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一类时滞不确定离散系统的鲁棒自适应滑模控制

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摘要

主要研究一类具有不确定项和外部扰动的离散时滞系统的鲁棒自适应控制,假设扰动和不确定项是有界的,而扰动和关于状态的不确定项的边界是不必知道的.设计了自适应控制器以保证系统能在有限时间内到达切换面,并且收敛到包含原点的一定的有界区域内.仿真的结果说明了该方法的有效性.

关键词

离散系统;自适应控制;时滞;滑模控制

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0 引言

由于滑模变结构控制在滑动模态时,对摄动和外部扰动具有良好的鲁棒性,因而成为处理各种不确定性系统控制的有效方法之一.另一方面,随着计算机的广泛使用,仿真研究和实时控制时由于采样周期的存在,使变结构滑动模态的性质、稳定性及到达条件都发生变化,连续时间系统的变结构控制方法难以直接应用于离散时间系统,因此,研究离散时间系统的变结构控制方法具有重要的理论价值和实际意义.近年来,针对离散时间系统的变结构控制理论与设计的文献逐渐增多[1-12],其中自适应变结构控制方法因其兼有自适应控制和变结构控制二者的优点而吸引了众多学者的关注[4-12].

本文主要研究一类具有不确定项和外部扰动的离散时滞系统的鲁棒自适应控制.假设扰动和不确定项是有界的,但是扰动和关于状态的不确定项的边界是不必知道的,线性切换面存在[12-14].设计了自适应控制器以保证系统能在有限时间内到达切换面并且收敛到包含原点的一定的有界区域内.仿真的结果说明了方法的有效性.

1 问题描述

考虑如下不确定时滞离散系统:

$$\begin{cases} \boldsymbol{x}(k+1)=(\boldsymbol{A}+\Delta\boldsymbol{A})\boldsymbol{x}(k)+(\boldsymbol{B}+\Delta\boldsymbol{B})\boldsymbol{x}(k-d(k))+(\boldsymbol{D}+\Delta\boldsymbol{D})\times \\ \qquad \Big(\boldsymbol{u}(k)+\boldsymbol{w}(\boldsymbol{x}(k),\boldsymbol{x}(k-d(k)),k)\Big), \\ \boldsymbol{x}(k)=\boldsymbol{\varphi}(k),\quad k=-d_M,\cdots,0. \end{cases} \tag{1}$$

其中,$\boldsymbol{x}(k)\in\mathbf{R}^n$ 是状态向量,$\boldsymbol{w}\in\mathbf{R}^m$ 是扰动向量,$\Delta\boldsymbol{A},\Delta\boldsymbol{B},\Delta\boldsymbol{D}$ 是不确定项系数矩阵,$\boldsymbol{u}(k)\in\mathbf{R}^m$ 是控制向量,$\boldsymbol{A},\boldsymbol{B},\boldsymbol{D}$ 是相应维数的实常阵,且 $\mathrm{rank}(\boldsymbol{D})=m$,$d(k)$是变时滞.

A1)假设不确定项系数矩阵 $\Delta\boldsymbol{A},\Delta\boldsymbol{B}$ 满足下式:

$$[\Delta\boldsymbol{A}\quad\Delta\boldsymbol{B}]=\boldsymbol{GF}(k)[\boldsymbol{H}_A\quad\boldsymbol{H}_B]. \tag{2}$$

其中,$\boldsymbol{G},\boldsymbol{H}_A,\boldsymbol{H}_B$ 是已知的相应维数的常矩阵,$\boldsymbol{F}(k)$是未知的但范数有界矩阵,满足 $\boldsymbol{F}^{\mathrm{T}}(k)\boldsymbol{F}(k)\leqslant\boldsymbol{I}$,$\Delta\boldsymbol{D}=\boldsymbol{D}\Delta\hat{\boldsymbol{D}}$,$\Delta\hat{\boldsymbol{D}}=\mathbf{R}^{m\times m}$.

选取非奇异矩阵 $\boldsymbol{T}$,使得

$$\boldsymbol{TD}=\begin{pmatrix}\boldsymbol{0}_{(n-m)\times m}\\ \boldsymbol{D}_2\end{pmatrix},$$

其中,$\boldsymbol{D}_2\in\mathbf{R}^{m\times m}$为非奇异阵.矩阵 $\boldsymbol{D}$ 可做如下分解:

$$D=(T_1\quad T_2)\begin{pmatrix}\mathbf{0}_{(n-m)\times m}\\ \Sigma\end{pmatrix}J^{\mathrm{T}},$$

其中,$\Sigma\in\mathbf{R}^{m\times m}$是对角正定矩阵,$J\in\mathbf{R}^{m\times m}$是酉矩阵,$T_1,T_2$ 是酉矩阵的 2 个子块,所以,取

$$T=\begin{pmatrix}T_1^{\mathrm{T}}\\ T_2^{\mathrm{T}}\end{pmatrix},$$

作非奇异线性变换,令

$$z=Tx,$$

则系统(1)可以变换成如下形式:

$$\begin{cases}z(k+1)=(\hat{A}+\Delta\hat{A})z(k)+(\hat{B}+\Delta\hat{B})z(k-d(k))+\\ \quad\begin{pmatrix}\mathbf{0}_{(n-m)\times m}\\ D_2\end{pmatrix}(I+\Delta\hat{D})\Big(u(k)+\\ \quad w(z(k),z(k-d(k)),k)\Big),\\ z(k)=\hat{\varphi}(k),\quad k=-d_M,\cdots,0.\end{cases}\tag{3}$$

其中:

$$\hat{A}=TAT^{-1},\quad \Delta\hat{A}=T\Delta AT^{-1},\quad \hat{B}=TBT^{-1},$$
$$\Delta\hat{B}=T\Delta BT^{-1},\quad \hat{\varphi}(k)=T\varphi(k),\quad D_2=\Sigma J^{\mathrm{T}}.$$

