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一类偶数阶 Sturm-Liouvile 四点边值问题多个正解的存在性

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摘要

讨论了一类偶数阶四点边值问题正解的存在性,借助 Leggett-Williams 不动点定理和不等式技巧,得到了该边值问题至少存在三个和任意奇数个正解的充分条件.

关键词

正解; 锥; 不动点定理; 边值问题

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0 引言

Ilin 等[1]首次开展了关于常微分方程多点边值问题的研究[2],此后一些学者相继研究了各种类型的多点边值问题[3-10].作为一类特殊的多点边值问题,四点边值问题得到了广泛的关注[2,10-14],文献[2]研究了如下二阶非线性常微分方程

$$\begin{cases} x''(t)+h(t)f(t,x(t),x'(t))=0,\ 0\leqslant t\leqslant 1,\\ x'(0)-ax(\xi)=0, \qquad x'(1)+bx(\eta)=0. \end{cases} \tag{1}$$

作者不仅得到了一个正解的存在性,而且给出了充分的条件保证了至少三个正解的存在性.

近年来,一些学者研究了一般的偶数阶常微分方程[15-20].文献[20]借助于 Leggett-Williams 不动点定理讨论了如下偶数阶 Sturm-Liouvile 二点常微分方程边值问题:

$$\begin{cases} (-1)^m y^{(2m)}(t)=f(t,y), & 0\leqslant t\leqslant 1,\\ \alpha_{i+1}y^{(2i)}(0)-\beta_{i+1}y^{(2i+1)}(0)=0, & \\ \gamma_{i+1}y^{(2i)}(1)+\delta_{i+1}y^{(2i+1)}(1)=0, & i=0,1,\cdots,m-1, \end{cases} \tag{2}$$

并且建立了该问题存在三个及任意奇数个正解的充分条件.

受到以上文献的启发,本文将讨论如下的偶数阶四点边值问题:

$$\begin{cases} (-1)^m u^{(2m)}(t)=g(t)f(t,u(t)), & 0\leqslant t\leqslant 1,\\ u^{(2i+1)}(0)-\alpha_{i+1}u^{(2i)}(\xi)=0, & \\ u^{(2i+1)}(1)+\beta_{i+1}u^{(2i)}(\eta)=0, & i=0,1,\cdots,m-1. \end{cases} \tag{3}$$

其中 $0<\alpha_i<1/\xi, 0<\xi<\eta<1, \alpha_i\beta_i\eta-\alpha_i\beta_i\xi+\alpha_i+\beta_i>0, i=1,2,\cdots,m, g:(0,1)\to[0,+\infty)$ 在 $(0,1)$ 上连续,且可能在 $t=0$ 或 $t=1$ 奇异,$f:[0,1]\times[0,+\infty)\to[0,+\infty)$ 上连续.

本文利用 Leggett-Williams 不动点定理,建立了边值问题(3)至少存在三个和任意奇数个正解的充分条件.

1 预备知识

为了证明主要结果,给出一些必要的预备知识.

定义1 令 $E=(E,\|\cdot\|)$ 是一个 Banach 空间,$K\subset E$ 为非空凸闭集,K 被称之为一个锥如果它满足以下两个条件:

1) $\forall u \in K, \tau \geqslant 0$ 有 $\tau u \in K$;

2) $u \in K, -u \in K$ 有 $u = 0$.

定义 2 若映射 $\varphi: K \to [0,\infty)$ 是连续的且对任意 $x, y \in K, 0 < \tau < 1, \varphi(\tau x + (1-\tau)y) \geqslant \tau\varphi(x) + (1-\tau)\varphi(y)$,则称 φ 是一个锥 K 上的非负连续凹泛函.

由锥 K 及凹泛函 φ 的定义,本文引入下面的记号. 令 $0 < a < b, r > 0$,记 $K_r = \{u \in K \mid \|u\| < r\}$ 以及 $K(\varphi, a, b) = \{u \in K \mid a \leqslant \varphi(u), \|u\| \leqslant b\}$,其中 $\|u\| = \max\limits_{0 \leqslant t \leqslant 1} |u(t)|$.

