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不确定奇异系统的时滞相关鲁棒 H_∞ 滤波器设计

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摘要

为了设计出全阶的滤波器,同时使得滤波误差动态系统是正则的、无脉冲的,并且满足一定的 H_∞ 性能指标,研究了连续奇异系统的全阶 H_∞ 滤波器的设计问题.利用 Lyapunov-Krasovskii 泛函及二次型的积分不等式方法获得了滤波误差动态系统的 H_∞ 性能时滞相关的判据,给出了奇异系统的鲁棒 H_∞ 滤波器存在的时滞依赖的充分条件.最后的数值例子说明了该方法的有效性.

关键词

奇异系统;不确定性;时滞相关判据;鲁棒 H_∞ 滤波

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0 引言

Introduction

近年来,系统的 H_∞ 滤波器设计问题被许多研究者关注[1-3].H_∞ 滤波器对于干扰噪声没有统计特性上的要求,而且拥有更好的鲁棒性[4-5],这些良好的特性也使得它在实际应用中有着很好的前景.许多文献中都提出了有关 H_∞ 滤波器的研究[6-8].

由于奇异系统在电力系统的建模和控制及在经济上和其他领域中的广泛应用,使它得到了深入的研究.关于奇异系统的滤波问题也被许多学者所研究[9-10].文献[11]给出了不确定时滞系统的鲁棒 H_∞ 保性能滤波,但该文中需要假定时滞变化率小于1.本文由文献[11]研究结果得到启发,讨论了不确定时滞奇异系统的鲁棒 H_∞ 滤波器设计问题.通过 Lyapunov-Krasovskii 泛函及二次型的积分不等式方法获得了滤波误差动态系统的 H_∞ 性能时滞相关的判据,给出了奇异系统的鲁棒 H_∞ 滤波器存在的时滞依赖的充分条件.值得指出的是,本文所提的方法没有对时变时滞项进行 $\dot{d}_i(t) < 1(i=1,2)$ 的限制.

本文采用以下记号:如果 $\boldsymbol{X}$ 是对称矩阵,$\boldsymbol{X}\geqslant 0(\boldsymbol{X}>0,\boldsymbol{X}<0,\boldsymbol{X}\leqslant 0)$表示 $\boldsymbol{X}$ 为半正定矩阵(正定矩阵,负定矩阵,半负定矩阵);矩阵 $\boldsymbol{M}^{\mathrm{T}}$ 表示矩阵 $\boldsymbol{M}$ 的转置矩阵;"$*$"表示对称矩阵的主对角线上块矩阵的转置;$\boldsymbol{I}$ 和 $\boldsymbol{0}$ 分别表示适当阶数的单位矩阵和零矩阵.

1 问题描述

Description of the problem

考虑下述不确定性时滞奇异系统:

$$\begin{cases}\boldsymbol{E}\dot{\boldsymbol{x}}(t)=(\boldsymbol{A}+\Delta\boldsymbol{A})\boldsymbol{x}(t)+(\boldsymbol{A}_1+\Delta\boldsymbol{A}_1)\boldsymbol{x}(t-d_1(t))+\\ \qquad(\boldsymbol{A}_2+\Delta\boldsymbol{A}_2)\boldsymbol{x}(t-d_2(t))+\boldsymbol{B}\boldsymbol{\omega}(t),\\ \boldsymbol{y}(t)=(\boldsymbol{C}+\Delta\boldsymbol{C})x(t)+(\boldsymbol{C}_1+\Delta\boldsymbol{C}_1)\boldsymbol{x}(t-d_1(t))+\\ \qquad(\boldsymbol{C}_2+\Delta\boldsymbol{C}_2)\boldsymbol{x}(t-d_2(t))+\boldsymbol{D}\boldsymbol{\omega}(t),\\ \boldsymbol{z}(t)=\boldsymbol{H}\boldsymbol{x}(t),\\ \boldsymbol{x}(t)=\boldsymbol{\varphi}_1(t),\quad\forall t\in[-d,0].\end{cases}\tag{1}$$

式中:$\boldsymbol{x}(t)\in\mathbf{R}^n$、$\boldsymbol{y}(t)\in\mathbf{R}^m$、$\boldsymbol{z}(t)\in\mathbf{R}^q$ 分别为系统的状态、测量控制输出和待估计的信号;$\boldsymbol{\omega}(t)\in\mathbf{R}^P$ 为 $L_2[0,\infty)$ 空间内的干扰输入;$\boldsymbol{E}$、$\boldsymbol{A}$、$\boldsymbol{A}_1$、$\boldsymbol{A}_2$、$\boldsymbol{B}$、$\boldsymbol{C}$、$\boldsymbol{C}_1$、$\boldsymbol{C}_2$、$\boldsymbol{D}$、$\boldsymbol{H}$ 为适当维数的已知实常阵,其中 $\boldsymbol{E}$ 可能是

奇异的,不失一般性,假设 $\mathrm{rank}\boldsymbol{E} = r \leqslant n$;$\Delta\boldsymbol{A}$、$\Delta\boldsymbol{A}_1$、$\Delta\boldsymbol{A}_2$、$\Delta\boldsymbol{C}$、$\Delta\boldsymbol{C}_1$、$\Delta\boldsymbol{C}_2$ 是具有适当维数的不确定时变矩阵,假设具有如下形式:

$$\begin{bmatrix} \Delta\boldsymbol{A} & \Delta\boldsymbol{A}_1 & \Delta\boldsymbol{A}_2 \\ \Delta\boldsymbol{C} & \Delta\boldsymbol{C}_1 & \Delta\boldsymbol{C}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{L}_1 \\ \boldsymbol{L}_2 \end{bmatrix} \boldsymbol{F}(t)[\boldsymbol{M}_1 \quad \boldsymbol{M}_2 \quad \boldsymbol{M}_3]. \quad (2)$$

其中:$\boldsymbol{L}_1$、$\boldsymbol{L}_2$、$\boldsymbol{M}_1$、$\boldsymbol{M}_2$、$\boldsymbol{M}_3$ 是具有适当维数的已知实常阵;$\boldsymbol{F}(t)$ 为有界不确定函数阵,满足

$$\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{F}(t) \leqslant \boldsymbol{I}. \quad (3)$$

$\boldsymbol{\varphi}_1(t) \in C([-d,0],\mathbf{R}^n)$ 函数,表示系统的初始状态;$d_i(t)(i = 1,2)$ 为系统的可微的有界时变时滞,且满足

$$0 < d_1(t) < d_2(t) < d,$$
$$0 < \dot{d}_i(t) < d^*, \quad i = 1,2. \quad (4)$$

首先给出关于奇异系统的几个定义. 对于奇异系统

$$\boldsymbol{E}\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) \quad (5)$$

有如下引理和定义:

引理 1[12] 系统(5)正则的、无脉冲的充分必要条件是存在矩阵 $\boldsymbol{Q} \in \mathbf{R}^{n\times n}$, 使得

$$\boldsymbol{E}^{\mathrm{T}}\boldsymbol{Q} = \boldsymbol{Q}^{\mathrm{T}}\boldsymbol{E} \geqslant 0, \quad \boldsymbol{A}^{\mathrm{T}}\boldsymbol{Q} + \boldsymbol{Q}^{\mathrm{T}}\boldsymbol{A} < 0.$$

定义 1[12]

1) 奇异系统(5)是正则、无脉冲的,
2) 奇异系统(5)是 Lyapunov 渐近稳定的,

则奇异系统(5)是鲁棒稳定的.

现在考虑如下关于信号 $\boldsymbol{z}(t)$ 估计的全阶滤波器:

$$\begin{cases} \dot{\hat{\boldsymbol{x}}}(t) = \boldsymbol{G}\hat{\boldsymbol{x}}(t) + \boldsymbol{K}\boldsymbol{y}(t), \quad \hat{\boldsymbol{x}}_0 = 0; \\ \hat{\boldsymbol{z}}(t) = \hat{\boldsymbol{H}}\hat{\boldsymbol{x}}(t) + \hat{\boldsymbol{D}}\boldsymbol{y}(t). \end{cases} \quad (6)$$

其中 $\boldsymbol{G}$、$\boldsymbol{K}$、$\hat{\boldsymbol{H}}$、$\hat{\boldsymbol{D}}$ 为待定的滤波器参数矩阵.

定义增广状态向量

$$\begin{cases} \tilde{\boldsymbol{x}}^{\mathrm{T}}(t) = [\boldsymbol{x}^{\mathrm{T}}(t), \hat{\boldsymbol{x}}^{\mathrm{T}}(t)]; \\ \tilde{\boldsymbol{z}}(t) = \boldsymbol{z}(t) - \hat{\boldsymbol{z}}(t). \end{cases} \quad (7)$$

且令

$$\widetilde{\boldsymbol{A}} = \boldsymbol{A} + \Delta\boldsymbol{A}, \quad \widetilde{\boldsymbol{A}}_1 = \boldsymbol{A}_1 + \Delta\boldsymbol{A}_1,$$
$$\widetilde{\boldsymbol{A}}_2 = \boldsymbol{A}_2 + \Delta\boldsymbol{A}_2, \quad \widetilde{\boldsymbol{C}} = \boldsymbol{C} + \Delta\boldsymbol{C},$$
$$\widetilde{\boldsymbol{C}}_1 = \boldsymbol{C}_1 + \Delta\boldsymbol{C}_1, \quad \widetilde{\boldsymbol{C}}_2 = \boldsymbol{C}_2 + \Delta\boldsymbol{C}_2. \quad (8)$$

