymbol{ML}\\\boldsymbol{LM}^{\mathrm{T}}&\boldsymbol{\Theta}\end{bmatrix},$$

$$\boldsymbol{\Theta}=\mathrm{diag}\left(\sum_{k=1}^{n}\boldsymbol{v}_{k1},\sum_{k=1}^{n}\boldsymbol{v}_{k2},\cdots,\sum_{k=1}^{n}\boldsymbol{v}_{kn}\right),$$

则(5)的零解均方指数稳定.

证明 令 $V(t)=V(t,v(t,\boldsymbol{x}),v_{j\cdot t}=v_j(t-\tau_j(t),\boldsymbol{x})$,选择 Lyapunov 函数为

$$V(t)=\frac{1}{2}\int_{\mathscr{O}}\sum_{i=1}^{n}v_i^2(t,\boldsymbol{x})\mathrm{d}\boldsymbol{x},$$

由 Itô 公式,可得 $V(t)$ 的 Itô 微分方程:

$$\mathrm{d}V(t)=\int_{\mathscr{O}}\sum_{i=1}^{n}v_i(t,\boldsymbol{x})\left\{\sum_{k=1}^{m}\frac{\partial}{\partial x_k}\left(D_ik\frac{\partial v_i(t,\boldsymbol{x})}{\partial x_k}\right)-c_iv_i(t,\boldsymbol{x})+\sum_{j=1}^{n}a_{ij}(f_j(v_j(t,\boldsymbol{x})))+\sum_{j=1}^{n}(b_{ij}+\sum_{l=1}^{n}(T_{ijl}+T_{ilj})\zeta_l)\times f_j(v_{j\cdot t})\right\}\mathrm{d}\boldsymbol{x}\mathrm{d}t+\frac{1}{2}\int_{\mathscr{O}}\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}^2(t,v_j,v_{j\cdot t})\mathrm{d}\boldsymbol{x}\mathrm{d}t+\int_{\mathscr{O}}\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}(t,v_j,v_{j\cdot t})\mathrm{d}\boldsymbol{x}\mathrm{d}w_j(t).\tag{6}$$

现将上述的 Itô 微分方程转化为与之等价的 Itô 积分方程:式(6)两边同时从 t 到 $t+\delta$ 积分并取数学期望,有

$$EV(t+\delta)-EV(t)=E\left[\int_t^{t+\delta}\int_{\mathscr{O}}\sum_{i=1}^{n}v_i(t,\boldsymbol{x})\left\{\sum_{k=1}^{m}\frac{\partial}{\partial x_k}\left(D_ik\frac{\partial v_i(t,\boldsymbol{x})}{\partial x_k}\right)-c_iv_i(t,x)+\sum_{j=1}^{n}a_{ij}(f_j(v_j(t,\boldsymbol{x})))+\sum_{j=1}^{n}(b_{ij}+\sum_{l=1}^{n}(T_{ijl}+T_{ilj})\zeta_l)\times f_j(v_{j\cdot t})\right\}\mathrm{d}\boldsymbol{x}\mathrm{d}t\right]+E\left[\int_t^{t+\delta}\frac{1}{2}\int_{\mathscr{O}}\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}^2(t,v_j(t,\boldsymbol{x}),v_{j\cdot t})\mathrm{d}\boldsymbol{x}\mathrm{d}t\right].\tag{7}$$

计算 $EV(t)$ 沿式(5)的 Dini 导数 $D^+EV(t)$ 得

$$D^+EV(t)=E\left[\int_{\mathscr{O}}\sum_{i=1}^{n}v_i(t,\boldsymbol{x})\left\{\sum_{k=1}^{m}\frac{\partial}{\partial x_k}\left(D_ik\frac{\partial v_i(t,\boldsymbol{x})}{\partial x_k}\right)-c_iv_i(t,\boldsymbol{x})+\sum_{j=1}^{n}a_{ij}(f_j(v_j(t,\boldsymbol{x})))+\sum_{j=1}^{n}(b_{ij}+\sum_{l=1}^{n}(T_{ijl}+T_{ilj})\zeta_l)\times f_j(v_{j\cdot t})\right\}\mathrm{d}\boldsymbol{x}\right]+\frac{1}{2}E\int_{\mathscr{O}}\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}^2(t,v_j(t,\boldsymbol{x}),v_{j\cdot t})\mathrm{d}\boldsymbol{x},\tag{8}$$

