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不确定时变时滞系统的鲁棒最优 H_∞ 控制

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摘要

针对一类参数不确定时变时滞线性系统,研究了鲁棒最优 H_∞ 控制器的设计问题.首先利用积分不等式和引入自由权矩阵的方法,得到了系统稳定及 H_∞ 反馈控制器存在的充分条件;然后将其转化为线性矩阵不等式(LMI)表示,通过线性矩阵不等式的可行解构造控制器,保证了闭环系统渐近稳定且满足一定的 H_∞ 干扰抑制水平,得到的稳定化条件是依赖时滞大小且不要求时滞函数的导数信息,即适用于时滞快速变化的系统.算例表明了该方法的可行性.

关键词

鲁棒 H_∞ 控制;不确定系统;时滞依赖;线性矩阵不等式

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0 引言
Introduction

在实际应用中,往往要求控制系统具备稳定性且满足相应的性能指标,而影响系统稳定的最主要因素包括时滞和不确定性.关于不确定时滞系统的研究,已取得了很多有意义的结论[1-3].文献[4]讨论了一类含有不确定参数的多时滞线性系统的鲁棒 H_∞ 控制问题,分别给出了全维鲁棒 H_∞ 动态输出反馈控制器和静态状态反馈控制器的设计.文献[5]使用 LMI 方法研究了一类不确定时滞系统的鲁棒 H_∞ 反馈控制器的分析与设计.然而上述关于时变时滞系统的鲁棒 H_∞ 控制的研究都是基于时滞函数导数存在,且满足一给定的上界,而对于时滞函数非光滑或导数界不存在的系统却不适用,但在一些实际系统中是很难确定时滞导数存在与否或其上界的.文献[6]研究了一类不确定时变时滞系统的鲁棒非脆弱 H_∞ 控制,给出了具有 H_∞ 干扰抑制的非脆弱控制器的设计方法,此方法不要求时滞函数的导数信息,因此快时滞也适用.

本文主要针对一类状态和控制输入同时存在不确定性且同时具有时变时滞的系统,通过定义新的 Lyapunov 函数,使用文献[6]的 LMI 方法研究其最优 H_∞ 反馈控制器的分析与设计,给出具有 H_∞ 干扰抑制的 H_∞ 控制器存在的一个充分条件,并转化成易于利用 Matlab 求解的线性矩阵不等式.此方法只要时滞函数上方有界,不用考虑时滞函数的导数信息,给设计过程带来方便也可扩大适用范围.

1 问题描述
Description of the problem

考虑如下不确定时变时滞系统

$$\begin{cases}\dot{\boldsymbol{x}}(t) = (\boldsymbol{A}_0 + \Delta\boldsymbol{A}_0)\boldsymbol{x}(t) + (\boldsymbol{A}_1 + \Delta\boldsymbol{A}_1)\boldsymbol{x}(t-d_1(t)) + \\ \qquad (\boldsymbol{B}_0 + \Delta\boldsymbol{B}_0)\boldsymbol{u}(t) + (\boldsymbol{B}_1 + \Delta\boldsymbol{B}_1)\boldsymbol{u}(t-d_2(t)) + \boldsymbol{D}_0\boldsymbol{\omega}(t), \\ \boldsymbol{z}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{D}\boldsymbol{\omega}(t), \\ \boldsymbol{y}(t) = \boldsymbol{C}_1\boldsymbol{x}(t) + \boldsymbol{D}_1\boldsymbol{\omega}(t), \\ \boldsymbol{x}(t) = \boldsymbol{\varphi}(t), \quad t \in [-\max(d_1,d_2),0].\end{cases} \tag{1}$$

其中:$\boldsymbol{x}(t) \in \mathbf{R}^n$ 是系统的状态向量;$\boldsymbol{u}(t) \in \mathbf{R}^m$ 是控制输入向量;$\boldsymbol{\omega}(t) \in \mathbf{R}^r$ 为干扰输入,且 $\boldsymbol{\omega}(t) \in L_2[0,+\infty)$;$\boldsymbol{z}(t) \in \mathbf{R}^q$ 为控制输出

向量;$\boldsymbol{y}(t)\in\mathbf{R}^l$ 为测量输出向量;时滞 $d_i(t)(i=1,2)$ 是任意的有界函数,且满足 $0<d_i(t)\leqslant d_i<\infty(i=1,2)$;$\boldsymbol{A}_0,\boldsymbol{A}_1,\boldsymbol{B}_0,\boldsymbol{B}_1,\boldsymbol{D}_0,\boldsymbol{C},\boldsymbol{B},\boldsymbol{D},\boldsymbol{C}_1$ 和 $\boldsymbol{D}_1$ 是已知具有适当维数的定常矩阵;$\boldsymbol{\varphi}(t)\in C[-\max(d_i,d_2),0]$ 为向量值初始函数. 假定 $(\boldsymbol{A}_0,\boldsymbol{B}_0,\boldsymbol{C}_1)$ 是能稳能检测的,$\Delta\boldsymbol{A}_0,\Delta\boldsymbol{A}_1,\Delta\boldsymbol{B}_0,\Delta\boldsymbol{B}_1$ 分别为不确定时变矩阵.