则由式(1)与式(6)得到的滤波误差动态系统就可以表示为以下形式:

$$\begin{cases} \overline{\boldsymbol{E}}\dot{\tilde{\boldsymbol{x}}}(t) = \overline{\boldsymbol{A}}\tilde{\boldsymbol{x}}(t) + \overline{\boldsymbol{A}}_1\tilde{\boldsymbol{x}}(t - d_1(t)) + \\ \qquad \overline{\boldsymbol{A}}_2\tilde{\boldsymbol{x}}(t - d_2(t)) + \overline{\boldsymbol{B}}\boldsymbol{\omega}(t); \\ \tilde{\boldsymbol{z}}(t) = \overline{\boldsymbol{C}}\tilde{\boldsymbol{x}}(t) + \overline{\boldsymbol{C}}_1\tilde{\boldsymbol{x}}(t - d_1(t)) + \\ \qquad \overline{\boldsymbol{C}}_2\tilde{\boldsymbol{x}}(t - d_2(t)) + \overline{\boldsymbol{D}}\boldsymbol{\omega}(t); \\ \tilde{\boldsymbol{x}}^{\mathrm{T}}(t) = [\boldsymbol{\varphi}^{\mathrm{T}}(t), 0], \quad \forall t \in [-d, 0]. \end{cases} \quad (9)$$

其中:

$$\overline{\boldsymbol{E}} = \begin{bmatrix} \boldsymbol{E} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix}; \quad \overline{\boldsymbol{A}} = \begin{bmatrix} \widetilde{\boldsymbol{A}} & \boldsymbol{0} \\ \boldsymbol{K}\widetilde{\boldsymbol{C}} & \boldsymbol{G} \end{bmatrix}; \quad \overline{\boldsymbol{A}}_1 = \begin{bmatrix} \widetilde{\boldsymbol{A}}_1 & \boldsymbol{0} \\ \boldsymbol{K}\widetilde{\boldsymbol{C}}_1 & \boldsymbol{0} \end{bmatrix};$$
$$\overline{\boldsymbol{A}}_2 = \begin{bmatrix} \widetilde{\boldsymbol{A}}_2 & \boldsymbol{0} \\ \boldsymbol{K}\widetilde{\boldsymbol{C}}_2 & \boldsymbol{0} \end{bmatrix}; \quad \overline{\boldsymbol{B}} = \begin{bmatrix} \boldsymbol{B} \\ \boldsymbol{K}\boldsymbol{D} \end{bmatrix};$$
$$\overline{\boldsymbol{C}} = [\boldsymbol{H} - \hat{\boldsymbol{D}}\widetilde{\boldsymbol{C}} \quad -\hat{\boldsymbol{H}}]; \quad \overline{\boldsymbol{C}}_1 = [-\hat{\boldsymbol{D}}\widetilde{\boldsymbol{C}}_1 \quad \boldsymbol{0}];$$
$$\overline{\boldsymbol{C}}_1 = [-\hat{\boldsymbol{D}}\widetilde{\boldsymbol{C}}_1 \quad \boldsymbol{0}]; \quad \overline{\boldsymbol{D}} = -\hat{\boldsymbol{D}}\boldsymbol{D}. \quad (10)$$

引入一个性能指标

$$\boldsymbol{J} = \int_0^\infty [\tilde{\boldsymbol{z}}^{\mathrm{T}}(t)\tilde{\boldsymbol{z}}(t) - \gamma^2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{\omega}(t)]\mathrm{d}t. \quad (11)$$

定义 2 考虑不确定时滞奇异系统(1)及性能指标式(11),如果对给定的衰减度 $\gamma > 0$ 和任意满足式(2)和(4)的参数不确定性,存在一个形如式(6)的滤波器使得

1) 滤波误差动态系统(9)是正则、无脉冲,
2) 滤波误差动态系统(9)渐近稳定,
3) H_∞ 性能指标满足 $\boldsymbol{J} < 0$,

则称滤波器(6)是系统(1)的一个鲁棒 H_∞ 滤波器.

本文的研究目的是为系统(1)设计一个形如(6)的全阶鲁棒 H_∞ 滤波器.

引理 2[13] 给定适当维数矩阵 $\boldsymbol{\Omega}$、$\boldsymbol{\Gamma}$ 和 $\boldsymbol{\Xi}$,其中 $\boldsymbol{\Omega}$ 是对称的,则

$$\boldsymbol{\Omega} + \boldsymbol{\Gamma}\boldsymbol{F}(t)\boldsymbol{\Xi} + \boldsymbol{\Xi}^{\mathrm{T}}\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{\Gamma}^{\mathrm{T}} < 0$$

对所有满足 $\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{F}(t) \leqslant \boldsymbol{I}$ 的矩阵 $\boldsymbol{F}(t)$ 成立,当且仅当存在一个常数 $\varepsilon > 0$, 使得

$$\boldsymbol{\Omega} + \varepsilon^{-1}\boldsymbol{\Gamma}\boldsymbol{\Gamma}^{\mathrm{T}} + \varepsilon\boldsymbol{\Xi}^{\mathrm{T}}\boldsymbol{\Xi} < 0.$$

引理 3[13](Schur 补引理) 对给定的对称矩阵 $\boldsymbol{S} = \begin{bmatrix} \boldsymbol{S}_{11} & \boldsymbol{S}_{12} \\ \boldsymbol{S}_{21} & \boldsymbol{S}_{22} \end{bmatrix}$,其中 $\boldsymbol{S}_{11} \in \mathbf{R}^{r\times r}$, 以下 3 个条件是等价的:

1) $\boldsymbol{S} < 0$;
2) $\boldsymbol{S}_{11} < 0, \boldsymbol{S}_{22} - \boldsymbol{S}_{12}^{\mathrm{T}}\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{12} < 0$;
3) $\boldsymbol{S}_{22} < 0, \boldsymbol{S}_{11} - \boldsymbol{S}_{12}\boldsymbol{S}_{22}^{-1}\boldsymbol{S}_{12}^{\mathrm{T}} < 0$.

2 鲁棒 H_∞ 性能分析

Robust H_∞ performance analysis

首先给出滤波误差动态系统的鲁棒 H_∞ 性能时滞相关的判据. 为此,引入如下的变量:

$$\boldsymbol{\eta}^{\mathrm{T}}(t) = [\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\tilde{\boldsymbol{x}}^{\mathrm{T}}(t - d_1(t))\tilde{\boldsymbol{x}}^{\mathrm{T}}(t - d_2(t))\boldsymbol{\omega}^{\mathrm{T}}(t)],$$

那么

$$\overline{\boldsymbol{E}}\dot{\tilde{\boldsymbol{x}}}(t) = [\overline{\boldsymbol{A}} \quad \overline{\boldsymbol{A}}_1 \quad \overline{\boldsymbol{A}}_2 \quad \overline{\boldsymbol{B}}]\boldsymbol{\eta}(t).$$

对向量 $\boldsymbol{\eta}(t)$ 和 $\dot{\tilde{\boldsymbol{x}}}(t)$，引入下面的积分不等式，该积分不等式在后面的定理证明中起到重要的作用.

引理 4[14]（积分不等式） 设 $\tilde{\boldsymbol{x}}(t) \in \mathbf{R}^{2n}$ 具有一阶连续导数，则对任意已知的常数矩阵 $N_1 \in \mathbf{R}^{n\times n}, N_2 \in \mathbf{R}^{n\times n}, N_3 \in \mathbf{R}^{n\times n}, \boldsymbol{W} \in \boldsymbol{R}^{n\times p}$，时变时滞 $d_1(t) > 0, d_2(t) > 0$，有以下积分不等式成立:

$$-\int_{t-d_2(t)}^{t-d_1(t)} \dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}(s)\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{E}}\dot{\tilde{\boldsymbol{x}}}(s)\mathrm{d}s \leqslant$$
$$\boldsymbol{\eta}^{\mathrm{T}}(t)[\boldsymbol{\Pi} + (d_2(t) - d_1(t))\boldsymbol{Y}^{\mathrm{T}}\boldsymbol{Y}]\boldsymbol{\eta}(t).$$

其中:

$$\boldsymbol{Y} = [\bar{\boldsymbol{N}}_1 \quad \bar{\boldsymbol{N}}_2 \quad \bar{\boldsymbol{N}}_3 \quad \bar{\boldsymbol{W}}]; \quad \bar{\boldsymbol{N}}_1 = \begin{bmatrix} N_1 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix};$$

$$\bar{\boldsymbol{N}}_2 = \begin{bmatrix} N_2 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}; \quad \bar{\boldsymbol{N}}_3 = \begin{bmatrix} N_3 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}; \quad \bar{\boldsymbol{W}} = \begin{bmatrix} \boldsymbol{W} \\ \boldsymbol{0} \end{bmatrix};$$

$$\boldsymbol{\Pi} = \begin{bmatrix} \boldsymbol{0} & \bar{\boldsymbol{N}}_1^{\mathrm{T}}\bar{\boldsymbol{E}} & -\bar{\boldsymbol{N}}_1^{\mathrm{T}}\bar{\boldsymbol{E}} & \boldsymbol{0} \\ * & \bar{\boldsymbol{N}}_2^{\mathrm{T}}\bar{\boldsymbol{E}} + \bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{N}}_2 & \bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{N}}_3 - \bar{\boldsymbol{N}}_2^{\mathrm{T}}\bar{\boldsymbol{E}} & \bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{W}} \\ * & * & -\bar{\boldsymbol{N}}_3^{\mathrm{T}}\bar{\boldsymbol{E}} - \bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{N}}_3 & -\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{W}} \\ * & * & * & \boldsymbol{0} \end{bmatrix}.$$

根据引理4，以下定理给出了滤波误差动态系统(9)鲁棒 H_∞ 性能分析的充分条件.