由 Green 公式和引理 1 可得

$$\int_{\mathscr{O}}\sum_{k=1}^{m}v_i(t,\boldsymbol{x})\frac{\partial}{\partial x_k}\left(D_ik\frac{\partial v_i(t,\boldsymbol{x})}{\partial x_k}\right)\mathrm{d}x=$$

$$\int_{\mathscr{O}}v_i(t,\boldsymbol{x})\nabla\cdot\left(D_ik\frac{\partial(v_i(t,\boldsymbol{x}))}{\partial x_k}\right)_{k=1}^{m}\mathrm{d}x=$$

$$\int_{\mathscr{O}}\nabla\cdot\left(v_i(t,\boldsymbol{x})D_ik\frac{\partial v_i(t,\boldsymbol{x})}{\partial x_k}\right)_{k=1}^{m}\mathrm{d}x-$$

$$\int_{\mathscr{O}}\left(D_ik\frac{\partial(v_i(t,\boldsymbol{x}))}{\partial x_k}\right)_{k=1}^{m}\nabla\cdot v_i(t,\boldsymbol{x})\mathrm{d}x=$$

$$\int_{\partial\mathscr{O}}\left(v_i(t,\boldsymbol{x})D_ik\frac{\partial v_i(t,\boldsymbol{x})}{\partial x_k}\right)_{k=1}^{m}\mathrm{d}\boldsymbol{x}-\sum_{k=1}^{m}\int_{\mathscr{O}}D_ik\left(\frac{\partial v_i(t,\boldsymbol{x})}{\partial x_k}\right)^2\mathrm{d}\boldsymbol{x}=$$

$$-\sum_{k=1}^{m}\int_{\mathscr{O}}D_{ik}\left(\frac{\partial v_i(t,\boldsymbol{x})}{\partial x_k}\right)^2\mathrm{d}\boldsymbol{x}\leqslant-\frac{q}{l^2}\sum_{i=1}^{n}\gamma_i\int_{\Omega}v_i^2(t,\boldsymbol{x})\mathrm{d}\boldsymbol{x},\tag{9}$$

其中 $l=\max_{1\leqslant k\leqslant q}\{l_k\}$,$\nabla=\left(\dfrac{\partial}{\partial x_1},\cdots,\dfrac{\partial}{\partial x_m}\right)^{\mathrm{T}}$ 为梯度算子,

$$\left(D_{ik}\frac{\partial v_i(t,\boldsymbol{x})}{\partial x_k}\right)_{k=1}^{m}=\left(D_i1\frac{\partial v_i(t,\boldsymbol{x})}{\partial x_1},\cdots,D_{im}\frac{\partial v_i(t,\boldsymbol{x})}{\partial x_m}\right)^{\mathrm{T}}.$$

因而有

$$D^+EV(t)\leqslant E\left[\int_{\mathscr{O}}\sum_{i=1}^{n}\left(-\frac{q}{l^2}\gamma_iv_i^2(t,\boldsymbol{x})-c_iv_i^2(t,\boldsymbol{x})+\frac{1}{2}\sum_{j=1}^{n}\mu_{ji}v_i^2(t,\boldsymbol{x})\right)\mathrm{d}x\right]+E\left[\int_{\mathscr{O}}\sum_{i=1}^{n}\left(a_{ii}^+L_iv_i^2(t,\boldsymbol{x})+\sum_{j\neq i}^{n}|a_{ij}|L_j|v_i(\boldsymbol{x},t)||v_j(\boldsymbol{x},t)|\right)\mathrm{d}x\right],$$