假定参数不确定性满足匹配和范数有界条件,且满足

$$\begin{aligned}&[\Delta\boldsymbol{A}_0\ \ \Delta\boldsymbol{B}_0\ \ \Delta\boldsymbol{A}_1\ \ \Delta\boldsymbol{B}_1]=\\&[\Delta\boldsymbol{A}_0(t)\ \ \Delta\boldsymbol{B}_0(t)\ \ \Delta\boldsymbol{A}_1(t)\ \ \Delta\boldsymbol{B}_1(t)]=\\&\boldsymbol{HF}(t)(\boldsymbol{E}_1\ \ \boldsymbol{E}_2\ \ \boldsymbol{E}_3\ \ \boldsymbol{E}_4).\end{aligned}\tag{2}$$

其中:$\boldsymbol{H},\boldsymbol{E}_1,\boldsymbol{E}_2,\boldsymbol{E}_3,\boldsymbol{E}_4$ 是已知具有适当维数的定常矩阵;$\boldsymbol{F}(t)\in\mathbf{R}^{i\times j}$ 是一个具有 Lebesgue 可测元的不确定矩阵且满足

$$\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{F}(t)\leqslant\boldsymbol{I}.\tag{3}$$

本文的目的是对于给定的常数 γ,设计无记忆状态反馈控制律

$$\boldsymbol{u}(t)=\boldsymbol{Kx}(t).\tag{4}$$

其中 $\boldsymbol{K}\in\mathbf{R}^{m\times n}$ 是一个常数矩阵,使得式(1)对所有允许的不确定性,所导出的闭环系统

$$\begin{cases}\dot{\boldsymbol{x}}(t)=\bar{\boldsymbol{A}}\boldsymbol{x}(t)+\bar{\boldsymbol{A}}_1\boldsymbol{x}(t-d_1(t))+\\\qquad\bar{\boldsymbol{B}}_1\boldsymbol{x}(t-d_2(t))+\boldsymbol{D}_0\boldsymbol{\omega}(t),\\\boldsymbol{z}(t)=(\boldsymbol{C}+\boldsymbol{BK})\boldsymbol{x}(t)+\boldsymbol{D\omega}(t)\end{cases}\tag{5}$$

具有 H_∞ 性能 γ,其中:

$$\bar{\boldsymbol{A}}=\boldsymbol{A}_0+\Delta\boldsymbol{A}_0+(\boldsymbol{B}_0+\Delta\boldsymbol{B}_0)\boldsymbol{K};$$
$$\bar{\boldsymbol{A}}_1=\boldsymbol{A}_1+\Delta\boldsymbol{A}_1;\quad\bar{\boldsymbol{B}}_1=\boldsymbol{B}_1+\Delta\boldsymbol{B}_1.$$

则控制律(4)称为时滞系统(1)的一个 γ-次优状态反馈 H_∞ 控制律.

引理 1[1] 对任意常数矩阵 $\boldsymbol{N}>0$ 和常数 $\sigma>0$,向量函数 $\boldsymbol{x}(t):[0,\sigma]\to\mathbf{R}^n$ 可积,则有下面的不等式成立

$$\left[\int_{t-\sigma}^{t}\boldsymbol{x}(s)\mathrm{d}s\right]^{\mathrm{T}}\boldsymbol{N}\left[\int_{t-\sigma}^{t}\boldsymbol{x}(s)\mathrm{d}s\right]\leqslant\sigma\int_{t-\sigma}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Nx}(s)\mathrm{d}s.$$

引理 2[7] 对给定的对称矩阵 $\boldsymbol{S}=\begin{bmatrix}\boldsymbol{S}_{11}&\boldsymbol{S}_{12}\\\boldsymbol{S}_{12}^T&\boldsymbol{S}_{22}\end{bmatrix}$,其中 $\boldsymbol{S}_{11}\in\mathbf{R}^{r\times r}$,以下 3 个条件等价:

1) $\boldsymbol{S}<0$;

2) $\boldsymbol{S}_{11}<0,\boldsymbol{S}_{22}-\boldsymbol{S}_{12}^{\mathrm{T}}\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{12}<0$;

3) $\boldsymbol{S}_{22}<0,\boldsymbol{S}_{11}-\boldsymbol{S}_{12}\boldsymbol{S}_{22}^{-1}\boldsymbol{S}_{12}^{\mathrm{T}}<0$.

引理 3[8] 对任意满足 $\boldsymbol{F}^{\mathrm{T}}\boldsymbol{F}\leqslant\boldsymbol{I}$ 的适当维数矩阵 $\boldsymbol{F}$ 以及适当维数矩阵 $\boldsymbol{M}=\boldsymbol{M}^{\mathrm{T}},\boldsymbol{S}$ 和 $\boldsymbol{N}$,下面 2 式等价:

1) $\boldsymbol{M}+\boldsymbol{SFN}+\boldsymbol{N}^{\mathrm{T}}\boldsymbol{FS}<0$;

2) 存在 $\rho<0$,使得 $\begin{bmatrix}\boldsymbol{M}&\rho\boldsymbol{S}&\boldsymbol{N}^{\mathrm{T}}\\\rho\boldsymbol{S}^{\mathrm{T}}&-\rho\boldsymbol{I}&0\\\boldsymbol{N}&0&-\rho\boldsymbol{I}\end{bmatrix}<0$.

本文中“ * ”代表一个对称矩阵主对角线下方的元素,矩阵 $\boldsymbol{X}>0(\boldsymbol{X}<0)$ 意指矩阵 $\boldsymbol{X}$ 为正定(负定).

2 主要结论
Main conclusions

定理 1 对于闭环系统式(5),若存在矩阵 $\boldsymbol{P}>0,\boldsymbol{Q}>0,\boldsymbol{R}>0,\boldsymbol{M}>0$ 以及矩阵 $\boldsymbol{K}$ 和常数 $\gamma>0$,使得下面的矩阵不等式成立

$$\begin{bmatrix}\boldsymbol{N}_1&\boldsymbol{P}\bar{\boldsymbol{A}}_1+\boldsymbol{Q}&\boldsymbol{P}\bar{\boldsymbol{B}}_1+\boldsymbol{R}&\bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{M}&\boldsymbol{PD}_0&(\boldsymbol{C}+\boldsymbol{BK})^{\mathrm{T}}\\ *&-\boldsymbol{Q}&0&\bar{\boldsymbol{A}}_1^{\mathrm{T}}\boldsymbol{M}&0&0\\ *&*&-\boldsymbol{R}&\bar{\boldsymbol{B}}_1^{\mathrm{T}}\boldsymbol{M}&0&0\\ *&*&*&\boldsymbol{N}_2&\boldsymbol{MD}_0&0\\ *&*&*&*&-\gamma\boldsymbol{I}&\boldsymbol{D}^{\mathrm{T}}\\ *&*&*&*&*&-\gamma\boldsymbol{I}\end{bmatrix}<0.\tag{6}$$