定理 1 对任意满足(4)的时变时滞 $d_i(t)(i = 1,2)$ 和任意给定的常数 $\gamma > 0$，如果存在正定矩阵 $\boldsymbol{P} > 0$、$\boldsymbol{Q}_1 > 0$、$\boldsymbol{Q}_2 > 0$ 和矩阵 $\bar{\boldsymbol{S}}$、$\bar{\boldsymbol{N}}_1$、$\bar{\boldsymbol{N}}_2$、$\bar{\boldsymbol{N}}_3$、$\bar{\boldsymbol{W}}$ 使得:

$$\boldsymbol{\Omega} = \begin{bmatrix} \boldsymbol{\Omega}_{11} & \boldsymbol{\Omega}_{12} & \boldsymbol{\Omega}_{13} & \boldsymbol{\Omega}_{14} & \bar{\boldsymbol{N}}_1^{\mathrm{T}} & \bar{\boldsymbol{A}}^{\mathrm{T}} & \bar{\boldsymbol{C}}^{\mathrm{T}} \\ * & \boldsymbol{\Omega}_{22} & \boldsymbol{\Omega}_{23} & \bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{W}} & \bar{\boldsymbol{N}}_2^{\mathrm{T}} & \bar{\boldsymbol{A}}_1^{\mathrm{T}} & \bar{\boldsymbol{C}}_1^{\mathrm{T}} \\ * & * & \boldsymbol{\Omega}_{33} & -\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{W}} & \bar{\boldsymbol{N}}_3^{\mathrm{T}} & \bar{\boldsymbol{A}}_2^{\mathrm{T}} & \bar{\boldsymbol{C}}_2^{\mathrm{T}} \\ * & * & * & -\gamma^2\boldsymbol{I} & \bar{\boldsymbol{W}}^{\mathrm{T}} & \bar{\boldsymbol{B}}^{\mathrm{T}} & \bar{\boldsymbol{D}}^{\mathrm{T}} \\ * & * & * & * & -\bar{d}\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & * & * & -\bar{d}\boldsymbol{I} & \boldsymbol{0} \\ * & * & * & * & * & * & -\boldsymbol{I} \end{bmatrix} < \boldsymbol{0}, \quad (12)$$

其中:

$$\boldsymbol{\Omega}_{11} = \bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{E}} + \bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{A}} + \bar{\boldsymbol{A}}^{\mathrm{T}}\bar{\boldsymbol{R}}\bar{\boldsymbol{S}}^{\mathrm{T}} + \bar{\boldsymbol{S}}\bar{\boldsymbol{R}}^{\mathrm{T}}\bar{\boldsymbol{A}} + \boldsymbol{Q}_1 + \boldsymbol{Q}_2,$$
$$\boldsymbol{\Omega}_{12} = \bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{A}}_1 + \bar{\boldsymbol{S}}\bar{\boldsymbol{R}}^{\mathrm{T}}\bar{\boldsymbol{A}}_1 + \bar{\boldsymbol{N}}_1^{\mathrm{T}}\bar{\boldsymbol{E}},$$
$$\boldsymbol{\Omega}_{13} = \bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{A}}_2 + \bar{\boldsymbol{S}}\bar{\boldsymbol{R}}^{\mathrm{T}}\bar{\boldsymbol{A}}_2 - \bar{\boldsymbol{N}}_1^{\mathrm{T}}\bar{\boldsymbol{E}},$$
$$\boldsymbol{\Omega}_{14} = \bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\bar{B} + \bar{\boldsymbol{S}}\bar{\boldsymbol{R}}^{\mathrm{T}}\bar{B},$$
$$\boldsymbol{\Omega}_{22} = -(1 - d^*)\boldsymbol{Q}_1 + \bar{\boldsymbol{N}}_2^{\mathrm{T}}\bar{\boldsymbol{E}} + \bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{N}}_2,$$
$$\boldsymbol{\Omega}_{23} = \bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{N}}_3 - \bar{\boldsymbol{N}}_2^{\mathrm{T}}\bar{\boldsymbol{E}},$$
$$\boldsymbol{\Omega}_{33} = -(1 - d^*)\boldsymbol{Q}_2 - \bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{N}}_3 - \bar{\boldsymbol{N}}_3^{\mathrm{T}}\bar{\boldsymbol{E}},$$

且 $\bar{\boldsymbol{R}} \in \bar{\mathbf{R}}^{2n\times(n-r)}$ 为任意满足 $\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{R}} = 0$ 的列满秩矩阵，则滤波误差动态系统(9)为鲁棒稳定且满足 H_∞ 性能 γ.

证明 先证明滤波误差动态系统(9)的正则、无脉冲性. 令 $\boldsymbol{T} = \boldsymbol{P}\bar{\boldsymbol{E}} + \bar{\boldsymbol{R}}\bar{\boldsymbol{S}}^{\mathrm{T}}$，由 $\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{R}} = 0$ 得

$$\bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{T} = \boldsymbol{T}^{\mathrm{T}}\bar{\boldsymbol{E}} \geqslant 0.$$

由于 $\boldsymbol{\Omega} < 0$ 且 $\boldsymbol{Q}_1 > 0$、$\boldsymbol{Q}_2 > 0$ 很容易得到以下不等式成立:

$$\bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{E}} + \bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{A}} + \bar{\boldsymbol{A}}^{\mathrm{T}}\bar{\boldsymbol{R}}\bar{\boldsymbol{S}}^{\mathrm{T}} + \bar{\boldsymbol{S}}\bar{\boldsymbol{R}}^{\mathrm{T}}\bar{\boldsymbol{A}} = \bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{T} + \boldsymbol{T}^{\mathrm{T}}\bar{\boldsymbol{A}} < 0.$$

由引理 1 得到滤波误差动态系统(9)的正则、无脉冲性.

下面，证明滤波误差动态系统(9)渐近稳定.

取如下 Lyapunov-Krasovskii 泛函

$$V(\tilde{\boldsymbol{x}}(t)) = V_1(\tilde{\boldsymbol{x}}(t)) + V_2(\tilde{\boldsymbol{x}}(t)) + V_3(\tilde{\boldsymbol{x}}(t)) + V_4(\tilde{\boldsymbol{x}}(t)). \quad (13)$$

其中:

$$V_1(\tilde{\boldsymbol{x}}(t)) = \tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{E}}\tilde{\boldsymbol{x}}(t);$$
$$V_2(\tilde{\boldsymbol{x}}(t)) = \int_{t-d_1(t)}^{t} \tilde{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{Q}_1\tilde{\boldsymbol{x}}(s)\mathrm{d}s;$$
$$V_3(\tilde{\boldsymbol{x}}(t)) = \int_{t-d_2(t)}^{t} \tilde{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{Q}_2\tilde{\boldsymbol{x}}(s)\mathrm{d}s;$$
$$V_4(\tilde{\boldsymbol{x}}(t)) = \int_{-d}^{0}\int_{t+\theta}^{t} \dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}(s)\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{E}}\dot{\tilde{\boldsymbol{x}}}(s)\mathrm{d}s\mathrm{d}\theta.$$

则 $V(\tilde{\boldsymbol{x}}(t))$ 沿着滤波误差动态系统(9)的导数为

$$\dot{V}(\tilde{\boldsymbol{x}}(t)) = \dot{V}_1(\tilde{\boldsymbol{x}}(t)) + \dot{V}_2(\tilde{\boldsymbol{x}}(t)) + \dot{V}_3(\tilde{\boldsymbol{x}}(t)) + \dot{V}_4(\tilde{\boldsymbol{x}}(t)). \quad (14)$$

其中:

$$\dot{V}_1(\tilde{\boldsymbol{x}}(t)) =$$
$$\dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}(t)\bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{E}}\tilde{\boldsymbol{x}}(t) + \tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{E}}\dot{\tilde{\boldsymbol{x}}}(t) =$$
$$\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)(\bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{E}} + \bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{A}})\tilde{\boldsymbol{x}}(t) + 2\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{A}}_1\tilde{\boldsymbol{x}}(t - d_1(t)) +$$
$$2\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{A}}_2\tilde{\boldsymbol{x}}(t - d_2(t)) + 2\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{B}}\boldsymbol{\omega}(t);$$

$$\dot{V}_2(\tilde{\boldsymbol{x}}(t)) =$$
$$\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Q}_1\tilde{\boldsymbol{x}}(t) - (1 - \dot{d}_1(t))\tilde{\boldsymbol{x}}^{\mathrm{T}}(t - d_1(t))\boldsymbol{Q}_1\tilde{\boldsymbol{x}}(t - d_1(t));$$

$$\dot{V}_3(\tilde{\boldsymbol{x}}(t)) =$$
$$\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Q}_2\tilde{\boldsymbol{x}}(t) - (1 - \dot{d}_2(t))\tilde{\boldsymbol{x}}^{\mathrm{T}}(t - d_2(t))\boldsymbol{Q}_2\tilde{\boldsymbol{x}}(t - d_2(t));$$

$$\dot{V}_4(\tilde{\boldsymbol{x}}(t)) =$$
$$d\dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}(t)\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{E}}\dot{\tilde{\boldsymbol{x}}}(t) - \int_{t-d}^{t}\dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}(s)\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{E}}\dot{\tilde{\boldsymbol{x}}}(s)\mathrm{d}s \leqslant$$
$$d\dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}(t)\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{E}}\dot{\tilde{\boldsymbol{x}}}(t) - \int_{t-d_2(t)}^{t-d_1(t)}\dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}(s)\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{E}}\dot{\tilde{\boldsymbol{x}}}(s)\mathrm{d}s =$$
$$d\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\bar{\boldsymbol{A}}^{\mathrm{T}}\bar{\boldsymbol{A}}\tilde{\boldsymbol{x}}(t) + 2d\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\bar{\boldsymbol{A}}^{\mathrm{T}}\bar{\boldsymbol{A}}_1\tilde{\boldsymbol{x}}(t - d_1(t)) +$$
$$2d\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\bar{\boldsymbol{A}}^{\mathrm{T}}\bar{\boldsymbol{A}}_2\tilde{\boldsymbol{x}}(t - d_2(t)) + 2d\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\bar{\boldsymbol{A}}^{\mathrm{T}}\bar{\boldsymbol{B}}\boldsymbol{\omega}(t) +$$
$$d\tilde{\boldsymbol{x}}^{\mathrm{T}}(t - d_1(t))\bar{\boldsymbol{A}}_1^{\mathrm{T}}\bar{\boldsymbol{A}}_1\tilde{\boldsymbol{x}}(t - d_1(t)) +$$
$$2d\tilde{\boldsymbol{x}}^{\mathrm{T}}(t - d_1(t))\bar{\boldsymbol{A}}_1^{\mathrm{T}}\bar{\boldsymbol{A}}_2\tilde{\boldsymbol{x}}(t - d_2(t)) +$$
$$2d\tilde{\boldsymbol{x}}^{\mathrm{T}}(t - d_1(t))\bar{\boldsymbol{A}}_1^{\mathrm{T}}\bar{\boldsymbol{B}}\boldsymbol{\omega}(t) +$$
$$d\tilde{\boldsymbol{x}}^{\mathrm{T}}(t - d_2(t))\bar{\boldsymbol{A}}_2^{\mathrm{T}}\bar{\boldsymbol{A}}_2\tilde{\boldsymbol{x}}(t - d_2(t)) +$$
$$2d\tilde{\boldsymbol{x}}^{\mathrm{T}}(t - d_2(t))\bar{\boldsymbol{A}}_2^{\mathrm{T}}\bar{\boldsymbol{B}}\boldsymbol{\omega}(t) + d\boldsymbol{\omega}^{\mathrm{T}}(t)\bar{\boldsymbol{B}}^{\mathrm{T}}\bar{\boldsymbol{B}}\boldsymbol{\omega}(t) -$$

$$\int_{t-d_2(t)}^{t-d_1(t)} \dot{\tilde{x}}^{\mathrm{T}}(s)\bar{E}^{\mathrm{T}}\bar{E}\dot{\tilde{x}}(s)\,\mathrm{d}s.$$

