文章编号:1674-7070(2009)03-0252-06

一类非线性中立型系统的状态反馈时滞相关 H-infinity 控制

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摘要

研究了一类非线性中立型时滞系统的 H-infinity 控制问题. 利用了适当的参数待定的 Lyapunov-Krasovskii 泛函及 LMI 方法获得了中立型系统的稳定性时滞相关的判据,给出了 H-infinity 状态反馈控制器存在的时滞依赖的充分条件. 所采用的方法在设计、求解控制器时不需要做矩阵变换,求解简单,利用 Matlab 的 LMI 工具箱求解方便,并且给出的数值仿真实例说明了方法的有效性.

关键词

中立型系统;非线性;时滞相关;H-infinity 控制

中图分类号 TP13

文献标志码 A

收稿日期 2009-09-09

资助项目 内蒙古自然科学基金(200711020104).

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0 引言

Introduction

中立型系统的研究具有广泛的应用背景. 众所周知,在信息与控制理论、化学反应堆、电动力学、分布网络、热交换系统、无损传输线路、生物医学等[1-2]都有中立型系统的应用. 正因如此,近年来引起了人们的广泛关注[1-11]. 关于中立型系统的稳定与镇定问题及 H-infinity 控制问题的研究主要集中于时滞无关与时滞相关的判据的建立,人们已经认识到时滞相关判据更为有效、具有更小保守性. 因此,近来很多研究成果都是致力于保守性较小的判据的建立[5-11],得到了很多优秀的研究成果,所有这些研究成果都为进一步推动中立型系统的研究做出了贡献.

在文献[12-14]中,基于 LMI 方法,利用 Lyapunov-Krasovskii 泛函讨论了中立型方程

$$\frac{\mathrm{d}}{\mathrm{d}t}[\boldsymbol{x}(t)+p\boldsymbol{x}(t-\tau)]=-a\boldsymbol{x}(t)+b\tanh\boldsymbol{x}(t-\sigma) \tag{1}$$

的稳定性问题. 文献[14]对该方程做了研究,并改进了文献[12-13]的结果;之后,文献[15]给出了更为深刻的结论,改进了文献[14]的结果.

本文在文献[15]的基础上讨论了一类非线性中立型系统

$$\begin{cases}\frac{\mathrm{d}}{\mathrm{d}t}[\boldsymbol{x}(t)+\boldsymbol{P}\boldsymbol{x}(t-\tau)]=-\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{B}\boldsymbol{u}(t)+\\ \qquad \boldsymbol{B}_1\boldsymbol{f}(\boldsymbol{x}(t-\sigma))+\boldsymbol{B}_\omega\omega(t),\\ \boldsymbol{x}(t)=\boldsymbol{\varphi}(t),t\in[-\sigma,0],\boldsymbol{\varphi}\in\boldsymbol{C}([-\sigma,0],\mathbf{R}^n),\\ \boldsymbol{z}(t)=\boldsymbol{C}\boldsymbol{x}(t)+\boldsymbol{C}_1\boldsymbol{x}(t-\tau)+\boldsymbol{D}\boldsymbol{u}(t)+\boldsymbol{D}_1\boldsymbol{\omega}(t)\end{cases} \tag{2}$$

的 H-infinity 控制问题,给出了时滞相关的判定条件.

本文通篇沿用如下的符号:$\mathbf{R}^n$,$\mathbf{R}^{n\times m}$分别表示 n 维向量空间和 $n\times m$矩阵空间;用"$*$"表示对称矩阵中的对称部分的相应元素;用矩阵$\boldsymbol{X}>0$($\boldsymbol{X}<0$)表示对称正定(负定)矩阵;$\rho(\boldsymbol{X})$表示矩阵 $\boldsymbol{X}$ 的谱范数;$L_2[0,\infty]$表示在$[0,\infty)$上平方可积的函数空间.