$$E\left[\int_{\mathscr{O}}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(b_{ij}+\sum_{l=1}^{n}(T_{ijl}+T_{ilj})\zeta_l\right)\right.$$
$$\left.L_j\,|v_i(t,\boldsymbol{x})|\,|v_j(t-\tau_j(t),\boldsymbol{x})|\,\mathrm{d}\boldsymbol{x}\right]+$$
$$E\left[\int_{\mathscr{O}}\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\upsilon_{ij}v_j^2(t-\tau_j(t),\boldsymbol{x})\mathrm{d}\boldsymbol{x}\right]=$$
$$-E\int_{\mathscr{O}}|v(t,\boldsymbol{x})|^{\mathrm{T}}\hat{A}\,|v(t,\boldsymbol{x})|+$$
$$\frac{1}{2}E\int_{\mathscr{O}}[|v(t,\boldsymbol{x})|^{\mathrm{T}},|v(t-\tau(t),\boldsymbol{x})|^{\mathrm{T}}]^{\mathrm{T}}$$
$$\boldsymbol{S}\begin{bmatrix}|v(t,\boldsymbol{x})|\\|v(t-\tau(t),\boldsymbol{x})|\end{bmatrix}\mathrm{d}\boldsymbol{x}.\qquad(10)$$

其中 $|v(t,\boldsymbol{x})|=\left(|v_1(t,\boldsymbol{x})|,\ |v_2(t,\boldsymbol{x})|,\cdots,|v_n(t,\boldsymbol{x})|\right)^{\mathrm{T}}$,$|v(t-\tau(t),\boldsymbol{x})|=\left(|v_1(t-\tau_1(t),\boldsymbol{x})|,|v_2(t-\tau_2(t),\boldsymbol{x})|,\cdots,|v_n(t-\tau_n(t),\boldsymbol{x})|\right)^{\mathrm{T}}$. 因此得到

$$D^+EV(t)\leqslant$$
$$-\lambda_m(\hat{A})EV(t)+\mu EV(t)+\mu EV(t-\tau(t))\leqslant$$
$$-(\lambda_m(\hat{A})-\mu)EV(t)+\mu E\tilde{V}(t),\qquad(11)$$

其中 $E\tilde{V}(t)=\sup\limits_{t-\tau^*\leqslant s\leqslant t}EV(s)$. 由引理 2 中的 Halanay 不等式,对任意的 $t\geqslant t_0$ 有

$$V(t)\leqslant \mathrm{e}^{-\lambda(t-t_0)}\,\|V(t_0)\|_{\tau^*}.$$

由 $V(t)$ 的定义方式得

$$E\|v(t,\boldsymbol{x})\|^2\leqslant E\|\boldsymbol{\phi}\|^2\mathrm{e}^{-\lambda t},\quad t\geqslant 0,$$

其中 λ 是方程 $\lambda-\lambda_m(\hat{\boldsymbol{A}})+\mu\mathrm{e}^{\lambda\tau}=0$ 的唯一解. 因而定理的结论成立,证毕.

当 $\boldsymbol{\sigma}_{ij}(t,u_j(t,\boldsymbol{x}),u_j(t-\tau_j(t),\boldsymbol{x})=0$ 时,式(5)退化为

$$\frac{\partial v_i(t,\boldsymbol{x})}{\partial t}=\sum_{k=1}^{r}\frac{\partial}{\partial x_k}\left(D_ik\frac{\partial v_i(t,\boldsymbol{x})}{\partial x_k}\right)-c_iv_i(t,\boldsymbol{x})+$$
$$\sum_{j=1}^{n}a_{ij}g_j(v_j(t,\boldsymbol{x}))+\sum_{j=1}^{n}\left(b_{ij}+\sum_{l=1}^{n}(T_{ijl}+T_{ilj})\zeta_l\right)\times$$
$$f_j(v_j(t-\tau_j(t),\boldsymbol{x})),\qquad(12)$$