另外,注意到 $\bar{E}^{\mathrm{T}}\bar{R}=0$, 容易得到

$$0 = 2\dot{\tilde{x}}^{\mathrm{T}}(t)\bar{E}^{\mathrm{T}}\bar{R}\bar{S}^{\mathrm{T}}\tilde{x}(t) = 2[\bar{A}\tilde{x}(t)+\bar{A}_1\tilde{x}(t-d_1(t))+\bar{A}_2\tilde{x}(t-d_2(t))+\bar{B}\boldsymbol{\omega}(t)]^{\mathrm{T}}\bar{R}\bar{S}^{\mathrm{T}}\tilde{x}(t). \tag{15}$$

因此,根据式(15)和引理(4)可得

$$\begin{aligned}
&\dot{V}(\tilde{x}(t)) = \\
&\dot{V}_1(\tilde{x}(t))+\dot{V}_2(\tilde{x}(t))+\dot{V}_3(\tilde{x}(t))+\dot{V}_4(\tilde{x}(t)) \leqslant \\
&\tilde{x}^{\mathrm{T}}(t)[\bar{A}^{\mathrm{T}}P\bar{E}+\bar{E}^{\mathrm{T}}P\bar{A}+\bar{A}^{\mathrm{T}}\bar{R}\bar{S}^{\mathrm{T}}+\bar{S}\bar{R}^{\mathrm{T}}\bar{A}+Q_1+Q_2+ \\
&\quad d\bar{N}_1^{\mathrm{T}}\bar{N}_1+d\bar{A}^{\mathrm{T}}\bar{A}]\tilde{x}(t)+ \\
&\quad 2\tilde{x}^{\mathrm{T}}(t)[\bar{E}^{\mathrm{T}}P\bar{A}_1+\bar{S}\bar{R}^{\mathrm{T}}\bar{A}_1+\bar{N}_1^{\mathrm{T}}\bar{E}+d\bar{A}^{\mathrm{T}}\bar{A}_1+ \\
&\quad d\bar{N}_1^{\mathrm{T}}\bar{N}_2]\tilde{x}(t-d_1(t))+ \\
&\quad 2\tilde{x}^{\mathrm{T}}(t)[\bar{E}^{\mathrm{T}}P\bar{A}_2+\bar{S}\bar{R}^{\mathrm{T}}\bar{A}_2-\bar{N}_1^{\mathrm{T}}\bar{E}+d\bar{A}^{\mathrm{T}}\bar{A}_2+ \\
&\quad d\bar{N}_1^{\mathrm{T}}\bar{N}_3]\tilde{x}(t-d_2(t))+ \\
&\quad 2\tilde{x}^{\mathrm{T}}(t)[\bar{E}^{\mathrm{T}}P\bar{B}+\bar{S}\bar{R}^{\mathrm{T}}\bar{B}+d\bar{A}^{\mathrm{T}}\bar{B}+d\bar{N}_1^{\mathrm{T}}\bar{W}]\boldsymbol{\omega}(t)+ \\
&\quad \tilde{x}^{\mathrm{T}}(t-d_1(t))[-(1-d^*)Q_1+\bar{N}_2^{\mathrm{T}}\bar{E}+\bar{E}^{\mathrm{T}}\bar{N}_2+ \\
&\quad d\bar{A}_1^{\mathrm{T}}\bar{A}_1+d\bar{N}_2^{\mathrm{T}}\bar{N}_2]\tilde{x}(t-d_1(t))+ \\
&\quad 2\tilde{x}^{\mathrm{T}}(t-d_1(t))[\bar{E}^{\mathrm{T}}\bar{N}_3-\bar{N}_2^{\mathrm{T}}\bar{E}+d\bar{A}_1^{\mathrm{T}}\bar{A}_2+ \\
&\quad d\bar{N}_2^{\mathrm{T}}\bar{N}_3]\tilde{x}^{\mathrm{T}}(t-d_2(t))+ \\
&\quad 2\tilde{x}^{\mathrm{T}}(t-d_1(t))[\bar{E}^{\mathrm{T}}\bar{W}+d\bar{A}_1^{\mathrm{T}}\bar{B}+d\bar{N}_2^{\mathrm{T}}\bar{W}]\boldsymbol{\omega}(t)+ \\
&\quad \tilde{x}^{\mathrm{T}}(t-d_2(t))[-(1-d^*)Q_2-\bar{E}^{\mathrm{T}}\bar{N}_3-\bar{N}_3^{\mathrm{T}}\bar{E}+ \\
&\quad d\bar{A}_2^{\mathrm{T}}\bar{A}_2+d\bar{N}_3^{\mathrm{T}}\bar{N}_3]\tilde{x}(t-d_2(t))+ \\
&\quad 2\tilde{x}^{\mathrm{T}}(t-d_2(t))[-\bar{E}^{\mathrm{T}}\bar{W}+d\bar{A}_2^{\mathrm{T}}\bar{B}+ \\
&\quad d\bar{N}_3^{\mathrm{T}}\bar{W}]\boldsymbol{\omega}(t)+\boldsymbol{\omega}^{\mathrm{T}}(t)[d\bar{B}^{\mathrm{T}}\bar{B}+ \\
&\quad d\bar{W}^{\mathrm{T}}\bar{W}]\boldsymbol{\omega}(t).
\end{aligned} \tag{16}$$

当 $\boldsymbol{\omega}(t)=0$ 时,可得

$$\dot{V}(\boldsymbol{\xi}(t)) \leqslant \boldsymbol{v}^{\mathrm{T}}(t)\boldsymbol{\Sigma}\boldsymbol{v}(t). \tag{17}$$

其中:

$$\boldsymbol{\Sigma} = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} \\ * & \Sigma_{22} & \Sigma_{23} \\ * & * & \Sigma_{33} \end{bmatrix};$$

$$\boldsymbol{v}(t) = [\tilde{x}^{\mathrm{T}}(t), \tilde{x}^{\mathrm{T}}(t-d_1(t)), \tilde{x}^{\mathrm{T}}(t-d_2(t))]^{\mathrm{T}};$$

$$\Sigma_{11} = \bar{A}^{\mathrm{T}}P\bar{E}+\bar{E}^{\mathrm{T}}P\bar{A}+\bar{A}^{\mathrm{T}}\bar{R}\bar{S}^{\mathrm{T}}+\bar{S}\bar{R}^{\mathrm{T}}\bar{A}+Q_1+Q_2+d\bar{N}_1^{\mathrm{T}}\bar{N}_1+d\bar{A}^{\mathrm{T}}\bar{A};$$

$$\Sigma_{12} = \bar{E}^{\mathrm{T}}P\bar{A}_1+\bar{S}\bar{R}^{\mathrm{T}}\bar{A}_1+\bar{N}_1^{\mathrm{T}}\bar{E}+d\bar{A}^{\mathrm{T}}\bar{A}_1+d\bar{N}_1^{\mathrm{T}}\bar{N}_2;$$

$$\Sigma_{13} = \bar{E}^{\mathrm{T}}P\bar{A}_2+\bar{S}\bar{R}^{\mathrm{T}}\bar{A}_2-\bar{N}_1^{\mathrm{T}}\bar{E}+d\bar{A}^{\mathrm{T}}\bar{A}_2+d\bar{N}_1^{\mathrm{T}}\bar{N}_3;$$

$$\Sigma_{22} = -(1-d^*)Q_1+\bar{N}_2^{\mathrm{T}}\bar{E}+\bar{E}^{\mathrm{T}}\bar{N}_2+d\bar{A}_1^{\mathrm{T}}\bar{A}_1+d\bar{N}_2^{\mathrm{T}}\bar{N}_2;$$

$$\Sigma_{23} = \bar{E}^{\mathrm{T}}\bar{N}_3-\bar{N}_2^{\mathrm{T}}\bar{E}+d\bar{A}_1^{\mathrm{T}}\bar{A}_2+d\bar{N}_2^{\mathrm{T}}\bar{N}_3;$$

$$\Sigma_{33} = -(1-d^*)Q_2-\bar{E}^{\mathrm{T}}\bar{N}_3-\bar{N}_3^{\mathrm{T}}\bar{E}+d\bar{A}_2^{\mathrm{T}}\bar{A}_2+d\bar{N}_3^{\mathrm{T}}\bar{N}_3.$$

且 $\bar{R}\in\bar{\mathbf{R}}^{2n\times(n-r)}$ 为任意满足 $\bar{E}^{\mathrm{T}}\bar{R}=0$ 的列满秩矩阵.