由以上假设条件,系统(3)可以写成如下形式:

$$\begin{cases}z_1(k+1)=(\hat{A}_{11}+\Delta\hat{A}_{11})z_1(k)+(\hat{A}_{12}+\Delta\hat{A}_{12})z_2(k)+\\ \quad(\hat{B}_{11}+\Delta\hat{B}_{11})z_1(k-d(k))+\\ \quad(\hat{B}_{12}+\Delta\hat{B}_{12})z_2(k-d(k)),\\ z_2(k+1)=(\hat{A}_{21}+\Delta\hat{A}_{21})z_1(k)+(\hat{A}_{22}+\Delta\hat{A}_{22})z_2(k)+\\ \quad(\hat{B}_{21}+\Delta\hat{B}_{21})z_1(k-d(k))+(\hat{B}_{22}+\\ \quad\Delta\hat{B}_{22})z_2(k-d(k))+D_2(I+\Delta\hat{D})\times\\ \quad(u(k)+w(z(k),z(k-d(k)),k)),\\ z_1(k)=\hat{\varphi}_1(k),\quad k=-d_M,\cdots,0,\\ z_2(k)=\hat{\varphi}_2(k),\quad k=-d_M,\cdots,0.\end{cases}\tag{4}$$

选取切换面:

$$s(k)=[C\quad I]z(k)=Cz_1(k)+z_2(k)=0.\tag{5}$$

其中,$C\in\mathbf{R}^{m\times(n-m)}$,由式(5)知 $z_2=-Cz_1$,并把它代入方程组(4)的第 1 个方程中,得到

$$\begin{cases}z_1(k+1)=(\hat{A}_{11}+\Delta\hat{A}_{11}-\hat{A}_{12}C-\Delta\hat{A}_{12}C)z_1(k)+\\ \quad(\hat{B}_{11}+\Delta\hat{B}_{11}-\hat{B}_{12}C-\Delta\hat{B}_{12}C)z_1(k-d(k)),\\ z_1(k)=\hat{\varphi}(k),\quad k=-d_M,\cdots,0.\end{cases}\tag{6}$$

为了方便书写,可以把方程组(6)写成下式形式:

$$\begin{cases}z_1(k+1)=\bar{A}_1z_1(k)+\bar{B}_1z_1(k-d(k)),\\ z_1(k)=\hat{\varphi}_1(k),k=-d_M,\cdots,0.\end{cases}\tag{7}$$

其中

$$\bar{A}=\tilde{A}_1+\Delta\tilde{A}_1,\tilde{A}_1=\hat{A}_{11}-\hat{A}_{12}C,\Delta\tilde{A}_1=\Delta\hat{A}_{11}-\Delta\hat{A}_{12}C,$$
$$\bar{B}=\tilde{B}_1+\Delta\tilde{B}_1,\tilde{B}_1=\hat{B}_{11}-\hat{B}_{12}C,\Delta\tilde{B}_1=\Delta\hat{B}_{11}-\Delta\hat{B}_{12}C.$$

引理 1[15] 对于适当维数的矩阵 $X,\tilde{G},\tilde{H}$,其中 X 是对称阵,则对所有满足 $F^{\mathrm{T}}(k)F(k)\leqslant I$ 的$F(k)$,下式成立

$$X+\tilde{G}F(k)\tilde{H}+\tilde{H}^{\mathrm{T}}F^{\mathrm{T}}(k)\tilde{G}^{\mathrm{T}}\leqslant\mathbf{0},$$

当且仅当 $\exists\alpha>0$(α 是常数),使 $X+\alpha\tilde{G}\tilde{G}^{\mathrm{T}}+\alpha^{-1}\tilde{H}^{\mathrm{T}}\tilde{H}\leqslant\mathbf{0}$ 成立.

2 主要结果

A2)变时滞 $d(k)$满足下式所给出的条件:

$$0\leqslant d_m\leqslant d(k)\leqslant d_M.\tag{8}$$

A3)假设外部扰动满足下式所给出的条件:

$$\|w(x(k),x(k-d(k)),k)\|\leqslant l+m_{01}\|x(k)\|+m_{02}\|x(k-d(k))\|,\tag{9}$$

其中,l,m_{01},m_{02}是未知正常数.

引理 2[12] 对于已知常数 d_M,d_m 以及 $d_0=d_M-d_m$,简化的系统(7)二次稳定,若存在常数 $\beta>0$,矩阵 $C\in\mathbf{R}^{n\times(n-m)}$,对称正定矩阵 $P,L\in\mathbf{R}^{(n-m)\times(n-m)}$,对称半正定矩阵 $Z_1,Z_2,R_1,R_2,Q_1,Q_2,Q_3\in\mathbf{R}^{(n-m)\times(n-m)}$和矩阵

$$M=\begin{pmatrix}M_1\\ M_2\end{pmatrix},\quad N=\begin{pmatrix}N_1\\ N_2\end{pmatrix},\quad S=\begin{pmatrix}S_1\\ S_2\end{pmatrix},$$

$$X=\begin{pmatrix}X_{11}&X_{12}\\ X_{21}&X_{22}\end{pmatrix}\geqslant\mathbf{0},\quad Y=\begin{pmatrix}Y_{11}&Y_{12}\\ Y_{21}&Y_{22}\end{pmatrix}\geqslant\mathbf{0}$$