其中:

$$\boldsymbol{N}_1=\bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}\bar{\boldsymbol{A}}-\boldsymbol{Q}-\boldsymbol{R};$$
$$\boldsymbol{N}_2=d_1^2\boldsymbol{Q}+d_2^2\boldsymbol{R}-2\boldsymbol{M}.$$

那么存在状态反馈控制器式(4),使得系统是渐近稳定的,且满足 H_∞ 性能指标,即 $\|\boldsymbol{z}\|_2<\gamma\|\boldsymbol{\omega}\|_2$.

证明 先来讨论系统(5)的镇定问题. 构造如下 Lyapunov 泛函

$$V(\boldsymbol{x},t)=V_1(\boldsymbol{x},t)+V_2(\boldsymbol{x},t)+V_3(\boldsymbol{x},t).\tag{7}$$

其中:

$$V_1(\boldsymbol{x},t)=\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Px}(t);$$
$$V_2(\boldsymbol{x},t)=d_1\int_{t-d_1}^{1}(s-t+d_1)\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{Q}\dot{\boldsymbol{x}}(s)\mathrm{d}s;$$
$$V_3(\boldsymbol{x},t)=d_2\int_{t-d_2}^{1}(s-t+d_2)\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}\dot{\boldsymbol{x}}(s)\mathrm{d}s.$$

$V(\boldsymbol{x},t)$ 沿闭环系统(5)的任意轨线,它对时间的导数

$$\dot{V}(\boldsymbol{x},t)\Big|_{(5)}=\dot{V}_1(\boldsymbol{x},t)+\dot{V}_2(\boldsymbol{x},t)+\dot{V}_3(\boldsymbol{x},t),\tag{8}$$

$$\begin{aligned}\dot{V}_1(\boldsymbol{x},t)\Big|_{(5)}=&\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Px}(t)+\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\dot{\boldsymbol{x}}(t)=\\&\boldsymbol{x}^{\mathrm{T}}(t)\bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{Px}(t)+\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\bar{\boldsymbol{A}}\boldsymbol{x}(t)+\\&\boldsymbol{x}^{\mathrm{T}}(t-d_1(t))\bar{\boldsymbol{A}}_1^{\mathrm{T}}\boldsymbol{Px}(t)+\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\bar{\boldsymbol{A}}_1\boldsymbol{x}(t-d_1(t))+\\&\boldsymbol{x}^{\mathrm{T}}(t-d_2(t))\bar{\boldsymbol{B}}_1^{\mathrm{T}}\boldsymbol{Px}(t)+\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\bar{\boldsymbol{B}}_1\boldsymbol{x}(t-d_2(t))+\\&\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{D}_0^{\mathrm{T}}\boldsymbol{Px}(t)+\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{PD}_0\boldsymbol{\omega}(t).\end{aligned}\tag{9}$$

由 $0<d_i(t)\leqslant d_i<\infty(i=1,2)$ 及引理 1 有

$$\begin{aligned}
\dot V_2(\boldsymbol{x},t)\Big|_{(5)} = {} & d_1^2\dot{\boldsymbol{x}}^{\mathrm T}(t)\boldsymbol{Q}\dot{\boldsymbol{x}}(t) - d_1\int_{t-d_1}^{t}\dot{\boldsymbol{x}}^{\mathrm T}(s)\boldsymbol{Q}\dot{\boldsymbol{x}}(s)\,\mathrm{d}s \leqslant \\
& d_1^2\dot{\boldsymbol{x}}^{\mathrm T}(t)\boldsymbol{Q}\dot{\boldsymbol{x}}(t) - d_1\int_{t-d_1(t)}^{t}\dot{\boldsymbol{x}}^{\mathrm T}(s)\boldsymbol{Q}\dot{\boldsymbol{x}}(s)\,\mathrm{d}s \leqslant \\
& d_1^2\dot{\boldsymbol{x}}^{\mathrm T}(t)\boldsymbol{Q}\dot{\boldsymbol{x}}(t) - \Big(\int_{t-d_1(t)}^{t}\dot{\boldsymbol{x}}^{\mathrm T}(s)\,\mathrm{d}s\Big)\boldsymbol{Q}\Big(\int_{t-d_1(t)}^{t}\dot{\boldsymbol{x}}(s)\,\mathrm{d}s\Big) = \\
& d_1^2\dot{\boldsymbol{x}}^{\mathrm T}(t)\boldsymbol{Q}\dot{\boldsymbol{x}}(t) - \boldsymbol{x}^{\mathrm T}(t)\boldsymbol{Q}\boldsymbol{x}(t) - \\
& \boldsymbol{x}^{\mathrm T}(t-d_1(t))\boldsymbol{Q}\boldsymbol{x}(t-d_1(t)) + \\
& \boldsymbol{x}^{\mathrm T}(t)\boldsymbol{Q}\boldsymbol{x}(t-d_1(t)) + \boldsymbol{x}^{\mathrm T}(t-d_1(t))\boldsymbol{Q}\boldsymbol{x}(t),
\end{aligned}\tag{10}$$