与定理 1 的证明相仿.易得到下列推论:

推论 1 若假设 1 成立且下列条件满足:

(A_1) $0<\mu<\lambda_m(\hat{\boldsymbol{A}})$;

(A_2) let $\lambda>0$ 使得 $\lambda-\lambda_m(\hat{\boldsymbol{A}})+\mu\mathrm{e}^{\lambda\tau}=0$,

其中

$$C=\mathrm{diag}(c_1,c_2,\cdots,c_n),$$
$$m_{ij}=|b_{ij}|+\sum_{l=1}^{n}|T_{ijl}+T_{ilj}|\chi_l,\quad \boldsymbol{M}=(m_{ij})_{n\times n},$$
$$\boldsymbol{T}_i=(T_{ijl})_{n\times n},\quad \boldsymbol{\chi}=(\chi_1,\cdots\chi_n)^{\mathrm{T}},$$
$$\boldsymbol{L}=\mathrm{diag}(L_1,\cdots,L_n),\quad a_{jj}^+=\max\{0,a_{jj}\},$$
$$\hat{\boldsymbol{A}}=(\hat{A}_{ij})=\begin{cases}l\dfrac{q}{l^2}\gamma_j+c_j-\dfrac{1}{2}\sum\limits_{k=1}^{n}\mu_kj-L_ja_{jj}^+,j=i,\\-(|a_{ij}|L_j+|a_{ji}|L_i)/2,\quad j\neq i,\end{cases}$$
$$\boldsymbol{S}=\begin{bmatrix}0&\boldsymbol{ML}\\\boldsymbol{LM}^{\mathrm{T}}&0\end{bmatrix},\quad \mu=\lambda_M(s),$$

则(12)的零解均方指数稳定.

3 数值算例

考虑如下的具反应扩散的二维高阶随机时滞 Hopfield 神经网络系统

$$\mathrm{d}v_i(t,\boldsymbol{x})=D_i\frac{\partial^2v_i(t,\boldsymbol{x})}{\partial\boldsymbol{x}^2}-c_iv_i(t,\boldsymbol{x})+$$
$$\sum_{j=1}^{2}a_{ij}g_j(v_j(t,\boldsymbol{x}))+\sum_{j=1}^{2}b_{ij}g_j(v_j(t-\tau_j(t),\boldsymbol{x}))+$$
$$\sum_{j=1}^{2}\sum_{l=1}^{2}T_{ijl}g_j(v_j(t-\tau_j(t),\boldsymbol{x}))\times$$
$$\left.g_l(v_l(t-\tau_l(t),\boldsymbol{x}))\right]\mathrm{d}t+$$
$$\sum_{j=1}^{2}\sigma_{ij}(t,v_j(t,\boldsymbol{x}),v_j(t-\tau_j(t),\boldsymbol{x})\mathrm{d}w_j(t),\qquad(13)$$

初值条件取为 $v_1(t,0)=v_2(t,0)=v_1(t,6)=v_2(t,6)$, $t\geqslant 0$. $\mathscr{O}=[0,6]\subset\mathbf{R}$,这里 $q=1,l=6$,选取 $D_{11}=6.5,D_{21}=4.5$, 即 $\gamma_1=6.5,\gamma_2=4.5$. $g_1(v_1)=\tanh(0.53v_1),g_2(v_2)=\tanh(0.67v_2),\tau_j(t)=0.5\mathrm{e}^{-t}$, $j=1,2$,