引理 1 (Leggett-Williams 不动点定理) 设 $T: \overline{K_c} \to \overline{K_c}$ 是全连续的且 φ 是 K 上的非负连续凹泛函,满足 $\varphi(u) \leqslant \|u\|, \forall u \in \overline{K_c}$. 又设存在常数 $0 < d < a < b \leqslant c$ 使得下面的条件成立:

(C1) $\{u \in K(\varphi, a, b) \mid \varphi(u) > a\} \neq \varnothing$,当 $u \in K(\varphi, a, b)$ 时,有 $\varphi(Tu) > a$;

(C2) 当 $\|u\| \leqslant d$ 时,有 $\|Tu\| < d$;

(C3) 当 $u \in K(\varphi, a, c)$ 且 $\|Tu\| > b$ 时,有 $\varphi(Tu) > a$.

那么 T 至少存在三个不动点 u_1, u_2 和 u_3 满足

$\|u_1\| < d, a < \varphi(u_2), \|u_3\| > d$ 且 $\varphi(u_3) < a$.

引理 2[2] 令 $0 < \alpha_i < 1/\xi, 0 < \beta_i < 1/(1-\eta), 0 < \xi < \eta < 1$,则 $\sigma_i = \alpha_i\beta_i\eta - \alpha_i\beta_i\xi + \alpha_i + \beta_i > 0, i = 1, 2, \cdots, m$. 令 $G_i(t,s)$ 是如下二阶微分方程边值问题

$$\begin{cases} -u''(t) = f(t), & 0 \leqslant t \leqslant 1, \\ u'(0) - \alpha_i u(\xi) = 0, & u'(1) + \beta_i u(\eta) = 0 \end{cases}$$

的 Green 函数. 则

$$G_i(t,s) = \begin{cases} G_{1i}(t,s), & 0 \leqslant s \leqslant \min\{t,\xi\} \leqslant 1, \\ G_{2i}(t,s), & 0 \leqslant t \leqslant s \leqslant \xi, \\ G_{3i}(t,s), & \xi \leqslant s \leqslant \min\{t,\eta\} \leqslant 1, \\ G_{4i}(t,s), & 0 \leqslant \max\{\xi,t\} \leqslant s \leqslant \eta, \\ G_{5i}(t,s), & \eta \leqslant s \leqslant t \leqslant 1, \\ G_{6i}(t,s), & 0 \leqslant \max\{\eta,t\} \leqslant s \leqslant 1. \end{cases}$$

其中:

$G_{1i}(t,s) = (\beta_i\eta + 1 - \beta_i t)/\sigma_i$,

$G_{2i}(t,s) = (\beta_i\eta + 1 - \beta_i s)/\sigma_i + (s-t)(\alpha_i\beta_i\xi - \alpha_i\beta_i\eta - \alpha_i)/\sigma_i$,

$G_{3i}(t,s) = (\beta_i\eta + 1 - \beta_i t)(\alpha_i s + 1 - \alpha_i\xi)/\sigma_i$,

$G_{4i}(t,s) = (\beta_i\eta + 1 - \beta_i s)(\alpha_i t + 1 - \alpha_i\xi)/\sigma_i$,

$G_{5i}(t,s) = (\alpha_i t + 1 - \alpha_i\xi)/\sigma_i + (s-t)$,

$G_{6i}(t,s) = (\alpha_i t + 1 - \alpha_i\xi)/\sigma_i$.