如果 $\boldsymbol{\Sigma}<0$, 则由 Schur 补引理得到等价的不等式:

$$\boldsymbol{\Gamma} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \bar{N}_1^{\mathrm{T}} & \bar{A}^{\mathrm{T}} \\ * & \Gamma_{22} & \Gamma_{23} & \bar{N}_2^{\mathrm{T}} & \bar{A}_1^{\mathrm{T}} \\ * & * & \Gamma_{33} & \bar{N}_2^{\mathrm{T}} & \bar{A}_2^{\mathrm{T}} \\ * & * & * & -\bar{d}I & \mathbf{0} \\ * & * & * & * & -\bar{d}I \end{bmatrix} < 0. \tag{18}$$

其中:

$$\Gamma_{11} = \bar{A}^{\mathrm{T}}P\bar{E}+\bar{E}^{\mathrm{T}}P\bar{A}+\bar{A}^{\mathrm{T}}\bar{R}\bar{S}^{\mathrm{T}}+\bar{S}\bar{R}^{\mathrm{T}}\bar{A}+Q_1+Q_2;$$

$$\Gamma_{12} = \bar{E}^{\mathrm{T}}P\bar{A}_1+\bar{S}\bar{R}^{\mathrm{T}}\bar{A}_1+\bar{N}_1^{\mathrm{T}}\bar{E};$$

$$\Gamma_{13} = \bar{E}^{\mathrm{T}}P\bar{A}_2+\bar{S}\bar{R}^{\mathrm{T}}\bar{A}_2-\bar{N}_1^{\mathrm{T}}\bar{E};$$

$$\Gamma_{22} = -(1-d^*)Q_1+\bar{N}_2^{\mathrm{T}}\bar{E}+\bar{E}^{\mathrm{T}}\bar{N}_2;$$

$$\Gamma_{23} = \bar{E}^{\mathrm{T}}\bar{N}_3-\bar{N}_2^{\mathrm{T}}\bar{E};$$

$$\Gamma_{33} = -(1-d^*)Q_2-\bar{E}^{\mathrm{T}}\bar{N}_3-\bar{N}_3^{\mathrm{T}}\bar{E}; \quad \bar{d}=d^{-1}.$$

且 $\bar{R}\in\bar{\mathbf{R}}^{2n\times(n-r)}$ 为任意满足 $\bar{E}^{\mathrm{T}}\bar{R}=0$ 的列满秩矩阵. 由高等代数的知识可知,如果 $\boldsymbol{\Omega}<0$,则 $\boldsymbol{\Gamma}<0$. 即式(14)是滤波误差动态系统(9)渐近稳定的充分条件.

下面证明滤波误差动态系统(9)在零初始条件 $\left(V(\tilde{x}(t))\Big|_{t=0}=0\right)$ 满足 $J<0$.

考虑到滤波误差动态系统(9)是鲁棒稳定的 $\left(V(\tilde{x}(t))\Big|_{t\to\infty}\geqslant 0\right)$,那么对于任意非零的 $\boldsymbol{\omega}(t)\in L_2[0,\infty)$ 有

$$\begin{aligned}
J &= \int_0^{\infty}[\tilde{z}^{\mathrm{T}}(t)\tilde{z}(t)-\gamma^2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{\omega}(t)]\mathrm{d}t = \\
&\int_0^{\infty}[\tilde{z}^{\mathrm{T}}(t)\tilde{z}(t)-\gamma^2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{\omega}(t)+\dot{V}(\tilde{x}(t))]\mathrm{d}t + \\
&\quad V(\tilde{x}(t))\Big|_{t=0}-V(\tilde{x}(t))\Big|_{t=\infty} \leqslant \\
&\int_0^{\infty}[\tilde{z}^{\mathrm{T}}(t)\tilde{z}(t)-\gamma^2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{\omega}(t)+\dot{V}(\tilde{x}(t))]\mathrm{d}t.
\end{aligned} \tag{19}$$

其中:

$$\begin{aligned}
&\tilde{z}^{\mathrm{T}}(t)\tilde{z}(t) = \\
&\tilde{x}^{\mathrm{T}}(t)\bar{C}^{\mathrm{T}}\bar{C}\tilde{x}(t)+2\tilde{x}^{\mathrm{T}}(t)\bar{C}^{\mathrm{T}}\bar{C}_1\tilde{x}(t-d_1(t))+ \\
&\quad 2\tilde{x}^{\mathrm{T}}(t)\bar{C}^{\mathrm{T}}\bar{C}_2\tilde{x}(t-d_2(t))+2\tilde{x}^{\mathrm{T}}(t)\bar{C}^{\mathrm{T}}\bar{D}\boldsymbol{\omega}(t)+ \\
&\quad \tilde{x}^{\mathrm{T}}(t-d_1(t))\bar{C}_1^{\mathrm{T}}\bar{C}_1\tilde{x}(t-d_1(t))+ \\
&\quad 2\tilde{x}^{\mathrm{T}}(t-d_1(t))\bar{C}_1^{\mathrm{T}}\bar{C}_2\tilde{x}(t-d_2(t))+ \\
&\quad 2\tilde{x}^{\mathrm{T}}(t-d_1(t))\bar{C}_1^{\mathrm{T}}\bar{D}\boldsymbol{\omega}(t)+\tilde{x}^{\mathrm{T}}(t- \\
&\quad d_2(t))\bar{C}_2^{\mathrm{T}}\tilde{C}_2\tilde{x}(t-d_2(t))+2\tilde{x}^{\mathrm{T}}(t- \\
&\quad d_2(t))\bar{C}_2^{\mathrm{T}}\bar{D}\boldsymbol{\omega}(t)+\boldsymbol{\omega}^{\mathrm{T}}(t)\bar{D}^{\mathrm{T}}\bar{D}\boldsymbol{\omega}(t).
\end{aligned} \tag{20}$$

由此得

$$\tilde{z}^{\mathrm{T}}(t)\tilde{z}(t)-\gamma^2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{\omega}(t)+\dot{V}(\boldsymbol{\xi}(t)) = \boldsymbol{\eta}^{\mathrm{T}}(t)\boldsymbol{\Theta}\boldsymbol{\eta}(t).$$

其中:

$$\boldsymbol{\Theta}=\begin{bmatrix}\boldsymbol{\Theta}_{11} & \boldsymbol{\Theta}_{12} & \boldsymbol{\Theta}_{13} & \boldsymbol{\Theta}_{14}\\ * & \boldsymbol{\Theta}_{22} & \boldsymbol{\Theta}_{23} & \boldsymbol{\Theta}_{24}\\ * & * & \boldsymbol{\Theta}_{33} & \boldsymbol{\Theta}_{34}\\ * & * & * & \boldsymbol{\Theta}_{44}\end{bmatrix}; \qquad (21)$$

$$\boldsymbol{\Theta}_{11}=\bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{E}}+\bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{A}}+\bar{\boldsymbol{A}}^{\mathrm{T}}\bar{\boldsymbol{R}}\bar{\boldsymbol{S}}^{\mathrm{T}}+\bar{\boldsymbol{S}}\bar{\boldsymbol{R}}^{\mathrm{T}}\bar{\boldsymbol{A}}+\boldsymbol{Q}_1+\boldsymbol{Q}_2+d\bar{\boldsymbol{N}}_1^{\mathrm{T}}\bar{\boldsymbol{N}}_1+d\bar{\boldsymbol{A}}^{\mathrm{T}}\bar{\boldsymbol{A}}+\bar{\boldsymbol{C}}^{\mathrm{T}}\bar{\boldsymbol{C}};$$

$$\boldsymbol{\Theta}_{12}=\bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{A}}_1+\bar{\boldsymbol{S}}\bar{\boldsymbol{R}}^{\mathrm{T}}\bar{\boldsymbol{A}}_1+\bar{\boldsymbol{N}}_1^{\mathrm{T}}\bar{\boldsymbol{E}}+d\bar{\boldsymbol{A}}^{\mathrm{T}}\bar{\boldsymbol{A}}_1+d\bar{\boldsymbol{N}}_1^{\mathrm{T}}\bar{\boldsymbol{N}}_2+\bar{\boldsymbol{C}}^{\mathrm{T}}\bar{\boldsymbol{C}}_1;$$

$$\boldsymbol{\Theta}_{13}=\bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{A}}_2+\bar{\boldsymbol{S}}\bar{\boldsymbol{R}}^{\mathrm{T}}\bar{\boldsymbol{A}}_2-\bar{\boldsymbol{N}}_1^{\mathrm{T}}\bar{\boldsymbol{E}}+d\bar{\boldsymbol{A}}^{\mathrm{T}}\bar{\boldsymbol{A}}_2+d\bar{\boldsymbol{N}}_1^{\mathrm{T}}\bar{\boldsymbol{N}}_3+\bar{\boldsymbol{C}}^{\mathrm{T}}\bar{\boldsymbol{C}}_2;$$

$$\boldsymbol{\Theta}_{14}=\bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\bar{B}+\bar{\boldsymbol{S}}\bar{\boldsymbol{R}}^{\mathrm{T}}\bar{\boldsymbol{B}}+d\bar{\boldsymbol{A}}^{\mathrm{T}}\bar{\boldsymbol{A}}_2+d\bar{\boldsymbol{N}}_1^{\mathrm{T}}\bar{\boldsymbol{N}}_3+\bar{\boldsymbol{C}}^{\mathrm{T}}\bar{\boldsymbol{D}};$$

$$\boldsymbol{\Theta}_{22}=-(1-d^*)\boldsymbol{Q}_1+\bar{\boldsymbol{N}}_2^{\mathrm{T}}\bar{\boldsymbol{E}}+\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{N}}_2+d\bar{\boldsymbol{A}}_1^{\mathrm{T}}\bar{\boldsymbol{A}}_1+d\bar{\boldsymbol{N}}_2^{\mathrm{T}}\bar{\boldsymbol{N}}_2+\bar{\boldsymbol{C}}_1^{\mathrm{T}}\bar{\boldsymbol{C}}_1;$$

$$\boldsymbol{\Theta}_{23}=\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{N}}_3-\bar{\boldsymbol{N}}_2^{\mathrm{T}}\bar{\boldsymbol{E}}+d\bar{\boldsymbol{A}}_1^{\mathrm{T}}\bar{\boldsymbol{A}}_2+d\bar{\boldsymbol{N}}_2^{\mathrm{T}}\bar{\boldsymbol{N}}_3+\bar{\boldsymbol{C}}_1^{\mathrm{T}}\bar{\boldsymbol{C}}_2;$$

$$\boldsymbol{\Theta}_{24}=\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{W}+d\bar{\boldsymbol{A}}_1^{\mathrm{T}}\bar{B}+d\bar{\boldsymbol{N}}_2^{\mathrm{T}}\bar{W}+\bar{\boldsymbol{C}}_1^{\mathrm{T}}\bar{\boldsymbol{D}};$$