$$\begin{aligned}
\dot V_3(\boldsymbol{x},t)\Big|_{(5)} = {} & d_2^2\dot{\boldsymbol{x}}^{\mathrm T}(t)\boldsymbol{Q}\dot{\boldsymbol{x}}(t) - d_2\int_{t-d_2}^{t}\dot{\boldsymbol{x}}^{\mathrm T}(s)\boldsymbol{R}\dot{\boldsymbol{x}}(s)\,\mathrm{d}s \leqslant \\
& d_2^2\dot{\boldsymbol{x}}^{\mathrm T}(t)\boldsymbol{R}\dot{\boldsymbol{x}}(t) - d_2\int_{t-d_2(t)}^{t}\dot{\boldsymbol{x}}^{\mathrm T}(s)\boldsymbol{R}\dot{\boldsymbol{x}}(s)\,\mathrm{d}s \leqslant \\
& d_2^2\dot{\boldsymbol{x}}^{\mathrm T}(t)\boldsymbol{R}\dot{\boldsymbol{x}}(t) - \Big(\int_{t-d_2(t)}^{t}\dot{\boldsymbol{x}}^{\mathrm T}(s)\,\mathrm{d}s\Big)\boldsymbol{R}\Big(\int_{t-d_2(t)}^{t}\dot{\boldsymbol{x}}(s)\,\mathrm{d}s\Big) = \\
& d_2^2\dot{\boldsymbol{x}}^{\mathrm T}(t)\boldsymbol{R}\dot{\boldsymbol{x}}(t) - \boldsymbol{x}^{\mathrm T}(t)\boldsymbol{R}\boldsymbol{x}(t) - \\
& \boldsymbol{x}^{\mathrm T}(t-d_2(t))\boldsymbol{R}\boldsymbol{x}(t-d_2(t)) + \\
& \boldsymbol{x}^{\mathrm T}(t)\boldsymbol{R}\boldsymbol{x}(t-d_2(t)) + \boldsymbol{x}^{\mathrm T}(t-d_2(t))\boldsymbol{R}\boldsymbol{x}(t).
\end{aligned}\tag{11}$$

由式(5),对任意 $\boldsymbol{M}>0$,易知下面等式成立

$$\begin{aligned}
0 = {} & \dot{\boldsymbol{x}}^{\mathrm T}(t)\boldsymbol{M}\Big(\bar{\boldsymbol{A}}\boldsymbol{x}(t) + \bar{\boldsymbol{A}}_1\boldsymbol{x}(t-d_1(t)) + \\
& \bar{\boldsymbol{B}}_1\boldsymbol{x}(t-d_2(t)) + \boldsymbol{D}_0\boldsymbol{\omega}(t) - \dot{x}(t)\Big) + \\
& \Big(\bar{\boldsymbol{A}}\boldsymbol{x}(t) + \bar{\boldsymbol{A}}_1\boldsymbol{x}(t-d_1(t)) + \bar{\boldsymbol{B}}_1\boldsymbol{x}(t-d_2(t)) + \\
& \boldsymbol{D}_0\boldsymbol{\omega}(t) - \dot{\boldsymbol{x}}(t)\Big)^{\mathrm T}\boldsymbol{M}\dot{\boldsymbol{x}}(t).
\end{aligned}\tag{12}$$

则

$$\begin{aligned}
\dot V(\boldsymbol{x},t)\Big|_{(5)} = {} & \dot V_1(\boldsymbol{x},t)\Big|_{(5)} + \dot V_2(\boldsymbol{x},t)\Big|_{(5)} + \dot V_3(\boldsymbol{x},t)\Big|_{(5)} + 0 \leqslant \\
& \boldsymbol{x}^{\mathrm T}(t)(\bar{\boldsymbol{A}}^{\mathrm T}\boldsymbol{P} + \boldsymbol{P}\bar{\boldsymbol{A}} - \boldsymbol{Q} - \boldsymbol{R})\boldsymbol{x}(t) + \\
& \boldsymbol{x}^{\mathrm T}(t)(\boldsymbol{P}\bar{\boldsymbol{A}}_1 + \boldsymbol{Q})\boldsymbol{x}(t-d_1(t)) + \\
& \boldsymbol{x}^{\mathrm T}(t-d_1(t))(\boldsymbol{P}\bar{\boldsymbol{A}}_1 + \boldsymbol{Q})^{\mathrm T}\boldsymbol{x}(t) + \\
& \boldsymbol{x}^{\mathrm T}(t)(\boldsymbol{P}\bar{\boldsymbol{B}}_1 + \boldsymbol{R})\boldsymbol{x}(t-d_2(t)) + \\
& \boldsymbol{x}^{\mathrm T}(t-d_2(t))(\boldsymbol{P}\bar{\boldsymbol{B}}_1 + \boldsymbol{R})^{\mathrm T}\boldsymbol{x}(t) - \\
& \boldsymbol{x}^{\mathrm T}(t-d_1(t))\boldsymbol{Q}\boldsymbol{x}(t-d_1(t)) - \\
& \boldsymbol{x}^{\mathrm T}(t-d_2(t))\boldsymbol{R}\boldsymbol{x}(t-d_2(t)) + \\
& \dot{\boldsymbol{x}}^{\mathrm T}(t)(d_1^2\boldsymbol{Q} + d_2^2\boldsymbol{R} - 2\boldsymbol{M})\dot{\boldsymbol{x}}(t) + \\
& \boldsymbol{x}^{\mathrm T}(t)\bar{\boldsymbol{A}}^{\mathrm T}\boldsymbol{M}\dot{\boldsymbol{x}}(t) + \dot{\boldsymbol{x}}^{\mathrm T}(t)\boldsymbol{M}\bar{\boldsymbol{A}}\boldsymbol{x}(t) + \\
& \boldsymbol{x}^{\mathrm T}(t-d_1(t))\bar{\boldsymbol{A}}_1^{\mathrm T}\boldsymbol{M}\dot{\boldsymbol{x}}(t) + \dot{\boldsymbol{x}}^{\mathrm T}(t)\boldsymbol{M}\bar{\boldsymbol{A}}_1\boldsymbol{x}(t-d_1(t)) + \\
& \boldsymbol{x}^{\mathrm T}(t-d_2(t))\bar{\boldsymbol{B}}_1^{\mathrm T}\boldsymbol{M}\dot{\boldsymbol{x}}(t) + \dot{\boldsymbol{x}}^{\mathrm T}(t)\boldsymbol{M}\bar{\boldsymbol{B}}_1\boldsymbol{x}(t-d_2(t)) + \\
& \boldsymbol{\omega}^{\mathrm T}(t)\boldsymbol{D}_0^{\mathrm T}\boldsymbol{M}\dot{\boldsymbol{x}}(t) + \dot{\boldsymbol{x}}^{\mathrm T}(t)\boldsymbol{M}\boldsymbol{D}_0\boldsymbol{\omega}(t) + \boldsymbol{\omega}^{\mathrm T}(t)\boldsymbol{D}_0^{\mathrm T}\boldsymbol{P}\boldsymbol{x}(t) + \\
& \boldsymbol{x}^{\mathrm T}(t)\boldsymbol{P}\boldsymbol{D}_0\boldsymbol{\omega}(t).
\end{aligned}\tag{13}$$