使以下不等式成立:

$$\begin{pmatrix}\Theta_{11}&\Theta_{12}&S_1&-N_1&\tilde{A}_1^{\mathrm{T}}&\Theta_{16}&\Theta_{17}&\Theta_{18}&\mathbf{0}\\ *&\Theta_{22}&S_2&-N_2&\tilde{B}_1^{\mathrm{T}}&d_M\tilde{B}_1^{\mathrm{T}}&d_0\tilde{B}_1^{\mathrm{T}}&\Theta_{28}&\mathbf{0}\\ *&*&-Q_1&\mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{0}\\ *&*&*&-Q_2&\mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{0}\\ *&*&*&*&-L&\mathbf{0}&\mathbf{0}&\mathbf{0}&\Theta_{59}\\ *&*&*&*&*&-d_MR_1&\mathbf{0}&\mathbf{0}&\Theta_{69}\\ *&*&*&*&*&*&-d_0R_2&\mathbf{0}&\Theta_{79}\\ *&*&*&*&*&*&*&-\beta I&\mathbf{0}\\ *&*&*&*&*&*&*&*&-\beta I\end{pmatrix}<\mathbf{0}\tag{10}$$

$$PL=I,\quad Z_1R_1=I,\quad Z_2R_2=I\tag{11}$$

$$\Psi_1=\begin{pmatrix}X&M\\ *&Z_1\end{pmatrix}\geqslant0,\quad \Psi_2=\begin{pmatrix}X+Y&N\\ *&Z_1+Z_2\end{pmatrix}\geqslant0,$$

$$\Psi_3=\begin{pmatrix}Y&S\\ *&Z_2\end{pmatrix}\geqslant0.\tag{12}$$

其中:

$$\Theta_{11}=Q_1+Q_2+(d_0+1)Q_3-P+M_1+M_1^{\mathrm{T}}+d_MX_{11}+d_0Y_{11},$$

$\boldsymbol{\Theta}_{22}=-\boldsymbol{Q}_3-\boldsymbol{M}_2-\boldsymbol{M}_2^{\mathrm{T}}-\boldsymbol{S}_2-\boldsymbol{S}_2^{\mathrm{T}}+\boldsymbol{N}_2+\boldsymbol{N}_2^{\mathrm{T}}+d_M\boldsymbol{X}_{22}+d_0\boldsymbol{Y}_{22}$,

$\boldsymbol{\Theta}_{12}=\boldsymbol{M}_2^{\mathrm{T}}-\boldsymbol{M}_1+\boldsymbol{N}_1-\boldsymbol{S}_1+d_M\boldsymbol{X}_{12}+d_0\boldsymbol{Y}_{12}$,

$\boldsymbol{\Theta}_{16}=d_M(\tilde{\boldsymbol{A}}_1^{\mathrm{T}}-\boldsymbol{I})$, $\boldsymbol{\Theta}_{17}=d_0(\tilde{\boldsymbol{A}}_1^{\mathrm{T}}-\boldsymbol{I})$,

$\boldsymbol{\Theta}_{18}=(\boldsymbol{H}_A\boldsymbol{T}_1-\boldsymbol{H}_A\boldsymbol{T}_2\boldsymbol{C})^{\mathrm{T}}$,

$\boldsymbol{\Theta}_{28}=(\boldsymbol{H}_B\boldsymbol{T}_1-\boldsymbol{H}_B\boldsymbol{T}_2\boldsymbol{C})^{\mathrm{T}}$, $\boldsymbol{\Theta}_{59}=\beta\boldsymbol{T}_1^{\mathrm{T}}\boldsymbol{G}$

$\boldsymbol{\Theta}_{69}=\beta d_M\boldsymbol{T}_1^{\mathrm{T}}\boldsymbol{G}$, $\boldsymbol{\Theta}_{79}=\beta d_0\boldsymbol{T}_1^{\mathrm{T}}\boldsymbol{G}$.

A4）假设

$\|\Delta\hat{\boldsymbol{D}}\|\leqslant 1-\delta$, $\|\boldsymbol{D}_2\Delta\hat{\boldsymbol{D}}\boldsymbol{D}_2^{-1}\|\leqslant 1-\delta, 0<\delta<1$,

$\|\boldsymbol{D}_2^{-1}\hat{\boldsymbol{C}}\Delta\boldsymbol{A}\boldsymbol{z}(k)\|\leqslant(m_{11}-m_{01})\|\boldsymbol{z}(k)\|$,

$\|\boldsymbol{D}_2^{-1}\hat{\boldsymbol{C}}\Delta\boldsymbol{B}\boldsymbol{z}(k-d(k))\|\leqslant(m_{21}-m_{02})\|\boldsymbol{z}(k-d(k))\|$,

$\|\Delta\hat{\boldsymbol{D}}\boldsymbol{D}_2^{-1}\boldsymbol{C}\boldsymbol{A}\boldsymbol{z}(k)\|\leqslant(m_{12}-m_{01})\|\boldsymbol{z}(k)\|$,

$\|\Delta\hat{\boldsymbol{D}}\boldsymbol{D}_2^{-1}\hat{\boldsymbol{C}}\boldsymbol{B}\boldsymbol{z}(k-d(k))\|\leqslant(m_{22}-m_{02})\|\boldsymbol{z}(k-d(k))\|$,

其中，$\hat{\boldsymbol{C}}=[\boldsymbol{C}\quad\boldsymbol{I}]$，令 $m_1=m_{11}+m_{12}-m_{01}$，$m_2=m_{21}+m_{22}-m_{12}$，由于 $\|\boldsymbol{x}(k)\|=\|\boldsymbol{z}(k)\|$，$\|\boldsymbol{x}(k-d(k))\|=\|\boldsymbol{z}(k-d(k))\|$，则

$\|\boldsymbol{D}_2^{-1}\hat{\boldsymbol{C}}\Delta\boldsymbol{A}\boldsymbol{z}(k)\|+\|\boldsymbol{D}_2^{-1}\hat{\boldsymbol{C}}\Delta\boldsymbol{B}\boldsymbol{z}(k-d(k))\|+\|\boldsymbol{w}(\boldsymbol{z}(k),\boldsymbol{z}(k-d(k)),k)\|\leqslant l+m_1\|\boldsymbol{z}(k)\|+m_2\|\boldsymbol{z}(k-d(k))\|\triangleq\rho$,

$\|\Delta\hat{\boldsymbol{D}}\boldsymbol{D}_2^{-1}\hat{\boldsymbol{C}}\boldsymbol{A}\boldsymbol{z}(k)\|+\|\Delta\hat{\boldsymbol{D}}\boldsymbol{D}_2^{-1}\hat{\boldsymbol{C}}\boldsymbol{B}\boldsymbol{z}(k-d(k))\|+\|\boldsymbol{w}(\boldsymbol{z}(k),\boldsymbol{z}(k-d(k)),k)\|\leqslant l+m_1\|\boldsymbol{z}(k)\|+m_2\|\boldsymbol{z}(k-d(k))\|$,

其中，l, m_1, m_2, ρ 是关于不确定项和扰动上界的未知参数.