$$\boldsymbol{C}=\begin{bmatrix}1.80&0\\0&1.59\end{bmatrix},\quad \boldsymbol{A}=\begin{bmatrix}0.05&-0.15\\-0.20&0.31\end{bmatrix},$$
$$\boldsymbol{B}=\begin{bmatrix}0.09&0.25\\-0.21&0.45\end{bmatrix},$$
$$\boldsymbol{T}_1=(T_{1jl})_{2\times2}=\begin{bmatrix}0.05&0.14\\-0.06&0.05\end{bmatrix},$$
$$\boldsymbol{T}_2=(T_{2jl})_{2\times2}=\begin{bmatrix}0.29&0.10\\0.23&-0.14\end{bmatrix},$$
$$\boldsymbol{\sigma}(t,v,v_{\cdot t})=\begin{bmatrix}\dfrac{0.05v_1+\sqrt{0.1}v_{1\cdot t}}{\sqrt{2}}&\dfrac{0.1v_2+\sqrt{0.1}v_{2\cdot t}}{\sqrt{2}}\\\dfrac{0.04v_1+\sqrt{0.2}v_{1\cdot t}}{\sqrt{2}}&\dfrac{0.1v_2+\sqrt{0.3}v_{2\cdot t}}{\sqrt{2}}\end{bmatrix},$$
$$\boldsymbol{L}=\mathrm{diag}\{0.53,0.67\},$$

计算可得

$$\sigma_{11}\leqslant 0.002\,5v_1^2+0.1v_{1\cdot t}^2,\sigma_{12}\leqslant 0.01v_2^2+0.1v_{2\cdot t}^2,$$
$$\sigma_{21}\leqslant 0.001\,6^2v_1+0.2v_{1\cdot t}^2,\sigma_{22}\leqslant 0.01v_2^2+0.3^2v_{2\cdot t},$$

所以,$\mu_{11} = 0.0025$,$\mu_{21} = 0.0016$,$\mu_{12} = \mu_{22} = 0.01$,$v_{11} = v_{12} = 0.1$,$v_{21} = 0.2$,$v_{22} = 0.3$.
由定理 1 中参数的定义可得

$$\tau = 0.5, \quad \chi_i = 1, \quad \hat{\boldsymbol{A}} = \begin{bmatrix} 1.621 & -0.132 \\ -0.115 & 1.468 \end{bmatrix},$$

$$\lambda_m(\hat{\boldsymbol{A}}) = 1.7631, \quad \mu = 0.5886,$$

因而$\mu = 0.5886 < 1.7631 = \lambda_m(\hat{\boldsymbol{A}})$.由定理 1 可得(13)的平衡点是均方指数稳定的且指数收敛率大约为$\lambda = 0.03326$.这里λ是方程$\lambda - \lambda_m(\hat{\boldsymbol{A}}) + \mu e^{\lambda\tau} = 0$的唯一解.

4 结论

1)本文利用 Lyapunov 稳定性理论、积分不等式和 Halanay 不等式,研究了具分布参数高阶随机时滞 Hopfield 神经网络的均方指数稳定性,得到了保证系统指数稳定的充分性条件并给出了指数收敛率.同时放松了现有文献中对变时滞的要求,在一定程度上了改进了现有文献的结果.最后给出的数值算例验证了所得结果的有效性.

2)许多神经网络系统可以看做是本文所研究模型的特例,例如当$D_{ik}(t,\boldsymbol{x},\boldsymbol{u}) = 0$ ($i = 1,2,\cdots,n$, $k = 1,2,\cdots,r$)时,系统(1)退化为不带扩散项的高阶随机 Hopfield 神经网络,很多文献已有研究,当$\sigma_{ij} = 0$时系统(1)则退化为确定型系统.