设 $H_1(t,s) = G_1(t,s)$,当 $j = 2, 3, \cdots, m$ 时,

$$H_j(t,s) = \int_0^1 H_{j-1}(t,\tau) G_j(\tau,s) \mathrm{d}\tau, \quad 0 < t, s < 1,$$

则 $H_m(t,s)$ 是边值问题(3)的 Green 函数,且边值问题(3)的唯一解为

$$u(t) = \int_0^1 H_m(t,s) g(s) f(s, u(s)) \mathrm{d}s.$$

引理 3[2] 令 $0 < \xi < \eta < 1, 0 < \alpha_i < 1/\xi, 0 < \beta_i < 1/(1-\eta)$,则对 $i = 1, 2, \cdots, m$,有

(B1) $G_i(t,s) \geqslant 0, \quad t, s \in [0,1]$; (4)

(B2) $G_i(t,s) \leqslant G_i(s,s), \quad t, s \in [0,1]$; (5)

(B3) $\gamma_i G_i(s,s) \leqslant \min\limits_{\xi \leqslant t \leqslant \eta} G_i(t,s)$. (6)

其中 $\gamma_i = \min\left\{\dfrac{1}{1 + \beta_i\eta}, \dfrac{1}{1 + \alpha_i - \alpha_i\xi}\right\} \in (0,1)$.

2 引理

引理 4 令 $0 < \alpha_i < 1/\xi, 0 < \beta_i < 1/1-\eta, 0 < \xi < \eta < 1$,则 $\sigma_i = \alpha_i\beta_i\eta - \alpha_i\beta_i\xi + \alpha_i + \beta_i > 0, i = 1, 2, \cdots, m$,那么边值问题(3)的唯一解 $u(t)$ 满足 $\min\limits_{\xi \leqslant t \leqslant \eta} u(t) \geqslant M\|u\|$. 这里

$$M = \begin{cases} \gamma_1, & m = 1, \\ \gamma_m \prod\limits_{j=1}^{m-1} \dfrac{\gamma_j B_j}{A_j}, & m \geqslant 2. \end{cases}$$

证明 首先运用数学归纳法证明两个不等式

(L1) $0 \leqslant H_m(t,s) \leqslant \prod\limits_{j=1}^{m-1} A_j G_m(s,s), \quad 0 \leqslant t, s \leqslant 1$,

(L2) $H_m(t,s) \geqslant \gamma_m \prod\limits_{j=1}^{m-1} \gamma_j B_j G_m(s,s), \quad \xi \leqslant t \leqslant \eta, 0 \leqslant s \leqslant 1$.

其中 $A_i = \int_0^1 G_i(\tau,\tau) \mathrm{d}\tau, B_i = \int_\xi^\eta G_i(\tau,\tau) \mathrm{d}\tau, i = 1, 2, \cdots, m$. 由于(L2)的证明方法和过程与(L1)类似,只证(L1). 不妨令 $m \geqslant 2$,当 $m = 2$ 时,由不等式(5)得到

$$0 \leqslant H_2(t,s) = \int_0^1 G_1(t,\tau) G_2(\tau,s) \mathrm{d}\tau \leqslant \int_0^1 G_1(\tau,\tau) G_2(s,s) \mathrm{d}\tau = A_1 G_2(s,s).$$

假设 $m = n-1$ 时不等式(L1)成立,则当 $m = n$ 时,

$$H_n(t,s) = \int_0^1 H_{n-1}(t,\tau) G_n(\tau,s) \mathrm{d}\tau \leqslant \int_0^1 \prod_{j=1}^{n-2} A_j G_{n-1}(\tau,\tau) G_n(\tau,s) \mathrm{d}\tau \leqslant \int_0^1 \prod_{j=1}^{n-2} A_j G_{n-1}(\tau,\tau) G_n(s,s) \mathrm{d}\tau = \prod_{j=1}^{n-1} A_j G_n(s,s).$$

(L1) 证毕.

以下证明本引理的结论. 利用引理 2，当 $m = 1$ 时，由式(5) 和(6) 得

$$\min_{\xi\le t\le\eta} u(t) = \min_{\xi\le t\le\eta}\int_0^1 G_1(t,s)g(s)f(s,u(s))\mathrm{d}s \ge \gamma_1\int_0^1 G_1(s,s)g(s)f(s,u(s))\mathrm{d}s \ge \gamma_1\|u\| = M\|u\|.$$