定理 1 常矩阵如引理 2 所示，切换面由(5)式给出，则由下式给出的控制器可使系统(4)的轨迹能在有限时间内到达切换面并且收敛到原点附近：

$$\boldsymbol{u}(k)=-\boldsymbol{D}_2^{-1}[\lambda\boldsymbol{s}(k)+\hat{\boldsymbol{C}}\hat{\boldsymbol{A}}\boldsymbol{z}(k)+\hat{\boldsymbol{C}}\hat{\boldsymbol{B}}\boldsymbol{z}(k-d(k))]+\boldsymbol{u}_0,\tag{13}$$

$$\boldsymbol{u}_0=\begin{cases}-\dfrac{\boldsymbol{D}_2^{\mathrm{T}}\boldsymbol{s}(k)}{\|\boldsymbol{D}_2^{\mathrm{T}}\boldsymbol{s}(k)\|^2}\tilde{\rho}^2, & \text{如果 } \tilde{\rho}^2\|\boldsymbol{D}_2^{\mathrm{T}}\boldsymbol{s}(k)\|>\varepsilon,\\ -\dfrac{\boldsymbol{D}_2^{\mathrm{T}}\boldsymbol{s}(k)}{\varepsilon^2}\tilde{\rho}^2, & \text{如果 } \tilde{\rho}^2\|\boldsymbol{D}_2^{\mathrm{T}}\boldsymbol{s}(k)\|\leqslant\varepsilon.\end{cases}\tag{14}$$

自适应律为

$$\Delta\tilde{l}(k)=h_1(-\varepsilon_0\delta^2\tilde{l}(k)+\delta\|\boldsymbol{D}_2^{\mathrm{T}}\boldsymbol{s}(k)\|),\tag{15}$$

$$\Delta\tilde{m}_1(k)=h_2(-\varepsilon_1\delta^2\tilde{m}_1(k)+\delta\|\boldsymbol{D}_2^{\mathrm{T}}\boldsymbol{s}(k)\|\,\|\boldsymbol{z}(k)\|),\tag{16}$$

$$\Delta\tilde{m}_2(k)=h_3(-\varepsilon_2\delta^2\tilde{m}_2(k)+\delta\|\boldsymbol{D}_2^{\mathrm{T}}\boldsymbol{s}(k)\|\,\|\boldsymbol{z}(k-d(k))\|).\tag{17}$$

其中，$\lambda, h_1, h_2, h_3, \varepsilon, \varepsilon_0, \varepsilon_1, \varepsilon_2$ 是设计参数(正常数)，$\tilde{l}(k), \tilde{m}_1(k), \tilde{m}_2(k)$ 是不确定项和扰动的上界估计，而且

$\tilde{\rho}=\tilde{l}(k)+\tilde{m}_1(k)\|\boldsymbol{z}(k)\|+\tilde{m}_2(k)\|\boldsymbol{z}(k-d(k))\|$,

$\Delta\tilde{l}(k)=\tilde{l}(k+1)-\tilde{l}(k)$,

$\Delta\tilde{m}_1(k)=\tilde{m}_1(k+1)-\tilde{m}_1(k)$,

$\Delta\tilde{m}_2(k)=\tilde{m}_2(k+1)-\tilde{m}_2(k)$.

证明 考虑 Lyapunov 函数

$$V(k)=\frac{1}{2}\left[\boldsymbol{s}^{\mathrm{T}}(k)\boldsymbol{s}(k)+\frac{1}{h_1}\hat{l}^2(k)+\frac{1}{h_2}\hat{m}_1^2(k)+\frac{1}{h_3}\hat{m}_2^2(k)\right],\tag{18}$$

其中，$\hat{l}(k)=\dfrac{2l}{\delta}-\tilde{l}(k)$，$\hat{m}_1(k)=\dfrac{2m_1}{\delta}-\tilde{m}_1(k)$，$\hat{m}_2(k)=\dfrac{2m_2}{\delta}-\tilde{m}_2(k)$，$\hat{l}(k)=\dfrac{2l}{\delta}-\tilde{l}(k)\triangleq\hat{l}$，$\tilde{l}(k)\triangleq\tilde{l}$，

$\hat{m}_1(k)=\dfrac{2m_1}{\delta}-\tilde{m}_1(k)\triangleq\hat{m}_1$，$\tilde{m}_1(k)\triangleq\tilde{m}_1$，

$\hat{m}_2(k)=\dfrac{2m_2}{\delta}-\tilde{m}_2(k)\triangleq\hat{m}_2$，$\tilde{m}_2(k)\triangleq\tilde{m}_2$.

得到

$$\Delta V(k)=V(k+1)-V(k)=\boldsymbol{s}^{\mathrm{T}}(k)\Delta\boldsymbol{s}(k)+\frac{1}{h_1}\hat{l}\Delta\hat{l}(k)+\frac{1}{h_2}\hat{m}_1\Delta\hat{m}_1(k)+\frac{1}{h_3}\hat{m}_2\Delta\hat{m}_2(k)+\Psi_0.\tag{19}$$

其中，

$\Delta\boldsymbol{s}(k)=\boldsymbol{s}(k+1)-\boldsymbol{s}(k)$,

$\Delta\hat{l}(k)=\hat{l}(k+1)-\hat{l}(k)$,

$\Delta\hat{m}_1(k)=\hat{m}_1(k+1)-\hat{m}_1(k)$,

$\Delta\hat{m}_2(k)=\hat{m}_2(k+1)-\hat{m}_2(k)$,

$$\Psi_0=\frac{1}{2}\left[\Delta\boldsymbol{s}^{\mathrm{T}}(k)\Delta\boldsymbol{s}(k)+\frac{1}{h_1}(\Delta\hat{l}(k))^2+\frac{1}{h_2}(\Delta\hat{m}_1(k))^2+\frac{1}{h_3}(\Delta\hat{m}_2(k))^2\right].$$