1 问题的描述

Description of the problem

考虑中立型系统

$$\begin{cases}\frac{\mathrm{d}}{\mathrm{d}t}[\boldsymbol{x}(t)+\boldsymbol{P}\boldsymbol{x}(t-\tau)]=-\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{B}\boldsymbol{u}(t)+\\ \quad \boldsymbol{B}_1\boldsymbol{f}(\boldsymbol{x}(t-\sigma))+\boldsymbol{B}_\omega\boldsymbol{\omega}(t),\\ \boldsymbol{x}(t)=\boldsymbol{\varphi}(t),t\in[-\sigma,0],\boldsymbol{\varphi}\in \boldsymbol{C}([-\sigma,0],\mathbf{R}^n),\\ \boldsymbol{z}(t)=\boldsymbol{C}\boldsymbol{x}(t)+\boldsymbol{C}_1\boldsymbol{x}(t-\tau)+\boldsymbol{D}\boldsymbol{u}(t)+\boldsymbol{D}_1\boldsymbol{\omega}(t).\end{cases} \tag{2}$$

其中:$\boldsymbol{x}(t)\in\mathbf{R}^n$ 为系统状态向量;$\boldsymbol{u}(t)\in\mathbf{R}^m$ 为控制输入向量;$\boldsymbol{f}(\boldsymbol{x})\in\mathbf{R}^n$ 是满足 $\|\boldsymbol{f}(\boldsymbol{x})\|\leqslant\|\boldsymbol{x}^2(t)\|$ 的非线性向量函数;τ,σ 为常时滞,且 $0\leqslant\tau\leqslant\sigma$;$\boldsymbol{\omega}(t)\in L_2[0,\infty)$ 为 p 维扰动输入向量;$z(t)\in\mathbf{R}^q$ 为控制输出向量;$\boldsymbol{P}\in\mathbf{R}^{n\times n}$ 且 $\rho(\boldsymbol{P})<1$;$\boldsymbol{A},\boldsymbol{B},\boldsymbol{B}_1,\boldsymbol{B}_\omega,\boldsymbol{C},\boldsymbol{C}_1,\boldsymbol{D}$ 为维数适当的已知常数矩阵.

对系统(2),考虑以下的无记忆控制器

$$\boldsymbol{u}(t)=\boldsymbol{K}\boldsymbol{x}(t), \tag{3}$$

其中:增益矩阵 $\boldsymbol{K}\in\mathbf{R}^{m\times n}$.

定义算子 $\mathscr{P}_1:\boldsymbol{C}([-\sigma,0],\mathbf{R}^n)$ 为

$$\mathscr{P}_1(\boldsymbol{x}_t)=\boldsymbol{x}(t)+\boldsymbol{P}\boldsymbol{x}(t-\tau),\quad \boldsymbol{x}(t)\in\mathbf{R}^n. \tag{4}$$

算子 $\mathscr{P}_2:\boldsymbol{C}([-\sigma,0],\mathbf{R}^n)$ 为

$$\mathscr{P}_2(\boldsymbol{x}_t)=\boldsymbol{x}(t)+\boldsymbol{P}\boldsymbol{x}(t-\tau)+\alpha\int_{t-\tau}^{t}\boldsymbol{x}(s)\mathrm{d}s+ \boldsymbol{B}_1\int_{t-\sigma}^{t-\tau}\boldsymbol{f}(\boldsymbol{x}(s))\mathrm{d}s,\quad \boldsymbol{x}\in\mathbf{R}^n. \tag{5}$$

这里 α 是个待定的参数.

本文的目的就是要设计形如式(3)的无记忆控制器,使得闭环系统为

1)内部稳定;

2)在零初始条件下,具有给定的 H_∞ 扰动抑制水平 γ,即 $\|z\|_2\leqslant\gamma\|\boldsymbol{\omega}\|_2$.

在控制器(3)的作用下,系统(2)的闭环系统为

$$\begin{cases}\frac{\mathrm{d}}{\mathrm{d}t}[\boldsymbol{x}(t)+\boldsymbol{P}\boldsymbol{x}(t-\tau)]=[-\boldsymbol{A}+\boldsymbol{B}\boldsymbol{K}]\boldsymbol{x}(t)+\\ \quad \boldsymbol{B}_1\boldsymbol{f}(\boldsymbol{x}(t-\sigma))+\boldsymbol{B}_\omega\boldsymbol{\omega}(t),\\ \boldsymbol{x}(t)=\boldsymbol{\varphi}(t),t\in[-\sigma,0],\boldsymbol{\varphi}\in \boldsymbol{C}([-\sigma,0],\mathbf{R}^n),\\ \boldsymbol{z}(t)=[\boldsymbol{C}+\boldsymbol{D}\boldsymbol{K}]\boldsymbol{x}(t)+\boldsymbol{C}_1\boldsymbol{x}(t-\tau)+\boldsymbol{D}_1\boldsymbol{\omega}(t).\end{cases} \tag{6}$$