令

$$\boldsymbol{\xi}(t) = (\boldsymbol{x}^{\mathrm T}(t), \boldsymbol{x}^{\mathrm T}(t-d_1(t)), \boldsymbol{x}^{\mathrm T}(t-d_2(t)), \dot{\boldsymbol{x}}^{\mathrm T}(t))^{\mathrm T}$$

则当 $\boldsymbol{\omega}(t)=0$ 时,

$$\dot V(\boldsymbol{x},t)\Big|_{(5)} \leqslant \boldsymbol{\xi}^{\mathrm T}(t)\begin{bmatrix} \boldsymbol{N}_1 & \boldsymbol{P}\bar{\boldsymbol{A}}_1+\boldsymbol{Q} & \boldsymbol{P}\bar{\boldsymbol{B}}_1+\boldsymbol{R} & \bar{\boldsymbol{A}}^{\mathrm T}\boldsymbol{M} \\ * & -\boldsymbol{Q} & 0 & \bar{\boldsymbol{A}}_1^{\mathrm T}\boldsymbol{M} \\ * & * & -\boldsymbol{R} & \bar{\boldsymbol{B}}_1^{\mathrm T}\boldsymbol{M} \\ * & * & * & \boldsymbol{N}_2 \end{bmatrix}\boldsymbol{\xi}(t),\tag{14}$$

再结合定理条件式(6)成立时,

$$\dot V(\boldsymbol{x},t)\Big|_{(5)} < 0(\boldsymbol{\omega}(t)=0),$$

则由 Lyapunov 稳定性定理可知,闭环系统式(5)渐近稳定.

下面考虑系统(5)的 H_∞ 问题.

在零初始条件下,考虑

$$J = \int_0^{\tau}\Big(\gamma^{-1}\boldsymbol{z}^{\mathrm T}(t)\boldsymbol{z}(t) - \gamma\boldsymbol{\omega}^{\mathrm T}(t)\boldsymbol{\omega}(t)\Big)\mathrm{d}t,\tag{15}$$

则对任意非零的外部扰动

$$\boldsymbol{\omega}(t) \in L_2[0,+\infty],$$

利用 Lyapunov 泛函(7),可以导出

$$\begin{aligned}
J = {} & \int_0^{\tau}\Big(\gamma^{-1}\boldsymbol{z}^{\mathrm T}(t)\boldsymbol{z}(t) - \gamma\boldsymbol{\omega}^{\mathrm T}(t)\boldsymbol{\omega}(t) + \dot V(\boldsymbol{x},t)\Big)\mathrm{d}t - V(\boldsymbol{x},\tau) \leqslant \\
& \int_0^{\tau}\Big(\gamma^{-1}\boldsymbol{z}^{\mathrm T}(t)\boldsymbol{z}(t) - \gamma\boldsymbol{\omega}^{\mathrm T}(t)\boldsymbol{\omega}(t) + \dot V(\boldsymbol{x},t)\Big)\mathrm{d}t,
\end{aligned}\tag{16}$$

记

$$\boldsymbol{\zeta}(t) = \Big(\boldsymbol{x}^{\mathrm T}(t), \boldsymbol{x}^{\mathrm T}(t-d_1(t)), \boldsymbol{x}^{\mathrm T}(t-d_2(t)), \dot{\boldsymbol{x}}^{\mathrm T}(t), \boldsymbol{\omega}^{\mathrm T}(t)\Big)^{\mathrm T}.$$

由式(13)可知

$$\dot V(\boldsymbol{x},t) \leqslant \boldsymbol{\zeta}^{\mathrm T}(t)\begin{bmatrix} \boldsymbol{N}_1 & \boldsymbol{P}\bar{\boldsymbol{A}}_1+\boldsymbol{Q} & \boldsymbol{P}\bar{\boldsymbol{B}}_1+\boldsymbol{R} & \bar{\boldsymbol{A}}^{\mathrm T}\boldsymbol{M} & \boldsymbol{P}\boldsymbol{D}_0 \\ * & -\boldsymbol{Q} & 0 & \bar{\boldsymbol{A}}_1^{\mathrm T}\boldsymbol{M} & 0 \\ * & * & -\boldsymbol{R} & \bar{\boldsymbol{B}}_1^{\mathrm T}\boldsymbol{M} & 0 \\ * & * & * & \boldsymbol{N}_2 & \boldsymbol{M}\boldsymbol{D}_0 \\ * & * & * & * & 0 \end{bmatrix}\boldsymbol{\zeta}(t),\tag{17}$$