由于 $\Delta\hat{l}(k)=-\Delta\tilde{l}(k)$，$\Delta\hat{m}_1(k)=-\Delta\tilde{m}_1(k)$，$\Delta\hat{m}_2(k)=-\Delta\tilde{m}_2(k)$，因而我们有

$$\Delta V(k)=\boldsymbol{s}^{\mathrm{T}}(k)\Delta\boldsymbol{s}(k)-\frac{1}{h_1}\hat{l}\Delta\tilde{l}(k)-\frac{1}{h_2}\hat{m}_1\Delta\tilde{m}_1(k)-\frac{1}{h_3}\hat{m}_2\Delta\tilde{m}_2(k)+\Psi_0.\tag{20}$$

由 $\boldsymbol{s}(k)=[\boldsymbol{C}\quad\boldsymbol{I}]\boldsymbol{z}(k)$，得

$$\boldsymbol{s}(k+1)=-(\boldsymbol{I}+\boldsymbol{D}_2\Delta\hat{\boldsymbol{D}}\boldsymbol{D}_2^{-1})\lambda\boldsymbol{s}(k)+\hat{\boldsymbol{C}}(\Delta\hat{\boldsymbol{A}}\boldsymbol{z}(k)+\Delta\hat{\boldsymbol{B}}\boldsymbol{z}(k-d(k)))-\boldsymbol{D}_2\Delta\hat{\boldsymbol{D}}\boldsymbol{D}_2^{-1}[\hat{\boldsymbol{C}}\hat{\boldsymbol{A}}\boldsymbol{z}(k)+\hat{\boldsymbol{C}}\hat{\boldsymbol{B}}\boldsymbol{z}(k-d(k))]+\boldsymbol{D}_2(\boldsymbol{I}+\Delta\hat{\boldsymbol{D}})\boldsymbol{u}_0+\boldsymbol{D}_2(\boldsymbol{I}+\Delta\hat{\boldsymbol{D}})\boldsymbol{w}(\boldsymbol{z}(k),\boldsymbol{z}(k-d(k)),k).\tag{21}$$

当 $\tilde{\rho}^2\|\boldsymbol{D}_2^{\mathrm{T}}\boldsymbol{s}(k)\|>\varepsilon$ 时，由控制律(13)以及自适应律(15)—(17)得

$$\Delta V(k)=\boldsymbol{s}^{\mathrm{T}}(k)\Delta\boldsymbol{s}(k)-\frac{1}{h_1}\hat{l}\Delta\tilde{l}(k)-\frac{1}{h_2}\hat{m}_1\Delta\tilde{m}_2(k)-$$

$$\frac{1}{h_3}\hat{m}_2\Delta\tilde{m}_2(k)+\Psi_0=$$
$$\boldsymbol{s}^{\mathrm{T}}(k)[-(\lambda+1)\boldsymbol{s}(k)+\hat{\boldsymbol{C}}\Delta\hat{\boldsymbol{A}}\boldsymbol{z}(k)+\hat{\boldsymbol{C}}\Delta\hat{\boldsymbol{B}}\times$$
$$\boldsymbol{z}(k-d(k))]+\boldsymbol{s}^{\mathrm{T}}(k)\boldsymbol{D}_2\boldsymbol{u}_0+$$
$$\boldsymbol{s}^{\mathrm{T}}(k)\boldsymbol{D}_2\boldsymbol{w}(\boldsymbol{z}(k),\boldsymbol{z}(k-d(k)),k)-$$
$$\boldsymbol{s}^{\mathrm{T}}(k)\boldsymbol{D}_2\Delta\hat{\boldsymbol{D}}\boldsymbol{D}_2^{-1}[\lambda\boldsymbol{s}(k)+\hat{\boldsymbol{C}}\hat{\boldsymbol{A}}\boldsymbol{z}(k)+$$
$$\hat{\boldsymbol{C}}\hat{\boldsymbol{B}}\boldsymbol{z}(k-d(k))]+\boldsymbol{s}^{\mathrm{T}}(k)\boldsymbol{D}_2\Delta\hat{\boldsymbol{D}}\boldsymbol{u}_0+$$
$$\boldsymbol{s}^{\mathrm{T}}(k)\boldsymbol{D}_2\Delta\hat{\boldsymbol{D}}\boldsymbol{w}(\boldsymbol{z}(k),\boldsymbol{z}(k-d(k)),k)-\frac{1}{h_1}\hat{l}\Delta\tilde{l}(k)-$$
$$\frac{1}{h_2}\hat{m}_1\Delta\tilde{m}_1(k)-\frac{1}{h_3}\hat{m}_2\Delta\tilde{m}_2(k)+\Psi_0\leqslant$$
$$-\left[\delta\left(\lambda-\frac{\|\boldsymbol{D}_2\|^2}{4}\right)+1\right]\boldsymbol{s}^{\mathrm{T}}(k)\boldsymbol{s}(k)-\varepsilon_0(\delta\tilde{l}-l)^2-$$
$$\varepsilon_1(\delta\tilde{m}_1-m_1)^2-\varepsilon_2(\delta\tilde{m}_2-m_2)^2+\varepsilon_0 l^2+\varepsilon_1 m_1^2+$$
$$\varepsilon_2 m_2^2+\Psi_0-\delta\left(\tilde{\rho}-\frac{\|\boldsymbol{D}_2^{\mathrm{T}}\boldsymbol{s}(k)\|}{2}\right)^2\leqslant$$
$$-\left[\delta\left(\lambda-\frac{\|\boldsymbol{D}_2\|^2}{4}\right)+1\right]\boldsymbol{s}^{\mathrm{T}}(k)\boldsymbol{s}(k)+$$
$$\varepsilon_0 l^2+\varepsilon_1 m_1^2+\varepsilon_2 m_2^2+\Psi_0. \tag{22}$$