$$\boldsymbol{\Theta}_{33}=-(1-d^*)\boldsymbol{Q}_2-\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{N}}_3-\bar{\boldsymbol{N}}_3^{\mathrm{T}}\bar{\boldsymbol{E}}+d\bar{\boldsymbol{A}}_2^{\mathrm{T}}\bar{\boldsymbol{A}}_2+d\bar{\boldsymbol{N}}_3^{\mathrm{T}}\bar{\boldsymbol{N}}_3+\bar{\boldsymbol{C}}_2^{\mathrm{T}}\bar{\boldsymbol{C}}_2;$$

$$\boldsymbol{\Theta}_{34}=-\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{W}}+d\bar{\boldsymbol{A}}_2^{\mathrm{T}}\bar{B}+d\bar{\boldsymbol{N}}_3^{\mathrm{T}}\bar{\boldsymbol{W}}+\bar{\boldsymbol{C}}_2^{\mathrm{T}}\bar{\boldsymbol{D}};$$

$$\boldsymbol{\Theta}_{44}=-\gamma^2\boldsymbol{I}+d\bar{\boldsymbol{B}}^{\mathrm{T}}\bar{\boldsymbol{B}}+d\bar{\boldsymbol{W}}^{\mathrm{T}}\bar{\boldsymbol{W}}+\bar{\boldsymbol{D}}^{\mathrm{T}}\bar{\boldsymbol{D}}.$$

且 $\bar{\boldsymbol{R}}\in\bar{\mathbf{R}}^{2n\times(n-r)}$ 为任意满足 $\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{R}}=0$ 的列满秩矩阵.

如果 $\boldsymbol{\Theta}<0$, 则由 Schur 补引理得到式(12). 因此, 滤波误差动态系统(9)在零初始条件 $\left(V(\tilde{\boldsymbol{x}}(t))\Big|_{t=0}=0\right)$ 满足 $\boldsymbol{J}<0$. 证毕.

3 鲁棒 H_∞ 滤波器设计

Design of robust H_∞ filter

上述定理 1 只能从理论上说明鲁棒 H_∞ 滤波器存在的充分条件,具体的滤波器无法求出. 下面的定理给出了求具体滤波器的方法.

定理 2 对任意满足(4)的时变时滞 $d_i(t)(i=1,2)$,不确定时滞奇异系统(1)和性能指标式(11). 给定标量 $\gamma>0$,如果存在正定矩阵

$$\boldsymbol{P}=\begin{bmatrix}\boldsymbol{P}_1 & -\boldsymbol{P}_2\\ -\boldsymbol{P}_2 & \boldsymbol{P}_2\end{bmatrix}>0,$$

$$\boldsymbol{Q}_1=\begin{bmatrix}\boldsymbol{Q}_{1,11} & \boldsymbol{Q}_{1,12}\\ * & \boldsymbol{Q}_{1,22}\end{bmatrix}>0,$$

$$\boldsymbol{Q}_2=\begin{bmatrix}\boldsymbol{Q}_{2,11} & \boldsymbol{Q}_{2,12}\\ * & \boldsymbol{Q}_{2,22}\end{bmatrix}>0$$

和矩阵 $\boldsymbol{S}$、$\boldsymbol{N}_1$、$\boldsymbol{N}_2$、$\boldsymbol{N}_3$、$\boldsymbol{W}$、$\boldsymbol{X}$、$\boldsymbol{Z}$ 以及标量 $\varepsilon_1>0$、$\varepsilon_2>0$ 使得

$$\boldsymbol{\Xi}=\begin{vmatrix}(\boldsymbol{\Xi}_1)_{6\times6} & (\boldsymbol{\Xi}_2)_{6\times7}\\ (\boldsymbol{\Xi}_2)_{6\times7}^{\mathrm{T}} & (\boldsymbol{\Xi}_3)_{7\times7}\end{vmatrix}<0. \qquad (22)$$

其中:

$$(\boldsymbol{\Xi}_1)_{6\times6}=\begin{vmatrix}\boldsymbol{\Xi}_{11} & \boldsymbol{\Xi}_{12} & \boldsymbol{\Xi}_{13} & \mathbf{0} & \boldsymbol{\Xi}_{15} & \mathbf{0}\\ * & \boldsymbol{\Xi}_{22} & \boldsymbol{\Xi}_{23} & \mathbf{0} & \boldsymbol{\Xi}_{25} & \mathbf{0}\\ * & * & \boldsymbol{\Xi}_{33} & \boldsymbol{\Xi}_{34} & \boldsymbol{\Xi}_{35} & \mathbf{0}\\ * & * & * & \boldsymbol{\Xi}_{44} & \mathbf{0} & \mathbf{0}\\ * & * & * & * & \boldsymbol{\Xi}_{55} & \boldsymbol{\Xi}_{56}\\ * & * & * & * & * & \boldsymbol{\Xi}_{66}\end{vmatrix};$$

$$(\boldsymbol{\Xi}_2)_{6\times7}=\begin{vmatrix}\boldsymbol{\Xi}_{17} & \boldsymbol{N}_1^{\mathrm{T}} & \boldsymbol{A}^{\mathrm{T}} & \boldsymbol{C}^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}} & (\boldsymbol{H}-\hat{\boldsymbol{D}}\boldsymbol{C})^{\mathrm{T}} & \boldsymbol{\Xi}_{112} & \mathbf{0}\\ \boldsymbol{\Xi}_{27} & \mathbf{0} & \mathbf{0} & \boldsymbol{G}^{\mathrm{T}} & -\hat{\boldsymbol{H}}^{\mathrm{T}} & \mathbf{0} & \boldsymbol{\Xi}_{213}\\ \boldsymbol{E}^{\mathrm{T}}\boldsymbol{W} & \boldsymbol{N}_2^{\mathrm{T}} & \boldsymbol{A}_1^{\mathrm{T}} & \boldsymbol{C}_1^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}} & -(\hat{\boldsymbol{D}}\boldsymbol{C}_1)^{\mathrm{T}} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ -\boldsymbol{E}^{\mathrm{T}}\boldsymbol{W} & \boldsymbol{N}_3^{\mathrm{T}} & \boldsymbol{A}_2^{\mathrm{T}} & \boldsymbol{C}_2^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}} & -(\hat{\boldsymbol{D}}\boldsymbol{C}_2)^{\mathrm{T}} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\end{vmatrix};$$

$$(\boldsymbol{\Xi}_3)_{7\times7}=\begin{vmatrix}-\gamma^2\boldsymbol{I} & \boldsymbol{W}^{\mathrm{T}} & \boldsymbol{B}^{\mathrm{T}} & \boldsymbol{D}^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}} & -(\hat{\boldsymbol{D}}\boldsymbol{D})^{\mathrm{T}} & \mathbf{0} & \mathbf{0}\\ * & -\bar{d}\boldsymbol{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ * & * & -\bar{d}\boldsymbol{I} & \mathbf{0} & \mathbf{0} & \boldsymbol{L}_1 & \mathbf{0}\\ * & * & * & -\bar{d}\boldsymbol{I} & \mathbf{0} & \boldsymbol{K}\boldsymbol{L}_2 & \mathbf{0}\\ * & * & * & * & -\boldsymbol{I} & -\hat{\boldsymbol{D}}\boldsymbol{L}_2 & \mathbf{0}\\ * & * & * & * & * & -\varepsilon_1\boldsymbol{I} & \mathbf{0}\\ * & * & * & * & * & * & -\varepsilon_2\boldsymbol{I}\end{vmatrix}.$$

其中:

$$\boldsymbol{\Xi}_{11}=\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{E}-\boldsymbol{C}^{\mathrm{T}}\boldsymbol{Z}^{\mathrm{T}}\boldsymbol{E}+\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{A}-\boldsymbol{E}^{\mathrm{T}}\boldsymbol{Z}\boldsymbol{C}+\boldsymbol{A}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{S}^{\mathrm{T}}+\boldsymbol{S}\boldsymbol{R}^{\mathrm{T}}\boldsymbol{A}+\boldsymbol{Q}_{1,11}+\boldsymbol{Q}_{2,11}+\varepsilon_1\boldsymbol{M}_1^{\mathrm{T}}\boldsymbol{M}_1+\varepsilon_2\boldsymbol{M}_1^{\mathrm{T}}\boldsymbol{M}_1;$$

$$\boldsymbol{\Xi}_{12}=\boldsymbol{C}^{\mathrm{T}}\boldsymbol{Z}^{\mathrm{T}}-\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}_2-\boldsymbol{E}^{\mathrm{T}}\boldsymbol{X}+\boldsymbol{Q}_{1,12}+\boldsymbol{Q}_{2,12};$$

$$\boldsymbol{\Xi}_{13}=\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{A}_1+\boldsymbol{S}\boldsymbol{R}^{\mathrm{T}}\boldsymbol{A}_1+\boldsymbol{N}_1^{\mathrm{T}}\boldsymbol{E}-\boldsymbol{E}^{\mathrm{T}}\boldsymbol{Z}\boldsymbol{C}_1+\varepsilon_1\boldsymbol{M}_1^{\mathrm{T}}\boldsymbol{M}_2+\varepsilon_2\boldsymbol{M}_1^{\mathrm{T}}\boldsymbol{M}_2;$$

$$\boldsymbol{\Xi}_{15}=\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{A}_2+\boldsymbol{S}\boldsymbol{R}^{\mathrm{T}}\boldsymbol{A}_2-\boldsymbol{E}^{\mathrm{T}}\boldsymbol{Z}\boldsymbol{C}_2-\boldsymbol{N}_1^{\mathrm{T}}\boldsymbol{E}+\varepsilon_1\boldsymbol{M}_1^{\mathrm{T}}\boldsymbol{M}_3+\varepsilon_2\boldsymbol{M}_1^{\mathrm{T}}\boldsymbol{M}_3;$$