由式(5)及式(6),得到系统方程的等价状态方程

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathscr{P}_2(\boldsymbol{x}_t)=[\alpha\boldsymbol{I}-\boldsymbol{A}+\boldsymbol{B}\boldsymbol{K}]\boldsymbol{x}(t)-\alpha\boldsymbol{x}(t-\tau)+ \boldsymbol{B}_1\boldsymbol{f}(\boldsymbol{x}(t-\tau))+\boldsymbol{B}_\omega\boldsymbol{\omega}(t). \tag{7}$$

引理 1[16] 对于任意给定的正常数 $\gamma>0$ 和正定矩阵 $\boldsymbol{M}=\boldsymbol{M}^{\mathrm{T}}>0$ 及向量函数 $\boldsymbol{\omega}:[0,\gamma]\to\mathbf{R}^n$,有下面的积分不等式成立

$$\gamma\int_0^\gamma\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{M}\boldsymbol{\omega}(t)\mathrm{d}t\geqslant\left(\int_0^\gamma\boldsymbol{\omega}^{\mathrm{T}}(t)\mathrm{d}t\right)\boldsymbol{M}\left(\int_0^\gamma\boldsymbol{\omega}(t)\mathrm{d}t\right).$$

2 闭环系统的内部稳定性

Stability inside the closed-loop system

下面考虑在控制器(3)的作用下,闭环系统(6)的内部稳定性问题.并给出如下定理.

定理 1 对于给定的正常数 $\tau>0,\sigma>0$,且 $\tau\leqslant\sigma$,系统(7)内部是可镇定的,如果矩阵 $\boldsymbol{P}$ 的谱范数 $\rho(\boldsymbol{P})<1$,且存在正常数 $\beta>0,\varepsilon>0$ 和常数 α 且 $|\alpha|\leqslant1$,以及矩阵 $\boldsymbol{K}$ 和正定矩阵 $\boldsymbol{E}>0,\boldsymbol{\Theta}>0,\boldsymbol{\Gamma}>0,\boldsymbol{\Delta}>0$ 使得以下矩阵不等式成立.

$$\Xi=\begin{bmatrix}\boldsymbol{Q}_{11} & \boldsymbol{Q}_{12} & \boldsymbol{B}_1 & \boldsymbol{Q}_{14} & \boldsymbol{Q}_{15} & 0\\ * & -\boldsymbol{E}-2\alpha\boldsymbol{P} & \boldsymbol{P}^{\mathrm{T}}\boldsymbol{B}_1 & -\alpha\boldsymbol{I} & -\alpha\boldsymbol{B}_1 & 0\\ * & * & \boldsymbol{Q}_{33} & \boldsymbol{B}_1^{\mathrm{T}} & \boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{B}_1 & 0\\ * & * & * & -\boldsymbol{\Gamma} & 0 & 0\\ * & * & * & * & -\boldsymbol{\Theta} & 0\\ * & * & * & * & * & -\boldsymbol{\Delta}\end{bmatrix}<0. \tag{8}$$

其中:

$$\boldsymbol{Q}_{11}=2\alpha\boldsymbol{I}-\boldsymbol{A}-\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{B}\boldsymbol{K}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}+\boldsymbol{E}+\beta\boldsymbol{I}+\tau^2\boldsymbol{\Gamma};$$
$$\boldsymbol{Q}_{12}=(\alpha\boldsymbol{I}-\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}})\boldsymbol{P}-\alpha\boldsymbol{I};$$
$$\boldsymbol{Q}_{14}=\alpha\boldsymbol{I}-\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}};$$
$$\boldsymbol{Q}_{15}=(\alpha\boldsymbol{I}-\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}})\boldsymbol{B}_1;$$
$$\boldsymbol{Q}_{33}=\boldsymbol{\Delta}-\beta\boldsymbol{I}+(\sigma-\tau)^2\boldsymbol{\Theta}.$$