同理,当 $\tilde{\rho}^2\|\boldsymbol{D}_2^{\mathrm{T}}\boldsymbol{s}(k)\|\leqslant\varepsilon$ 时,由控制律(13)以及自适应律(15)—(17)得

$$\Delta V(k)=\boldsymbol{s}^{\mathrm{T}}(k)\Delta\boldsymbol{s}(k)-\frac{1}{h_1}\hat{l}\Delta\tilde{l}(k)-$$
$$\frac{1}{h_2}\hat{m}_1\Delta\tilde{m}_1(k)-\frac{1}{h_3}\hat{m}_2\Delta\tilde{m}_2(k)+\Psi_0=$$
$$\boldsymbol{s}^{\mathrm{T}}(k)[-(\lambda+1)\boldsymbol{s}(k)+\hat{\boldsymbol{C}}\Delta\hat{\boldsymbol{A}}\boldsymbol{z}(k)+\hat{\boldsymbol{C}}\Delta\hat{\boldsymbol{B}}\times$$
$$\boldsymbol{z}(k-d(k))]+\boldsymbol{s}^{\mathrm{T}}(k)\boldsymbol{D}_2\boldsymbol{u}_0+$$
$$\boldsymbol{s}^{\mathrm{T}}(k)\boldsymbol{D}_2\boldsymbol{w}(\boldsymbol{z}(k),\boldsymbol{z}(k-d(k)),k)-$$
$$\boldsymbol{s}^{\mathrm{T}}(k)\boldsymbol{D}_2\Delta\hat{\boldsymbol{D}}\boldsymbol{D}_2^{-1}[\lambda\boldsymbol{s}(k)+\hat{\boldsymbol{C}}\hat{\boldsymbol{A}}\boldsymbol{z}(k)+$$
$$\hat{\boldsymbol{C}}\hat{\boldsymbol{B}}\boldsymbol{z}(k-d(k))]+\boldsymbol{s}^{\mathrm{T}}(k)\boldsymbol{D}_2\Delta\hat{\boldsymbol{D}}\boldsymbol{u}_0+$$
$$\boldsymbol{s}^{\mathrm{T}}(k)\boldsymbol{D}_2\Delta\hat{\boldsymbol{D}}\boldsymbol{w}(\boldsymbol{z}(k),\boldsymbol{z}(k-d(k)),k)-$$
$$\frac{1}{h_1}\hat{l}\Delta\tilde{l}(k)-\frac{1}{h_2}\hat{m}_1\Delta\tilde{m}_1(k)-$$
$$\frac{1}{h_3}\hat{m}_2\Delta\tilde{m}_2(k)+\Psi_0\leqslant$$
$$-[\delta\lambda+1]\boldsymbol{s}^{\mathrm{T}}(k)\boldsymbol{s}(k)-\varepsilon_0(\delta\tilde{l}-l)^2+\frac{\delta\varepsilon^2}{4}-$$
$$\varepsilon_2(\delta\tilde{m}_2-m_2)^2-\varepsilon_1(\delta\tilde{m}_1-m_1)^2+\varepsilon_1 m_1^2+$$
$$\varepsilon_0 l^2+\varepsilon_2 m_2^2+\Psi_0-\delta\left(\frac{\|\boldsymbol{D}_2^{\mathrm{T}}\boldsymbol{s}(k)\|\tilde{\rho}}{\varepsilon}-\frac{\varepsilon}{2}\right)^2\leqslant$$
$$-[\delta\lambda+1]\boldsymbol{s}^{\mathrm{T}}(k)\boldsymbol{s}(k)+\frac{\delta\varepsilon^2}{4}+\varepsilon_0 l^2+$$
$$\varepsilon_1 m_1^2+\varepsilon_2 m_2^2+\Psi_0. \tag{23}$$

虽然 $\Delta\boldsymbol{s}(k)$ 不是渐近收敛到 0 的,但是它是合理有界的. 由于 h_1,h_2,h_3 是参数,因而可以选择合适的 h_1,h_2,h_3,使 $(\Delta\hat{l}(k))^2,(\Delta\hat{m}_1(k))^2,(\Delta\hat{m}_2(k))^2$ 有界,由于 λ 是正常数,因而当 $\boldsymbol{s}(k)$ 不属于包含平衡点的一定的有界区域内时,可以选择充分大的 λ,使 $\Delta V(k)<0$. 因此系统(4)能在有限时间内到达切换面,并且收敛到包含原点的一定有界区域内.

3 仿真结果

考虑如下离散控制系统:

$$\begin{cases}\boldsymbol{x}(k+1)=(\boldsymbol{A}+\boldsymbol{\Delta A})\boldsymbol{x}(k)+(\boldsymbol{B}+\boldsymbol{\Delta B})\boldsymbol{x}(k-d(k))+\\ \quad(\boldsymbol{D}+\boldsymbol{\Delta D})(\boldsymbol{u}(k)+\boldsymbol{w}(\boldsymbol{x}(k),\boldsymbol{x}(k-d(k)),k)),\\ \boldsymbol{x}(k)=\boldsymbol{\varphi}(k),\quad k=-d_M,\cdots,0.\end{cases}$$

其中

$$\boldsymbol{A}=\begin{bmatrix}1&1\\0.6&0.9\end{bmatrix},\boldsymbol{B}=\begin{bmatrix}0.1&0.5\\0.4&0.2\end{bmatrix},\boldsymbol{G}=\begin{bmatrix}0.3\\0.1\end{bmatrix},$$
$$\boldsymbol{H}_A=\begin{bmatrix}0.1\\0.1\end{bmatrix}^{\mathrm{T}},\quad\boldsymbol{H}_B=\begin{bmatrix}0.1\\0.5\end{bmatrix}^{\mathrm{T}},\quad\boldsymbol{D}=\begin{bmatrix}0.5\\0.5\end{bmatrix},$$
$$\boldsymbol{\varphi}(k)=[0\quad 3]^{\mathrm{T}},\quad k=-5,\cdots,0.$$
$$\Delta\hat{\boldsymbol{D}}=0.1\sin(kT_s),\quad\boldsymbol{F}(k)=\sin(kT_s),$$

$\boldsymbol{w}(k)=0.1+0.1\sin(kT_s)$,$T_s=0.01$ 是采样周期.
令

$$\boldsymbol{T}=\begin{bmatrix}-0.7071&0.7071\\-0.7071&-0.7071\end{bmatrix},$$

得 $\boldsymbol{D}_2=-0.7071$.