参考文献
References

[1] Hopfield J J.Neural networks and physical systems with emergentcollective computational abilities[J].Proc Natl Acad Sci,1982,79:2554-2558
[2] Hopfield J J.Neurons with graded response have collectivecomputational properties like those of two-state neurons[J].Proc NatlAcad Sci,1984,81:3088-3092
[3] Liao X X,Yang S Z,Cheng S J.Stability of general neuralnetworks with reaction-diffusion[J].Science in China:F,2001,44(5):389-395
[4] Luo Q, Deng F Q, Bao J D, et al. Stabilization of stochastic Hopfield neural network with distributed parameters[J].Science in China:F,2004,47(6):752-762
[5] Lou X Y,Cui B T.Novel global stability criteria for high-orderHopfield-type neural networks with time-varying delays[J].Journal of Mathematical Analysis and Applications,2007,33(1):144-158
[6] Wang Z,Fang J,Liu X.Global stability of stochastic high-order neural networkswith discrete and distributed delays[J].Chaos,Solitons and Fractals,2008,36:388-396
[7] Cao J,Liang J.Exponential stability of high-orderbidirectional associative memory neural networks with time delays[J].Physica D,2004,199:425-436
[8] Liu X Z,Teo K L.Exponential stability of impulsive high-orderHopfield-type neural networks with time-varying delays[J].IEEE Transactions on Neural Networks,2005,16(6):1329-1339
[9] Liang J,Cao J.Global exponential stability of reaction-diffusionrecurrent neural networks with time-varying delays[J].Phys Lett A,2003,314:434-442
[10] Song Q K,Zhao Z J,Li Y M.Global exponential stability of BAM with distributed delays and reaction-diffusion terms[J].Phys Lett A,2005,335:213-225
[11] Cao J,Wang J.Absolute exponential stability of recurrent neuralnetworks with Lipschitz-continuous activation functions and timedelays[J].Neural Networks,2004,17(3):379-390
[12] Li C J,Li C D,Liao X F,et al.Impulsiven effects on stability of high-order BAM neural networks with time delays[J].Neurocomputing,2011,74:1541-1550
[13] Wang Z D, Fang J A, Liu, X H. Global stability of stochastic high-order neural networkswith discrete and distributed delays[J]. Chaos, Solitons and Fractals, 2008, 36:388-396
[14] Wang Y L, Cao J D. Exponential stability of stochastic higher-order BAM neural networks with reaction—diffusion terms and mixed time-varying delays[J].Neurocomputing,2013,119:192-200
[15] Zhou J P,Xu S Y,Zhang B Y,Zou Y,Shen H.robust exponential stability of uncertain stochastic neural networks with distributed delays and reaction-diffusions[J].IEEE Transactions on Neural Networks and Learning Systems, 2012,23(9):1407-1416
[16] Huang H Y, Du Q S, Kang X B. Global exponential stability of neutral high-order stochastic Hopfield neural networks with Markovian jump parameters andmixed time delays[J].ISA Transactions,2013,52:759-767
[17] Huang Z, Yang Q, Cao J. Stochastic stability and bifurcation analysis on Hopfieldneural networks with noise[J]. Expert Systems with Applications, 2011, 38: 10437-10445
[18] Zhou W N,Li M H.Mixed time-delays dependent exponential stability for uncertainstochastic high-order neural networks[J]. Applied Mathematics and Computation, 2009,215:503-513
[19] Qiu J L.Dynamics of high-order Hopfield neural networks with time delays[J].Neurocomputing,2010,73:820-826
[20] Zhang H,Wang W T,Xiao B.Exponential convergence for high-order recurrent neural networkswith a class of general activation functions[J]. Applied Mathematical Modelling,2011,35:123-129
[21] Wang L L. Dynamical analysis on the multistability of high-order neural networks[J]. Neurocomputing, 2013, 110:137-144
[22] Xu B J,Xu Y,He L M.LMI-based stability analysis of impulsive high-order Hopfield-type neural networks[J]. Mathematics and Computers in Simulation, 2012, 86: 67-77

Exponential stability of high-order stochastic Hopfield-type neuralnetworks with time-varying delays and distributed parameters

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Abstract In this paper，a generalized stochastic model of high-order Hopfield-typeneural networks with time-varying delays and distributed parametersis considered. The sufficientconditions ensuring the exponential stability of the systems are developed by using Lyapunovstability theory，an integral inequality and Halanay's inequality. The proposed conditions are diffusion-dependent due to the use of the new integral inequality. As a result，the obtained conditions may have some advantages over the those previously reported. As anillustration，an numerical example is worked out using the resultsobtained.

Key words exponential stability；Hopfield-typeneural networks；Halanay'sinequality；time-varying delays；distributed parameters