$$\boldsymbol{\Xi}_{17}=\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{B}+\boldsymbol{S}\boldsymbol{R}^{\mathrm{T}}\boldsymbol{B}-\boldsymbol{E}^{\mathrm{T}}\boldsymbol{Z}\boldsymbol{D};$$

$$\boldsymbol{\Xi}_{112}=(\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}_1+\boldsymbol{S}\boldsymbol{R}^{\mathrm{T}})\boldsymbol{L}_1-\boldsymbol{E}^{\mathrm{T}}\boldsymbol{Z}\boldsymbol{L}_2;$$

$$\boldsymbol{\Xi}_{22}=\boldsymbol{X}^{\mathrm{T}}+\boldsymbol{X}+\boldsymbol{Q}_{1,22}+\boldsymbol{Q}_{2,22};$$

$$\boldsymbol{\Xi}_{23}=\boldsymbol{Z}\boldsymbol{C}_1-\boldsymbol{P}_2\boldsymbol{A}_1;\boldsymbol{\Xi}_{25}=\boldsymbol{Z}\boldsymbol{C}_2-\boldsymbol{P}_2\boldsymbol{A}_2;$$

$$\boldsymbol{\Xi}_{27}=\boldsymbol{Z}\boldsymbol{D}-\boldsymbol{P}_2\boldsymbol{B};\boldsymbol{\Xi}_{213}=\boldsymbol{Z}\boldsymbol{L}_2-\boldsymbol{P}_2\boldsymbol{L}_1;$$

$$\boldsymbol{\Xi}_{33}=-(1-d^*)\boldsymbol{Q}_{1,11}+\boldsymbol{N}_2^{\mathrm{T}}\boldsymbol{E}+\boldsymbol{E}^{\mathrm{T}}\boldsymbol{N}_2+\varepsilon_1\boldsymbol{M}_2^{\mathrm{T}}\boldsymbol{M}_2+\varepsilon_2\boldsymbol{M}_2^{\mathrm{T}}\boldsymbol{M}_2;$$

$$\boldsymbol{\Xi}_{34}=-(1-d^*)\boldsymbol{Q}_{1,12};$$

$$\boldsymbol{\Xi}_{35}=\boldsymbol{E}^{\mathrm{T}}\boldsymbol{N}_3-\boldsymbol{N}_2^{\mathrm{T}}\boldsymbol{E}+\varepsilon_1\boldsymbol{M}_2^{\mathrm{T}}\boldsymbol{M}_3+\varepsilon_2\boldsymbol{M}_2^{\mathrm{T}}\boldsymbol{M}_3;$$

$$\boldsymbol{\Xi}_{44}=-(1-d^*)\boldsymbol{Q}_{1,22};$$

$$\boldsymbol{\Xi}_{55}=-(1-d^*)\boldsymbol{Q}_{2,11}-\boldsymbol{N}_3^{\mathrm{T}}\boldsymbol{E}-\boldsymbol{E}^{\mathrm{T}}\boldsymbol{N}_3+$$

$$\varepsilon_1 M_3^{\mathrm T} M_3 + \varepsilon_2 M_3^{\mathrm T} M_3.$$

$$\Xi_{56} = -(1-d^*)Q_{2,12};\quad \Xi_{66} = -(1-d^*)Q_{2,22}.$$

且 $R \in \mathbf{R}^{n\times(n-r)}$ 为任意满足 $E^{\mathrm T}R = 0$ 的列满秩矩阵,则式(6)为系统(1)的鲁棒 H_∞ 滤波器,既有 $J<0$ 且滤波器的参数矩阵为 $G = P_2^{-1}X, K = P_2^{-1}Z$,其他的两个参数矩阵 $\hat H$ 和 $\hat D$ 由式(22)可以解出.

证明 在定理1的基础上,式(21)中取

$$P = \begin{bmatrix} P_1 & -P_2 \\ -P_2 & P_2 \end{bmatrix} > 0,\quad Q_1 = \begin{bmatrix} Q_{1,11} & Q_{1,12} \\ * & Q_{1,22} \end{bmatrix} > 0,$$

$$Q_2 = \begin{bmatrix} Q_{2,12} & Q_{2,12} \\ * & Q_{2,22} \end{bmatrix} > 0,\quad \bar R = \begin{bmatrix} R \\ 0 \end{bmatrix},\quad \bar S = \begin{bmatrix} S \\ 0 \end{bmatrix}. \tag{23}$$

并考虑定义式(10),如果令

$$P_2 G = X,\quad P_2 K = Z. \tag{24}$$

则有

$$\Delta = \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & 0 & \Delta_{15} & 0 & \Delta_{17} \\ * & \Delta_{22} & \Delta_{23} & 0 & \Delta_{25} & 0 & \Delta_{27} \\ * & * & \Delta_{33} & \Xi_{34} & \Delta_{35} & 0 & \Delta_{37} \\ * & * & * & \Xi_{44} & 0 & 0 & 0 \\ * & * & * & * & \Delta_{55} & \Xi_{56} & \Delta_{47} \\ * & * & * & * & * & \Xi_{66} & 0 \\ * & * & * & * & * & * & \Delta_{77} \end{bmatrix}. \tag{25}$$

式中:

$$\Delta_{11} = \tilde A^{\mathrm T} P_1 E - \tilde C^{\mathrm T} Z^{\mathrm T} E + E^{\mathrm T} P_1 \tilde A - E^{\mathrm T} Z \tilde C + S R^{\mathrm T} \tilde A + \tilde A^{\mathrm T} R S^{\mathrm T} + Q_{1,11} + Q_{2,11} + d N_1^{\mathrm T} N_1 + d(\tilde A^{\mathrm T} \tilde A + \tilde C^{\mathrm T} K^{\mathrm T} K \tilde C) + (H - \hat C \tilde C)^{\mathrm T} (H - \hat C \tilde C);$$

$$\Delta_{12} = \tilde C^{\mathrm T} Z^{\mathrm T} - \tilde A^{\mathrm T} P_2 - E^{\mathrm T} X + Q_{1,12} + Q_{2,12} + d\tilde C^{\mathrm T} K^{\mathrm T} G - (H - \hat C \tilde C)^{\mathrm T} \hat H;$$

$$\Delta_{13} = E^{\mathrm T} P_1 \tilde A_1 + S R^{\mathrm T} \tilde A_1 - E^{\mathrm T} Z \tilde C_1 + N_1^{\mathrm T} E + d(\tilde A^{\mathrm T} \tilde A_1 + \tilde C^{\mathrm T} K^{\mathrm T} K \tilde C_1) + d N_1^{\mathrm T} N_2 - (H - \hat C \tilde C)^{\mathrm T} \hat C \tilde C_1;$$

$$\Delta_{15} = E^{\mathrm T} P_1 \tilde A_2 + S R^{\mathrm T} \tilde A_2 - E^{\mathrm T} Z \tilde C_2 - N_1^{\mathrm T} E + d(\tilde A^{\mathrm T} \tilde A_2 + \tilde C^{\mathrm T} K^{\mathrm T} K \tilde C_2) + d N_1^{\mathrm T} N_3 - (H - \hat C \tilde C)^{\mathrm T} \hat C \tilde C_2;$$

$$\Delta_{17} = E^{\mathrm T} P_1 B + S R^{\mathrm T} B - E^{\mathrm T} P_2 K D - (H - \hat C \tilde C)^{\mathrm T} \hat C D + d N_1^{\mathrm T} + d(\tilde A^{\mathrm T} B + \tilde C^{\mathrm T} K^{\mathrm T} K D);$$

$$\Delta_{22} = X^{\mathrm T} + X + Q_{1,22} + Q_{2,22} + d G^{\mathrm T} G + \hat H^{\mathrm T} \hat H;$$

$$\Delta_{23} = Z \tilde C_1 - P_2 \tilde A_1 + d G^{\mathrm T} K \tilde C_1 + \hat H^{\mathrm T} \hat C \tilde C_1;$$

$$\Delta_{25} = Z \tilde C_2 - P_2 \tilde A_2 + d G^{\mathrm T} K \tilde C_2 + \hat H^{\mathrm T} \hat C \tilde C_2;$$

$$\Delta_{27} = P_2 K D - P_2 B + d G^{\mathrm T} K D + \hat H^{\mathrm T} \hat C D;$$

$$\Delta_{33} = -(1-d^{\cdot})Q_{1,11} + E^{\mathrm T} N_2 + N_2^{\mathrm T} E + d(\tilde A_1^{\mathrm T} \tilde A_1 + \tilde C_1^{\mathrm T} K^{\mathrm T} K \tilde C_1) + d N_2^{\mathrm T} N_2 + (\hat C \tilde C_1)^{\mathrm T} (\hat C \tilde C_1);$$

$$\Delta_{35} = E^{\mathrm T} N_3 - N_2^{\mathrm T} E + d(\tilde A_1^{\mathrm T} \tilde A_2 + \tilde C_1^{\mathrm T} K^{\mathrm T} K \tilde C_2) + d N_2^{\mathrm T} N_3 + (\hat C \tilde C_1)^{\mathrm T} (\hat C \tilde C_2);$$

$$\Delta_{37} = E^{\mathrm T} W + (\hat C \tilde C_1)^{\mathrm T} (\hat C D) + d N_2^{\mathrm T} W + d(\tilde A_1^{\mathrm T} B + \tilde C_1^{\mathrm T} K^{\mathrm T} K D);$$

$$\Delta_{55} = -(1-d^{\cdot})Q_{2,11} - E^{\mathrm T} N_3 - N_3^{\mathrm T} E + d(\tilde A_2^{\mathrm T} \tilde A_2 + \tilde C_2^{\mathrm T} K^{\mathrm T} K \tilde C_2) + d N_3^{\mathrm T} N_3 + (\hat C \tilde C_2)^{\mathrm T} (\hat C \tilde C_2);$$

$$\Delta_{57} = -E^{\mathrm T} W + (\hat C \tilde C_2)^{\mathrm T} (\hat C D) + d N_3^{\mathrm T} W + d(\tilde A_2^{\mathrm T} B + \tilde C_2^{\mathrm T} K^{\mathrm T} K D);$$

$$\Delta_{77} = -\gamma^2 I + (\hat C D)^{\mathrm T} (\hat C D) + d W^{\mathrm T} W + d(B^{\mathrm T} B + D^{\mathrm T} K^{\mathrm T} K D).$$

且 $R \in \mathbf{R}^{n\times(n-r)}$ 为任意满足 $E^{\mathrm T}R = 0$ 的列满秩矩阵.