由引理 2,得 $\boldsymbol{C}=-0.4079$,因而线性切换面为 $\boldsymbol{s}(k)=[-0.4079\quad 1]\boldsymbol{z}(k)$,其中,$\boldsymbol{z}(k)=\boldsymbol{Tx}(k)$.

由定理 1,控制律为:

$$\boldsymbol{u}(k)=-\boldsymbol{D}_2^{-1}[\lambda\boldsymbol{s}(k)+\hat{\boldsymbol{C}}\hat{\boldsymbol{A}}\boldsymbol{z}(k)+\hat{\boldsymbol{C}}\hat{\boldsymbol{B}}\boldsymbol{z}(k-d(k))]+\boldsymbol{u}_0,$$

$$\boldsymbol{u}_0=\begin{cases}-\dfrac{\boldsymbol{D}_2^{\mathrm{T}}\boldsymbol{s}(k)}{\|\boldsymbol{D}_2^{\mathrm{T}}\boldsymbol{s}(k)\|^2}\tilde{\rho}^2,&\text{如果 }\tilde{\rho}^2\|D_2^{\mathrm{T}}\boldsymbol{s}(k)\|>\varepsilon,\\ -\dfrac{\boldsymbol{D}_2^{\mathrm{T}}\boldsymbol{s}(k)}{\varepsilon^2}\tilde{\rho}^2,&\text{如果 }\tilde{\rho}^2\|\boldsymbol{D}_2^{\mathrm{T}}\boldsymbol{s}(k)\|\leqslant\varepsilon,\end{cases}$$

其中,$\hat{\boldsymbol{C}}=[\boldsymbol{C}\quad\boldsymbol{I}]=[-0.4079\quad 1]$,$\hat{\boldsymbol{A}}=\boldsymbol{TAT}^{-1}$,$\hat{\boldsymbol{B}}=\boldsymbol{TBT}^{-1}$.

图 1—6,是在 $\lambda=0.5$,$\varepsilon=0.5$,$\varepsilon_i=2.7682(i=0,1,2)$,$h_j=0.001(j=1,2,3)$,初始条件 $\boldsymbol{x}(0)=[0\quad 3]^{\mathrm{T}}$,$\tilde{l}(0)=0.075$,$\tilde{m}_1(0)=0.275$,$\tilde{m}_2(0)=0.05$ 的情况下给出的.

4 结束语

本文主要研究一类具有不确定项和外部扰动的离散时滞系统的鲁棒自适应控制. 假设扰动和不确定项是有界的,但是扰动和关于状态的不确定项的

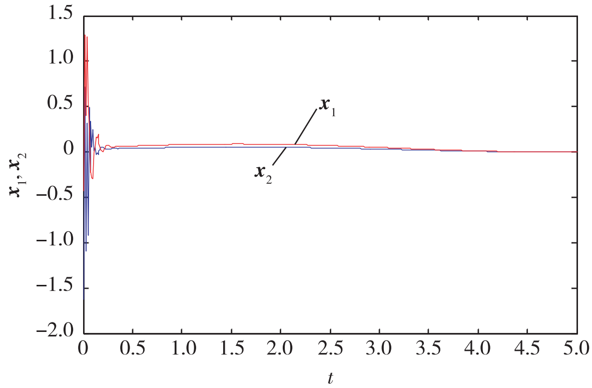

图 1 状态 x_1(红) x_2(蓝)变化曲线

Fig. 1 Changing curve of condition x_1(red) x_2(blue)