当 $m \ge 2$ 时，利用引理 2，由(L2) 和(L1) 得

$$\min_{\xi\le t\le\eta} u(t) = \min_{\xi\le t\le\eta}\int_0^1 H_m(t,s)g(s)f(s,u(s))\mathrm{d}s \ge \int_0^1 \gamma_m\prod_{j=1}^{m-1}\gamma_jB_jG_m(s,s)g(s)f(s,u(s))\mathrm{d}s \ge \gamma_m\prod_{j=1}^{m-1}\gamma_jB_j\int_0^1 G_m(s,s)g(s)f(s,u(s))\mathrm{d}s \ge \gamma_m\prod_{j=1}^{m-1}\gamma_jB_j\int_0^1\max_{0\le t\le1}\frac{H_m(t,s)}{\prod_{j=1}^{m-1}A_j}g(s)f(s,u(s))\mathrm{d}s \ge \gamma_m\prod_{j=1}^{m-1}\frac{\gamma_jB_j}{A_j}\|u\| = M\|u\|.$$

证毕.

下面作以下假设:

(A1) $g:(0,1)\to[0,\infty)$ 连续，且 $0 < \int_0^1 G_m(s,s)g(s)\mathrm{d}s < +\infty$；

(A2) $f:[0,1]\times[0,\infty)\to[0,\infty)$ 连续，定义锥为

$$C^+[0,1] = \{u\in C[0,1] \mid u(t)\ge 0, t\in[0,1]\};$$

$$K = \{u\in C^+[0,1] \mid \min_{\xi\le t\le\eta}u(t)\ge M\|u\|\},$$

定义算子为

$$Tu(t) = \int_0^1 H_m(t,s)g(s)f(s,u(s))\mathrm{d}s,$$

定义常数为

$$C = \min_{\xi\le t\le\eta}\int_\xi^\eta H_m(t,s)g(s)\mathrm{d}s,$$

$$D = \max_{0\le t\le1}\int_0^1 H_m(t,s)g(s)\mathrm{d}s.$$

定义非负凹泛函为

$$\varphi: K\to R^+, \varphi(u) = \min_{\xi\le t\le\eta}u(t).$$

引理 5 设(A1)，(A2) 成立，则 T 是全连续的，且 $T(K)\subset K$，T 在 K 中的每一个不动点是边值问题(3) 的正解.

证明 由引理 4 知 $T(K)\subset K$，下面证明算子 T 是全连续的. 对 $n\ge2$，定义 $g_n(t)$ 为

$$g_n(t) = \begin{cases}\inf\limits_{0\le s\le 1/n} g(s), & 0\le t\le 1/n,\\ g(t), & 1/n < t < 1-(1/n),\\ \inf\limits_{1-(1/n)\le s\le 1} g(s), & 1-(1/n)\le t < 1\end{cases}$$

及 $T_n: K\to K$ 为

$$T_nu(t) = \int_0^1\int_0^1 H_{m-1}(t,\tau)G_m(\tau,s)g_n(s)f(s,u(s))\mathrm{d}s\mathrm{d}\tau.$$

显然对任意 $n\ge2$，由 Ascoli-Arzela 定理[22] 知 T_n 在 K 上是全连续的. 令 $B_R = \{u\in K \mid \|u\|\le R\}$，那么当 $n\to\infty$ 时 T_n 在 B_R 上一致收敛于 T. 事实上，设

$$G_R = \max\{f(t,u) \mid 0\le t\le1, 0\le u\le R\},$$

$$G = \max\{H_{m-1}(\tau,\tau) \mid 0\le\tau\le1\},$$

那么 $G_R, G < \infty$. 因为 $0 < \int_0^1 G_m(s,s)g(s)\mathrm{d}s < +\infty$，由积分的绝对连续性可得

$$\lim_{n\to\infty}\int_{e(1/n)} G_m(s,s)g(s)\mathrm{d}s = 0.$$

这里 $e(1/n) = [0,1/n]\cup[1-(1/n),1]$，所以对任意 $0\le t\le1$，固定的 $R>0$ 及 $u\in B_R$，有

$$|T_nu(t) - Tu(t)| = \left|\int_0^1\int_0^1 H_{m-1}(t,\tau)G_m(\tau,s)[g(s)-g_n(s)]f(s,u(s))\mathrm{d}s\mathrm{d}\tau\right| \le GG_R\int_0^1 G_m(s,s)|g(s)-g_n(s)|\mathrm{d}s \le GG_R\int_{e(1/n)} G_m(s,s)g(s)\mathrm{d}s\to 0.$$

因此，当 $n\to\infty$ 时，全连续算子 T_n 在 K 的任意有界域上一致收敛于 T，故 T 是全连续的，证毕.