证明 选取待定 Lyapunov-Krasovskii 泛函

$$V(t,\boldsymbol{x}_t)=V_1(t,\boldsymbol{x}_t)+V_2(t,\boldsymbol{x}_t),$$

其中,

$$\begin{aligned}V_1(t,\boldsymbol{x}_t)=&\mathscr{P}_2^{\mathrm{T}}(\boldsymbol{x}_t)\mathscr{P}_2(\boldsymbol{x}_t)+\\ &\int_{t-\tau}^{t}\boldsymbol{x}_t^{\mathrm{T}}(s)\boldsymbol{E}\boldsymbol{x}_t(s)\mathrm{d}s+\\ &\tau\int_{t-\tau}^{t}(\tau-t+s)(\alpha\boldsymbol{x}(s))^{\mathrm{T}}\boldsymbol{\Gamma}(\alpha\boldsymbol{x}(s))\mathrm{d}s+\\ &(\sigma-\tau)\int_{t-\sigma}^{t-\tau}(\sigma-t+s)\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(s))\boldsymbol{\Theta}\boldsymbol{f}(\boldsymbol{x}(s))\mathrm{d}s+\\ &\beta\int_{t-\tau}^{t}\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(s))\boldsymbol{f}(\boldsymbol{x}(s))\mathrm{d}s+\\ &\int_{t-\sigma}^{t-\tau}\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(s))\boldsymbol{\Delta}\boldsymbol{f}(\boldsymbol{x}(s))\mathrm{d}s,\end{aligned} \tag{9}$$

$$V_2(t,\boldsymbol{x}_t)=\varepsilon\mathscr{P}_1^{\mathrm{T}}(\boldsymbol{x}_t)\mathscr{P}_1(\boldsymbol{x}_t). \tag{10}$$

首先,求关于 V_1,V_2 沿着系统(7)轨线的导数,有

$$\begin{aligned}\left.\frac{\mathrm{d}V_1}{\mathrm{d}t}\right|_{(7)}=&2\Big[\boldsymbol{x}(t)+\boldsymbol{P}\boldsymbol{x}(t-\tau)+\alpha\int_{t-\tau}^{t}\boldsymbol{x}(s)\mathrm{d}s+\\ &\boldsymbol{B}_1\int_{t-\sigma}^{t-\tau}\boldsymbol{f}(\boldsymbol{x}(s))\mathrm{d}s\Big]^{\mathrm{T}}\Big[(\alpha\boldsymbol{I}-\boldsymbol{A}+\boldsymbol{B}\boldsymbol{K})\boldsymbol{x}(t)-\\ &\alpha\boldsymbol{x}(t-\tau)+\boldsymbol{B}_1\boldsymbol{f}(\boldsymbol{x}(t-\tau))\Big]+\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{E}\boldsymbol{x}(t)-\\ &\boldsymbol{x}^{\mathrm{T}}(t-\tau)\boldsymbol{E}\boldsymbol{x}(t-\tau)+\tau^2\alpha^2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{\Gamma}\boldsymbol{x}(t)-\\ &\tau\int_{t-\tau}^{t}(\alpha\boldsymbol{x}(s))^{\mathrm{T}}\boldsymbol{\Gamma}(\alpha\boldsymbol{x}(s))\mathrm{d}s+\\ &(\sigma-\tau)^2\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau))\boldsymbol{\Theta}\boldsymbol{f}(\boldsymbol{x}(t-\tau))-\\ &(\sigma-\tau)\int_{t-\sigma}^{t-\tau}\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(s))\boldsymbol{\Theta}\boldsymbol{f}(\boldsymbol{x}(s))\mathrm{d}s+\end{aligned}$$

$$\beta \boldsymbol{f}^{\mathrm T}(\boldsymbol{x}(t))\boldsymbol{f}(\boldsymbol{x}(t)) - \beta \boldsymbol{f}^{\mathrm T}(\boldsymbol{x}(t-\tau))\boldsymbol{f}(\boldsymbol{x}(t-\tau)) + \beta \boldsymbol{f}^{\mathrm T}(\boldsymbol{x}(t))\boldsymbol{f}(\boldsymbol{x}(t)) + \boldsymbol{f}^{\mathrm T}(\boldsymbol{x}(t-\tau))\Delta \boldsymbol{f}(\boldsymbol{x}(t-\tau)) - \boldsymbol{f}^{\mathrm T}(\boldsymbol{x}(t-\sigma))\boldsymbol{\Delta}\boldsymbol{f}(\boldsymbol{x}(t-\sigma)), \tag{11}$$

$$\frac{\mathrm d}{\mathrm dt}V_2(t,\boldsymbol{x}_t)\Big|_{(7)} = 2\varepsilon[\boldsymbol{x}(t) + \boldsymbol{P}\boldsymbol{x}(t-\tau)]^{\mathrm T}[(-\boldsymbol{A} + \boldsymbol{B}\boldsymbol{K})\boldsymbol{x}(t) + \boldsymbol{B}_1\boldsymbol{f}(\boldsymbol{x}(t-\sigma))]. \tag{12}$$