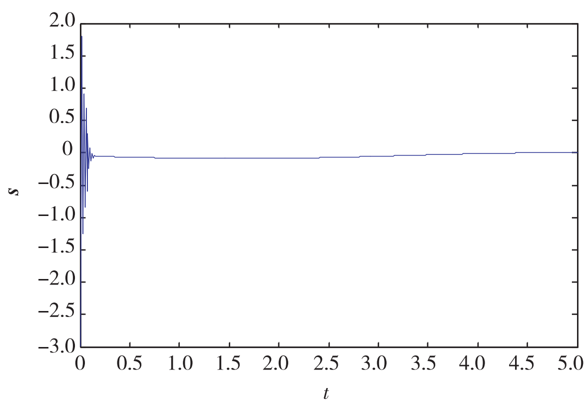

图 2 滑模面 s 的形状

Fig. 2 Shape of sliding surface s

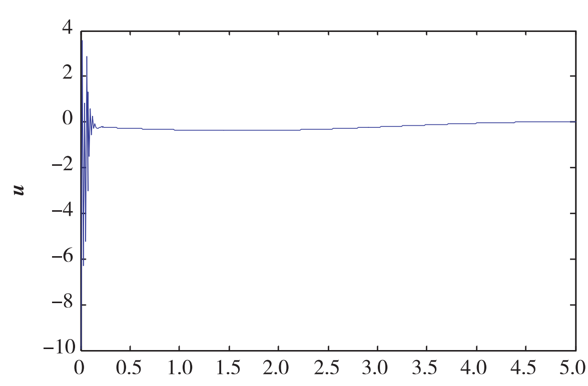

图 3 控制输入 u 的变化曲线

Fig. 3 Changing curve of control input u

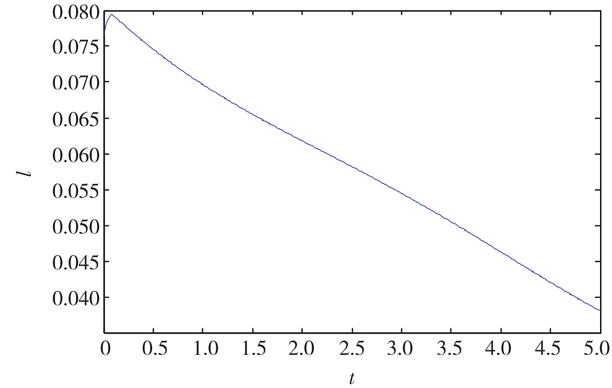

图 4 参数 $\tilde{l}$ 的变化情况

Fig. 4 Changing of parameter $\tilde{l}$

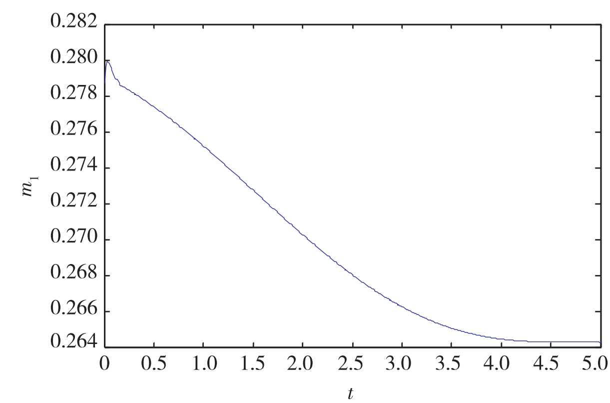

图 5 参数 $\tilde{m}_1$ 的变化情况

Fig. 5 Changing of parameter $\tilde{m}_1$

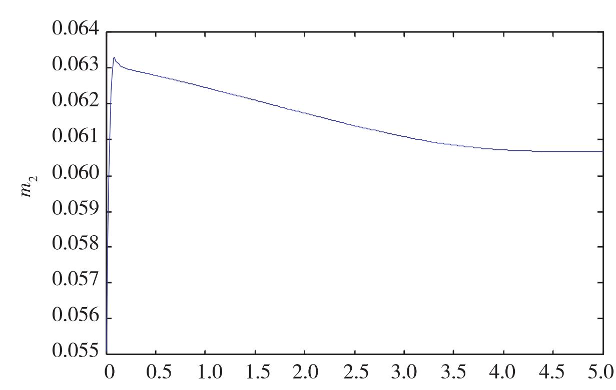

图 6 参数 $\tilde{m}_2$ 的变化情况

Fig. 6 Changing of parameter $\tilde{m}_2$

边界是不必知道的,设计了自适应控制器以保证状态变量能收敛到原点附近.仿真的结果说明了方法的有效性.

参考文献
References

[1] 姚琼荟,黄继起,吴汉松.变结构控制系统[M].重庆:重庆大学出版社,1997

YAO Qionghui, HUANG Jiqi, WU Hansong. Variable structure control system[M]. Chongqing:Chongqing University Press,1997
[2] 翟长连,吴智铭. 不确定离散时间系统的变结构控制设计[J]. 自动化学报,2000,26(2):184-191
ZHAI Changlian,WU Zhiming. Variable structure control design for uncertain discrete time systems[J]. Acta Automatica Sinica,2000,26(2):184-191
[3] 刘金琨,孙富春. 滑模变结构控制理论及其算法研究与进展[J]. 控制理论与应用,2007,43(3):407-418
LIU Jinkun,SUN Fuchun. Research and development on theory and algorithms of sliding mode control[J]. Control Theory & Applications,2007,43(3):407-418
[4] Chen X K,Fukuda T,Young K D. Adaptive quasi-sliding mode tracking control for discrete uncertain input-output systems[J]. IEEE Transactions on Industrial Electronics,2001,48(1):216-224
[5] Bartolini G,Ferrara A,Utkin V I. Adaptive sliding mode-control in discrete-time systems[J]. Automatica,1995,31(5):769-773
[6] Chan C Y. Discrete adaptive sliding-mode tracking controllers[J]. Automatica,1997,33(5):999-1002
[7] Zhang Y,Wen C Y,Soh Y C. Robust adaptive control of uncertain discrete-time systems [J]. Automatica,1999,35(2):321-329
[8] Chan C Y. Discrete adaptive sliding mode control of a state-space system with a bounded disturbance[J]. Automatica,1998,34(12):1631-1635
[9] Chen X,Fukuda T. Robust adaptive quasi-sliding mode controller for discrete-time systems[J]. Systems & Control Letters,1998,35 (5):165-173
[10] Mahmoud M S,Boukas E,Ismail A. Robust adaptive control of uncertain discrete-time state-delay systems[J]. Computers & Mathematics with Applications,2008,55(12):2887-2908
[11] Wheeler G,Su C,Stepanenko Y. A sliding mode controller with improved adaptation laws for the upper bounds on the norm of uncertainties[J]. Automatica,1998,34(12):1657-1661
[12] Xia Y Q,Zhu Z,Li C M,et al. Robust adaptive sliding mode control for uncertain discrete-time systems with time delay[J]. Journal of the Franklin Institute,2010,347:339-357
[13] Gao H,Lam J,Wang C,et al. Delay-dependent output-feedback stabilization of discrete-time systems with time-varying state delay[J]. IEEE Proceedings of Control Theory and Applications,2004,151(6):691-698
[14] He Y,Wu M,Liu G P,et al. Output feedback stabilization for discrete-time systems with a time-varying delay [J]. IEEE Transactions on Automatic Control,2008,53(10):2372-2377
[15] Shi P,Mahmoud M S,Yi J,et al. Worst case control of uncertain jumping systems with multi-state and input delay information [J]. Information Sciences,2006,176(2):186-200

Robust adaptive sliding mode control for a kind of uncertain discrete-time systems with time delay

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Abstract This paper focuses on robust adaptive sliding mode control for a kind of discrete-time state-delay systems with uncertainties and external disturbances. The uncertainties and disturbances are assumed to be norm-bounded but the bound of disturbances and uncertainties of state variables is not necessarily known. Especially,the uncertainties of state variables are mismatched. In this paper,a corresponding adaptive controller is designed by estimating the unknown upper bound of the disturbances and uncertainties of state variables. And it is designed to guarantee that the trajectory of the system can be driven on to the sliding surface in finite time and finally converges into a residual set of the origin. Also,simulation results are presented to illustrate the effectiveness of the control strategy.

Key words discrete-time systems;adaptive control;time delay;sliding mode control