3 主要结果

定理 1 假设条件(A1)，(A2) 成立，且存在常数 $0<\lambda<\mu$ 使得下列条件成立:

(H1) 当 $t\in[0,1]$，$u(t)\in[0,\lambda]$ 时，$f(t,u(t)) < \frac{\lambda}{D}$；

(H2) 下列情形之一满足:

(H21) $\limsup\limits_{u\to+\infty}\max\limits_{t\in[0,1]}\frac{f(t,u)}{u} < \frac{1}{D}$，

(H22) 存在常数 $\theta > \upsilon = \mu/M$，使得当 $t\in[0,1]$，$u\in[0,\theta]$ 时，有 $f(t,u)\le\frac{\theta}{D}$；

(H3) 当 $t\in[\xi,\eta]$，$u\in[\mu,\mu/M]$ 时，有 $f(t,u)\ge\frac{\mu}{Q}$，

那么边值问题(3) 至少存在三个正解 u_1，u_2 和 u_3 满足 $\|u_1\| < \lambda$，$\mu < \min\limits_{\xi\le t\le\eta}u_2(t)$ 和 $\|u_3\| > \lambda$，$\min\limits_{\xi\le t\le\eta}u_3(t) < \mu$.

证明 根据前文给出的定义和引理,下面将验证引理1的所有条件都成立. 首先证明由(H22)蕴含存在常数 θ 使得 $T:\bar{K}_\theta \to \bar{K}_\theta$.

若(H22)成立,则 $\forall u \in \bar{K}_\theta$,有

$$\|Tu\| = \max_{t\in[0,1]} \int_0^1 H_m(t,s)g(s)f(s,u(s))\,ds < \frac{\theta}{D}\max_{t\in[0,1]}\int_0^1 H_m(t,s)g(s)\,ds = \theta.$$

若(H21)成立,则存在 $N>0$ 和 $\varepsilon<1/D$ 使得

$$\forall u \geqslant N, t\in[0,1], 有 \frac{f(t,u(t))}{u} \leqslant \varepsilon.$$

记 $L=\max\{f(t,u)\mid t\in[0,1],u\in[0,N]\}$,则

$$f(t,u) \leqslant L+\varepsilon u,\quad u\geqslant 0. \tag{8}$$

现在选取常数 θ 使

$$\theta > \max\left\{\frac{LD}{1-\varepsilon D}, v\right\}, \tag{9}$$

那么,对 $\forall u \in \bar{K}_\theta$,由式(8)和(9)可得

$$\|Tu\| = \max_{t\in[0,1]}\int_0^1 H_m(t,s)g(s)f(s,u(s))\,ds \leqslant \max_{t\in[0,1]}\int_0^1 H_m(t,s)g(s)(L+\varepsilon u)\,ds \leqslant (L+\varepsilon\theta)\max_{t\in[0,1]}\int_0^1 H_m(t,s)g(s)\,ds = (L+\varepsilon\theta)D < \theta.$$

因此(H2)成立时,可以得到 $T:\bar{K}_\theta\to\bar{K}_\theta$.

同理,若取 $u\in\bar{K}_\lambda$,则由(H1)和以上类似的讨论,可以得到 $T:\bar{K}_\lambda\to\bar{K}_\lambda$,因此(C2)成立.