又

$$\begin{aligned}
\gamma^{-1}\boldsymbol{z}^{\mathrm T}(t)\boldsymbol{z}(t) - \gamma\boldsymbol{\omega}^{\mathrm T}(t)\boldsymbol{\omega}(t) = {} & \gamma^{-1}\boldsymbol{x}^{\mathrm T}(t)\boldsymbol{W}^{\mathrm T}\boldsymbol{W}\boldsymbol{x}(t) + \\
& \gamma^{-1}\boldsymbol{x}^{\mathrm T}(t)\boldsymbol{W}^{\mathrm T}\boldsymbol{D}\boldsymbol{\omega}(t) + \\
& \gamma^{-1}\boldsymbol{\omega}^{\mathrm T}(t)\boldsymbol{D}^{\mathrm T}\boldsymbol{W}\boldsymbol{x}(t) + \\
& \gamma^{-1}\boldsymbol{\omega}^{\mathrm T}(t)\boldsymbol{D}^{\mathrm T}\boldsymbol{D}\boldsymbol{\omega}(t) - \gamma\boldsymbol{\omega}^{\mathrm T}(t)\boldsymbol{\omega}(t) = \\
& \boldsymbol{\zeta}^{\mathrm T}(t)\begin{bmatrix} \gamma^{-1}\boldsymbol{W}^{\mathrm T}\boldsymbol{W} & 0 & 0 & 0 & \gamma^{-1}\boldsymbol{W}^{\mathrm T}\boldsymbol{D} \\ * & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & \gamma^{-1}\boldsymbol{D}^{\mathrm T}\boldsymbol{D} - \gamma\boldsymbol{I} \end{bmatrix}\boldsymbol{\zeta}(t),
\end{aligned}\tag{18}$$

其中:$\boldsymbol{W} = \boldsymbol{C} + \boldsymbol{B}\boldsymbol{K}$.

故

$$J \leqslant \int_0^\tau \left(\gamma^{-1} z^{\mathrm{T}}(t) z(t) - \gamma \omega^{\mathrm{T}}(t) \omega(t) + \dot{V}(x,t) \right) \mathrm{d}t = \int_0^\tau \zeta^{\mathrm{T}}(t) \cdot$$

$$\begin{bmatrix} N_1 + \gamma^{-1} W^{\mathrm{T}} W & G_1 & G_2 & \bar{A}^{\mathrm{T}} M & PD_0 + \gamma^{-1} W^{\mathrm{T}} D \\ * & -Q & 0 & \bar{A}_1^{\mathrm{T}} M & 0 \\ * & * & -R & \bar{B}_1^{\mathrm{T}} M & 0 \\ * & * & * & N_2 & MD_0 \\ * & * & * & * & \gamma^{-1} D^{\mathrm{T}} D - \gamma I \end{bmatrix} \zeta(t)\,\mathrm{d}t. \qquad (19)$$

其中:

$$W = C + BK; \quad G_1 = P\bar{A}_1 + Q; \quad G_2 = P\bar{B}_1 + R;$$
$$N_1 = \bar{A}^{\mathrm{T}} P + P\bar{A} - Q - R;$$
$$N_2 = d_1^2 Q + d_2^2 R - 2M.$$

由引理2(Schur 补),矩阵不等式(6)等价于

$$\begin{bmatrix} N_1 + \gamma^{-1} W^{\mathrm{T}} W & G_1 & G_2 & \bar{A}^{\mathrm{T}} M & PD_0 + \gamma^{-1} W^{\mathrm{T}} D \\ * & -Q & 0 & \bar{A}_1^{\mathrm{T}} M & 0 \\ * & * & -R & \bar{B}_1^{\mathrm{T}} M & 0 \\ * & * & * & N_2 & MD_0 \\ * & * & * & * & \gamma^{-1} D^{\mathrm{T}} D - \gamma I \end{bmatrix} < 0, \qquad (20)$$

其中:

$$W = C + BK; \quad G_1 = P\bar{A}_1 + Q; \quad G_2 = P\bar{B}_1 + R.$$

所以

$$\int_0^\tau z^{\mathrm{T}}(t) z(t)\,\mathrm{d}t \leqslant$$
$$\gamma^2 \int_0^\tau \omega^{\mathrm{T}}(t) \omega(t)\,\mathrm{d}t \leqslant$$
$$\gamma^2 \int_0^\infty \omega^{\mathrm{T}}(t) \omega(t)\,\mathrm{d}t$$

对所有的 τ 都成立. 定理得证.

定理2 对于闭环系统式(5)和给定 $\gamma > 0$,若存在矩阵 $X > 0$,$Q_1 > 0$,$R_1 > 0$ 以及矩阵 Y 和常数 $\rho > 0$,使得下面的线性矩阵不等式成立

$$\begin{bmatrix} N_3 & G_3 & G_4 & N_4 & D_0 & N_5 & \rho H & N_6 \\ * & -Q_1 & 0 & XA_1^{\mathrm{T}} & 0 & 0 & 0 & XE_3^{\mathrm{T}} \\ * & * & -R_1 & Y^{\mathrm{T}} B_1^{\mathrm{T}} & 0 & 0 & 0 & Y^{\mathrm{T}} E_4^{\mathrm{T}} \\ * & * & * & N_7 & D_0 & 0 & \rho H & 0 \\ * & * & * & * & -\gamma I & D^{\mathrm{T}} & 0 & 0 \\ * & * & * & * & * & -\gamma I & 0 & 0 \\ * & * & * & * & * & * & -\rho I & 0 \\ * & * & * & * & * & * & * & -\rho I \end{bmatrix} < 0. \qquad (21)$$

其中:

$$N_3 = (A_0 X + B_0 Y)^{\mathrm{T}} + (A_0 X + B_0 Y) - Q_1 - R_1;$$
$$N_4 = (A_0 X + B_0 Y)^{\mathrm{T}};$$
$$N_5 = (CX + BY)^{\mathrm{T}};$$
$$N_6 = (E_1 X + E_2 Y)^{\mathrm{T}};$$
$$N_7 = d_1^2 \varepsilon^2 Q_1 + d_2^2 \varepsilon^2 R_1 - 2\varepsilon X;$$
$$G_3 = A_1 X + Q_1; \quad G_4 = B_1 Y + R_1.$$