如果 $\Delta < 0$,则由 Schur 补引理得到等价的不等式:

$$\Lambda = \begin{vmatrix} (\Lambda_1)_{6\times 6} & (\Lambda_2)_{6\times 5} \\ (\Lambda_2)_{6\times 5}^{\mathrm T} & (\Lambda_3)_{5\times 5} \end{vmatrix} < 0. \tag{26}$$

其中

$$(\Lambda_1)_{6\times 6} = \begin{vmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & 0 & \Lambda_{15} & 0 \\ * & \Lambda_{22} & \Lambda_{23} & 0 & \Lambda_{25} & 0 \\ * & * & \Lambda_{33} & \Xi_{34} & \Lambda_{35} & 0 \\ * & * & * & \Xi_{44} & 0 & 0 \\ * & * & * & * & \Lambda_{55} & \Xi_{56} \\ * & * & * & * & * & \Xi_{66} \end{vmatrix};$$

$$(\Lambda_2)_{6\times 5} = \begin{vmatrix} \Lambda_{17} & N_1^{\mathrm T} & \tilde A^{\mathrm T} & \tilde C^{\mathrm T} K^{\mathrm T} & (H - \hat C \tilde C)^{\mathrm T} \\ \Lambda_{27} & 0 & 0 & G^{\mathrm T} & -\hat H^{\mathrm T} \\ E^{\mathrm T} W & N_2^{\mathrm T} & \tilde A_1^{\mathrm T} & \tilde C_1^{\mathrm T} K^{\mathrm T} & -(\hat C \tilde C_1)^{\mathrm T} \\ 0 & 0 & 0 & 0 & 0 \\ -E^{\mathrm T} W & N_3^{\mathrm T} & \tilde A_2^{\mathrm T} & \tilde C_2^{\mathrm T} K^{\mathrm T} & -(\hat C \tilde C_2)^{\mathrm T} \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix};$$

$$(\Lambda_3)_{5\times 5} = \begin{vmatrix} -\gamma^2 I & W^{\mathrm T} & B^{\mathrm T} & D^{\mathrm T} K^{\mathrm T} & -(\hat C D)^{\mathrm T} \\ * & -\bar d I & 0 & 0 & 0 \\ * & * & -\bar d I & 0 & 0 \\ * & * & * & -\bar d I & 0 \\ * & * & * & * & I \end{vmatrix}.$$

其中

$$\Lambda_{11} = \tilde A^{\mathrm T} P_1 E - \tilde C^{\mathrm T} Z^{\mathrm T} E + E^{\mathrm T} P_1 \tilde A - E^{\mathrm T} Z \tilde C + S R^{\mathrm T} \tilde A + \tilde A^{\mathrm T} R S^{\mathrm T} + Q_{1,11} + Q_{2,11};$$

$$\Lambda_{12} = \tilde C^{\mathrm T} Z^{\mathrm T} - \tilde A^{\mathrm T} P_2 - E^{\mathrm T} X + Q_{1,12} + Q_{2,12};$$

$$\Lambda_{13} = E^{\mathrm T} P_1 \tilde A_1 + S R^{\mathrm T} \tilde A_1 - E^{\mathrm T} Z \tilde C_1 + N_1^{\mathrm T} E;$$

$$\Lambda_{15} = E^{\mathrm T} P_1 \tilde A_2 + S R^{\mathrm T} \tilde A_2 - E^{\mathrm T} Z \tilde C_2 - N_1^{\mathrm T} E;$$

$\Lambda_{17} = E^{\mathrm{T}}P_1B + SR^{\mathrm{T}}B - E^{\mathrm{T}}P_2KD$;

$\Lambda_{22} = X^{\mathrm{T}} + X + Q_{1,22} + Q_{2,22}$; $\Lambda_{23} = Z\widetilde{C}_1 - P_2\widetilde{A}_1$;

$\Lambda_{25} = Z\widetilde{C}_2 - P_2\widetilde{A}_2$; $\Lambda_{27} = P_2KD - P_2B$;

$\Lambda_{33} = -(1-d^*)Q_{1,11} + E^{\mathrm{T}}N_2 + N_2^{\mathrm{T}}E$;

$\Lambda_{35} = E^{\mathrm{T}}N_3 - N_2^{\mathrm{T}}E$;

$\Lambda_{55} = -(1-d^*)Q_{2,11} - E^{\mathrm{T}}N_3 - N_3^{\mathrm{T}}E$.

且 $R \in \mathbf{R}^{n\times(n-r)}$ 为任意满足 $E^{\mathrm{T}}R = 0$ 的列满秩矩阵.

考虑到式(2)、(8),应用引理2得到等价的不等式(22).这说明了所设计的滤波器具有给定的 H_∞ 干抑制度 γ.

最后,由式(24)可得滤波器的两个参数矩阵 $G = P_2^{-1}X$ 和 $K = P_2^{-1}Z$,其他的两个参数矩阵 $\hat{H}$ 和 $\hat{C}$ 由矩阵不等式(22)可以解出.

证毕.

4 算例
Calculation example

以下通过一个数值例子来说明本文设计方法的有效性.

例1 考虑系统(1)具有如下参数:

$$E = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix};\quad A = \begin{bmatrix} 4 & -1.7 \\ -0.5 & 2.5 \end{bmatrix};$$

$$A_1 = \begin{bmatrix} -0.3 & 2.5 \\ 0.1 & -0.5 \end{bmatrix};\quad A_2 = \begin{bmatrix} 0.1 & -0.5 \\ -0.3 & -0.8 \end{bmatrix};$$

$$C = \begin{bmatrix} 0.1 & -0.1 \\ -0.1 & -0.2 \end{bmatrix};\quad C_1 = \begin{bmatrix} -0.1 & -0.1 \\ -0.5 & -0.2 \end{bmatrix};$$

$$C_2 = \begin{bmatrix} -1 & -0.3 \\ -0.6 & -0.4 \end{bmatrix};\quad B = \begin{bmatrix} -0.5 \\ -0.1 \end{bmatrix};$$

$$D = \begin{bmatrix} -4 \\ -1 \end{bmatrix};\quad H = [-1 \quad -0.1];$$

$$L_1 = L_2 = 0.02I;\quad M_1 = M_2 = M_3 = I;$$

$$\omega(t) \in L_2[0,\infty).$$

设计鲁棒 H_∞ 滤波器.

在该例题中,令 $R = [0 \quad 1]^{\mathrm{T}}$.根据本文定理2,对不同的时滞变化率 d^*,通过求解线性矩阵不等式(22)的可行性问题,表1给出了系统不同的最大允许的时滞上确界 d 和衰减度 γ 及相应的滤波器矩阵 G、K、$\hat{H}$、$\hat{C}$.

表1 不同的 d^* 值时所求得的最大时滞值和衰减度 γ 及相应的滤波器矩阵 G、K、$\hat{H}$、$\hat{C}$

Table 1 The maximal time-delay, attenuation degree γ and corresponding filter matrix G、K、$\hat{H}$、$\hat{C}$ for different d^*

d^*	d	γ	G	K	$\hat{H}$	$\hat{C}$
0.1	0.177 2	2.317 0	$\begin{pmatrix} -11.332\,0 & -0.181\,4 \\ -6.815\,0 & 15.905\,9 \end{pmatrix}$	$\begin{pmatrix} 0.563\,5 & -3.193\,6 \\ -0.364\,2 & 2.347\,7 \end{pmatrix}$	(0.129 1 0.116 2)	(0.051 1 −0.168 9)
0.3	0.201 6	2.130 9	$\begin{pmatrix} -19.889\,1 & 3.514\,0 \\ -6.651\,0 & 16.281\,2 \end{pmatrix}$	$\begin{pmatrix} 0.551\,3 & -3.145\,2 \\ -0.347\,8 & 2.062\,0 \end{pmatrix}$	(0.105 2 0.096 9)	(0.044 8 −0.141 0)
0.5	0.113 0	3.137 5	$\begin{pmatrix} -70.824\,0 & 40.056\,8 \\ 3.839\,8 & 8.181\,1 \end{pmatrix}$	$\begin{pmatrix} 1.807\,7 & -10.461\,9 \\ -0.663\,3 & 3.743\,7 \end{pmatrix}$	(0.132 4 0.121 7)	(0.047 1 −0.121 1)
1.3	0.031 4	5.693 4	$\begin{pmatrix} -1\,918.2 & 1\,658.8 \\ 971.3 & -852.3 \end{pmatrix}$	$\begin{pmatrix} -40.220\,5 & 128.002\,9 \\ 19.906\,9 & -63.203\,0 \end{pmatrix}$	(0.070 2 −0.086 0)	(0.063 4 −0.069 7)

5 结束语
Concluding remarks

本文研究了不确定奇异系统的时滞相关鲁棒 H_∞ 滤波器设计问题,通过 Lyapuno-Krasovskii 泛函及二次型的积分不等式方法获得了滤波误差动态系统的 H_∞ 性能时滞相关的判据,给出了奇异系统的鲁棒 H_∞ 滤波器存在的时滞依赖的充分条件.所给出的结果都采用严格线性矩阵不等式形式,利用 Matlab 的 LMI 工具箱求解方便简单.最后,通过数值仿真例子验证了本文所提出方法的有效性.

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Design of delay-dependent robust H_∞ filter for uncertain singular system

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Abstract The design problem of full order H_∞ filter for continuous singular system is addressed in this paper, in order to design satisfactory full order H_∞ filter with regular and impulse-free filter error dynamics. Lyapunov-Krasovskii functional and quadratic inequality method was used to get H_∞ time-delay criterion of filter error dynamics. A sufficient condition of robust H_∞ filter existence of singular system is provided. Finally, a numerical example is given to verify the effectiveness and practicability of the design method.

Key words singular system; uncertainly; delay-dependent criterion; robust H_∞ filter