以下验证条件(C1)成立. 易见 $u=\frac{\mu+v}{2}$ 是 $K\{\varphi,\mu,v\}$ 中的元素,且 $\varphi(u)=\varphi\left(\frac{\mu+v}{2}\right)>\mu$,因此 $\{u\in K\{\varphi,\mu,v\}\mid\varphi(u)>\mu\}\neq\varnothing$. 取 $u\in K\{\varphi,\mu,v\}$,则 $\mu<\min_{\xi\leqslant t\leqslant\eta}u(t)\leqslant u(t)\leqslant v,\xi\leqslant t\leqslant\eta$.

继而由(H3)得

$$\varphi(Tu) = \min_{\xi\leqslant t\leqslant\eta}\int_0^1 H_m(t,s)g(s)f(s,u(s))\,ds > \min_{\xi\leqslant t\leqslant\eta}\int_\xi^\eta H_m(t,s)g(s)f(s,u(s))\,ds \geqslant \frac{\mu}{Q}\min_{\xi\leqslant t\leqslant\eta}\int_\xi^\eta H_m(t,s)g(s)\,ds = \mu.$$

最后,验证(C3)成立. 设 $u\in K\{\varphi,\mu,\theta\}$,$\|Tu\|\geqslant v$,由(L2)和(L1)得

$$\varphi(Tu) = \min_{\xi\leqslant t\leqslant\eta}\int_0^1 H_m(t,s)g(s)f(s,u(s))\,ds \geqslant \gamma_m\prod_{j=1}^{m-1}\gamma_j B_j\int_0^1 G_m(s,s)g(s)f(s,u(s))\,ds \geqslant \gamma_m\prod_{j=1}^{m-1}\gamma_j B_j\int_0^1 \max_{t\in[0,1]}\frac{1}{\prod_{j=1}^{m-1}A_j}H_m(t,s)g(s)f(s,u(s))\,ds = M\|Tu\| > Mv = \mu.$$

因此,引理1的所有条件都成立,应用引理1可知定理1的结论成立,证毕.

由定理1可以看到,当形如(H1)—(H3)的假设适度地加之于 f,可以建立边值问题(3)存在更多正解的充分性条件,具体地说,有下面的结论.

定理2 设存在常数组 $\{\lambda_i\}_{i=1}^n$ 与 $\{\mu_i\}_{i=1}^n$,且 $0<\lambda_1<\mu_1<\frac{\mu_1}{M}<\lambda_2<\mu_2<\frac{\mu_2}{M}<\lambda_3<\cdots<\lambda_n$,$n\in N$ 使得下列条件成立:

(S1) 当 $t\in[0,1]$,$u\in[0,\lambda_i]$ 时,有 $f(t,u(t))<\frac{\lambda_i}{D}$;

(S2) 当 $t\in[\xi,\eta]$,$u\in[\mu_i,\mu_i/M]$ 时,有 $f(t,u(t))>\frac{\mu_i}{Q}$,

那么边值问题(3)至少存在 $2n-1$ 个正解.

证明 当 $n=1$ 时,由定理1的证明及(S1)可知 $T:\overline{K_{\lambda_1}}\to K_{\lambda_1}\subset\overline{K_{\lambda_1}}$,又 T 是全连续的,则由Schauder不动点定理知 T 在 $\overline{K_{\lambda_1}}$ 中至少存在一个不动点 u_1.

当 $n=2$ 时,由(S1)和(S2)知定理1成立,故至少可得到三个正解 u_1, u_2 和 u_3 满足 $\|u_1\|<\lambda_1$,$\mu_1<\min_{\xi\leqslant t\leqslant\eta}u_2(t)$ 和 $\|u_3\|>\lambda_1$,$\min_{\xi\leqslant t\leqslant\eta}u_3(t)<\mu_1$.

一般情况利用数学归纳法加以证明,从略.

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The existence of multiple positive solutions for an even order Sturm-Liouville boundary value problem

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Abstract In this paper,we discuss the existence of positive solutions for an even order four point boundary value problems. Sufficient conditions are obtained for the existence of three or arbitrary odd positive solutions of the boundary value problem by using Leggett-Williams fixed point theorem and inequality techniques.

Key words positive solution; cone; fixed point theorem; boundary value problem