那么存在状态反馈控制器式(4),使得闭环系统式(5)渐近稳定,且满足 H_∞ 性能指标,并且反馈增益矩阵为 $K = YX^{-1}$

证明 若令

$$\pi = \begin{bmatrix} \Omega & G_1 & G_5 & G_6^{\mathrm{T}} M & PD_0 & W^{\mathrm{T}} \\ * & -Q & 0 & A_1^{\mathrm{T}} M & 0 & 0 \\ * & * & -R & K^{\mathrm{T}} B_1^{\mathrm{T}} M & 0 & 0 \\ * & * & * & N_2 & MD_0 & 0 \\ * & * & * & * & -\gamma I & D^{\mathrm{T}} \\ * & * & * & * & * & -\gamma I \end{bmatrix}$$

$$\pi_1 = \begin{bmatrix} H^{\mathrm{T}} P & 0 & 0 & H^{\mathrm{T}} M & 0 & 0 \end{bmatrix}^{\mathrm{T}},$$

$$\pi_2 = \begin{bmatrix} E_1 + E_2 K & E_3 & E_4 K & 0 & 0 & 0 \end{bmatrix}.$$

其中:

$$\Omega = (A_0 + B_0 K)^{\mathrm{T}} P + P(A_0 + B_0 K) - Q - R;$$
$$G_1 = PA_1 + Q; \quad G_5 = PB_1 K + R;$$
$$G_6 = A_0 + B_0 K; \quad W = C + BK;$$
$$N_2 = d_1^2 Q + d_2^2 R - 2M.$$

则式(6)可表达为

$$\pi + \pi_1 F(t) \pi_2 + \pi_2^{\mathrm{T}} F^{\mathrm{T}}(t) \pi_1^{\mathrm{T}} < 0. \qquad (22)$$

由引理3,存在 $\rho > 0$,使得

$$\begin{bmatrix} \Omega & G_1 & G_5 & G_6^{\mathrm{T}} M & PD_0 & W^{\mathrm{T}} & \rho PH & (E_1 + E_2 K)^{\mathrm{T}} \\ * & -Q & 0 & A_1^{\mathrm{T}} M & 0 & 0 & 0 & E_3^{\mathrm{T}} \\ * & * & -R & KB_1^{\mathrm{T}} M & 0 & 0 & 0 & K^{\mathrm{T}} E_4^{\mathrm{T}} \\ * & * & * & N_2 & MD_0 & 0 & \rho MH & 0 \\ * & * & * & * & -\gamma I & D^{\mathrm{T}} & 0 & 0 \\ * & * & * & * & * & -\gamma I & 0 & 0 \\ * & * & * & * & * & * & -\rho I & 0 \\ * & * & * & * & * & * & * & -\rho I \end{bmatrix} < 0, \qquad (23)$$

对式(23)两端分别左乘和右乘矩阵

$$\mathrm{diag}\{ P^{-1} \quad P^{-1} \quad P^{-1} \quad M^{-1} \quad I \quad I \quad I \quad I \}$$

并记

$$X = P^{-1}, \quad Y = KP^{-1}, \quad Q_1 = P^{-1} Q P^{-1},$$
$$R_1 = P^{-1} R P^{-1}, \quad M^{-1} = \varepsilon P^{-1},$$

即可得到等价的矩阵不等式(21).

故存在状态反馈控制器式(4),使得闭环系统式(5)渐近稳定,且满足 H_∞ 性能指标,并且反馈增益矩阵为 $K = YX^{-1}$

以上定理中的矩阵不等式(21)是一个关于 X,Y,Q_1,R_1,ρ 的线性矩阵不等式,可借助于 LMI 工具

箱求解.

进一步,系统(1)基于无记忆的 γ-次优状态反馈 H_∞ 控制率存在条件(21),通过建立和求解以下的优化问题,s. t.

$$\min \delta$$

$$\begin{bmatrix} N_3 & G_3 & G_4 & N_4 & D_0 & N_5 & \rho H & N_6 \\ * & -Q_1 & 0 & XA_1^{\mathrm{T}} & 0 & 0 & 0 & XE_3^{\mathrm{T}} \\ * & * & -R_1 & Y^{\mathrm{T}}B_1^{\mathrm{T}} & 0 & 0 & 0 & Y^{\mathrm{T}}E_4^{\mathrm{T}} \\ * & * & * & N_7 & D_0 & 0 & \rho H & 0 \\ * & * & * & * & -\delta I & D^{\mathrm{T}} & 0 & 0 \\ * & * & * & * & * & -\delta I & 0 & 0 \\ * & * & * & * & * & * & -\rho I & 0 \\ * & * & * & * & * & * & * & -\rho I \end{bmatrix} < 0 \tag{24}$$

的最优解得到系统(1)的最优 H_∞ 控制器.

式(24)中:

$$N_3 = (A_0X + B_0Y)^{\mathrm{T}} + (A_0X + B_0Y) - Q_1 - R_1;$$

$$G_3 = A_1X + Q_1; \quad G_4 = B_1Y + R_1;$$

$$N_4 = (A_0X + B_0Y)^{\mathrm{T}};$$

$$N_5 = (CX + BY)^{\mathrm{T}};$$

$$N_6 = (E_1X + E_2Y)^{\mathrm{T}};$$

$$N_7 = d_1^2\varepsilon^2 Q_1 + d_2^2\varepsilon^2 R_1 - 2\varepsilon X;$$

$$X > 0; \quad Q_1 > 0; \quad R_1 > 0; \quad \rho > 0.$$

3 算例
Calculation examples

考虑下面的时滞系统

$$\begin{cases} \dot{x}(t) = (A_0 + \Delta A_0)x(t) + (A_1 + \Delta A_1)x(t - d_1(t)) + \\ \quad (B_0 + \Delta B_0)u(t) + (B_1 + \Delta B_1)u(t - d_2(t)) + \\ \quad D_0\omega(t), \\ z(t) = Cx(t) + Bu(t) + D\omega(t), \\ y(t) = C_1x(t) + D_1\omega(t), \\ x(t) = \varphi(t), \quad t \in [-\max(d_1, d_2), 0]. \end{cases}$$

其中:滞后时间

$$d_1(t) = 2 + 0.1\sin t, \quad d_2(t) = 3 + 0.1\cos t,$$

则 $d_1 = 2.1, d_2 = 3.1$,取 $\varepsilon = 1$,

$$A_0 = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}, \quad B_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 0 \\ 0.1 & 0.1 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, \quad \Delta A_0 = \begin{bmatrix} 0.1r(t) & 0.1r(t) \\ 0 & 0 \end{bmatrix},$$

$$\Delta B_0 = \begin{bmatrix} 0 \\ 0.1s(t) \end{bmatrix}, \quad \Delta A_1 = \begin{bmatrix} 0.1v(t) & 0.1v(t) \\ 0 & 0 \end{bmatrix},$$

$$\Delta B_1 = \begin{bmatrix} 0 \\ 0.1p(t) \end{bmatrix}, \quad D_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

$$|r(t)|, |s(t)|, |v(t)|, |p(t)| \leqslant 0.1.$$

定义

$$H = 0.1\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}, \quad E = 0.1\begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$E_2 = 0.1\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad E_3 = 0.1\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}, \quad E_4 = 0.1\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix},$$

$$F = 10 \times \mathrm{diag}\{r(t), s(t), v(t), p(t)\}$$

则参数不确定性满足(2)与(3),解式(24)可得

$$X = \begin{bmatrix} 0.7302 & -0.7308 \\ -0.7308 & 1.5637 \end{bmatrix},$$

$$Y = \begin{bmatrix} -1.1451 & 0.4653 \end{bmatrix}, \quad \delta = 5.3721,$$

得到系统(1)最优状态反馈 H_∞ 控制律的增益矩阵

$$K = YX^{-1} = [-2.3865 \quad -0.8178],$$

相应最小的扰动抑制度为 5.372 1.

4 结语
Concluding remarks

本文研究了一类不确定时变时滞系统的 H_∞ 反馈控制,得到了具有 H_∞ 干扰抑制的鲁棒最优 H_∞ 控制器存在的一个充分条件,并转化成易于利用 Matlab 求解的线性矩阵不等式.此方法不用考虑时滞函数的导数信息,给设计过程带来了方便也扩大了适用范围.

参考文献
References

[1] Kwon O M, Park J H. On improved delay-dependent robust control for uncertain time-delay systems[J]. IEEE Transactions on Automatic Control, 2004, 49(11): 1989-1999

[2] 付艳明,段广任.时变时滞不确定系统的鲁棒稳定性分析[J].黑龙江大学学报:自然科学报, 2005, 22(5): 655-658
FU Yanming, DUAN Guangren. The robust stability analysis for uncertain systems with time varying delays[J]. Journal of Natural Science of Heilongjiang University: Natural Science Edition, 2005, 22(5): 655-658

[3] 吕亮,李钟慎.不确定时变时滞系统 H_∞ 反馈控制器设计[J].计算技术与自动化, 2006, 25(1): 4-7
LÜ Liang, LI Zhongshen. Design of robust H_∞ feedback controller for uncertain systems with time varying delays[J]. Computing

Technology and Automation,2006,25(1):4-7
[4] 陆国平,施也冲,翟其亮.一类不确定时滞系统的鲁棒 H_∞ 控制[J].系统工程与电子技术,2002,24(5):41-44
LU Guoping,SHI Yechong,ZHAI Qiliang. The robust H_∞ control for a class of uncertain time-delay systems[J]. Journal of Systems Engineering and Electronics,2002,24(5):41-44
[5] ZHENG Zhiqiang,BAO Jundong. Design of robust H_∞ controller for uncertain systems with time varying delays[C]//Fourth International Conference on Impulsive and Hybrid Dynamical Systems. 2007:754-757
[6] 罗跃生,董晓璋,孙明丽.不确定变时滞系统的鲁棒非脆弱 H_∞ 控制[J].控制工程,2008,15(3):265-268
LUO Yuesheng,DONG Xiaozhang,SUN Mingli. The robust non-fragile H_∞ control for uncertain systems with time varying delays[J]. Control Engineering,2008,15(3):265-268
[7] 俞立.鲁棒控制——线性矩阵不等式处理方法[M].北京:清华大学出版社,2002
YU Li. The robust control:an approach to linear matrix inequalities[M]. Beijing:Tsinghua University Press,2002
[8] Kwon O M,Park J H. Guaranteed cost control of time-delay chaotic systems[J]. Chaos,Solitions and Fractals,2006,27(4):1108-1118

Robust H_∞ optimal control for uncertain time-varying delay systems

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Abstract The problem of robust H_∞ controller design is studied for uncertain time-varying linear systems with time-delay. A sufficient condition of existence of H_∞ feedback controller is obtained by using integral inequality and free-weighting matrix method. Then the sufficient condition is transformed to a linear matrix inequality(LMI) presentation. The controller is easily constructed by the feasible solution of the LMI. The obtained controller guarantees the robust stability and a prescribed H_∞ performance of the closed-loop system. The stabilization condition depends on the size of delay but requires no information about the derivate of time delay,which is applicable to the systems with fast time-varying delay. The example shows the feasibility of the approach.

Key words robust H_∞ control;uncertain systems;delay dependence;linear matrix inequality