记

$$\boldsymbol{u}(t) = \alpha\int_{t-\tau}^{t}\boldsymbol{x}(s)\,\mathrm ds, \boldsymbol{v}(t) = \int_{t-\sigma}^{t-\tau}\boldsymbol{f}(\boldsymbol{x}(s))\,\mathrm ds.$$

再应用引理 1 及条件 $\|\boldsymbol{f}(\boldsymbol{x}(t)\| \leqslant \|\boldsymbol{x}(t)\|$, 可以得到

$$\begin{aligned}\frac{\mathrm dV_1}{\mathrm dt}\Big|_{(7)} \leqslant\ & \boldsymbol{x}^{\mathrm T}(t)[2\alpha\boldsymbol{I} - \boldsymbol{A} - \boldsymbol{A}^{\mathrm T} + \boldsymbol{B}\boldsymbol{K} + \boldsymbol{K}^{\mathrm T}\boldsymbol{B} + \boldsymbol{E} + \beta\boldsymbol{I} + \tau^2\boldsymbol{\Gamma}]\boldsymbol{x}(t) + 2\boldsymbol{x}^{\mathrm T}(t)[(\alpha\boldsymbol{I} - \boldsymbol{A} + \boldsymbol{B}\boldsymbol{K})^{\mathrm T}\boldsymbol{P} - \alpha\boldsymbol{I}]\boldsymbol{x}(t-\tau) + 2\boldsymbol{x}^{\mathrm T}(t)\boldsymbol{B}_1\boldsymbol{f}(\boldsymbol{x}(t-\tau)) + \\ & 2\boldsymbol{x}^{\mathrm T}(t)[\alpha\boldsymbol{I} - \boldsymbol{A} + \boldsymbol{B}\boldsymbol{K}]^{\mathrm T}\boldsymbol{u}(t) + 2\boldsymbol{x}^{\mathrm T}(t)(\alpha\boldsymbol{I} - \boldsymbol{A} + \boldsymbol{B}\boldsymbol{K})^{\mathrm T}\boldsymbol{B}_1\boldsymbol{v}(t) - \boldsymbol{x}^{\mathrm T}(t-\tau)[\boldsymbol{E} + 2\alpha\boldsymbol{P}]\boldsymbol{x}(t-\tau) + 2\boldsymbol{x}^{\mathrm T}(t-\tau)\boldsymbol{P}^{\mathrm T}\boldsymbol{B}_1\boldsymbol{f}(\boldsymbol{x}(t-\tau)) - \\ & 2\alpha\boldsymbol{x}^{\mathrm T}(t-\tau)\boldsymbol{u}(t) - 2a\boldsymbol{x}^{\mathrm T}(t-\tau)\boldsymbol{B}_1\boldsymbol{v}(t) + \boldsymbol{f}^{\mathrm T}(\boldsymbol{x}(t-\tau))[(\sigma-\tau)^2\boldsymbol{\Theta} - \beta\boldsymbol{I} + \boldsymbol{\Delta}]\cdot \boldsymbol{f}(\boldsymbol{x}(t-\tau)) + 2\boldsymbol{f}^{\mathrm T}(\boldsymbol{x}(t-\tau))\boldsymbol{B}_1{}^{\mathrm T}\boldsymbol{u}(t) + \\ & 2\boldsymbol{f}^{\mathrm T}(\boldsymbol{x}(t-\tau))\boldsymbol{B}_1{}^{\mathrm T}\boldsymbol{B}_1\boldsymbol{v}(t) - \boldsymbol{u}^{\mathrm T}(t)\boldsymbol{\Gamma}\boldsymbol{u}(t) - \boldsymbol{v}^{\mathrm T}(t)\boldsymbol{\Theta}\boldsymbol{v}(t) - \boldsymbol{f}^{\mathrm T}(\boldsymbol{x}(t-\sigma))\boldsymbol{\Delta}\boldsymbol{f}(\boldsymbol{x}(t-\sigma)) \triangleq \boldsymbol{\zeta}^{\mathrm T}(t)\boldsymbol{\Omega}\boldsymbol{\zeta}(t).\end{aligned} \tag{13}$$

其中:

$$\boldsymbol{\zeta}(t) = (\boldsymbol{x}^{\mathrm T}(t), \boldsymbol{x}^{\mathrm T}(t-\tau), \boldsymbol{f}^{\mathrm T}(\boldsymbol{x}(t-\tau)), \boldsymbol{u}^{\mathrm T}(t), \boldsymbol{v}^{\mathrm T}(t), \boldsymbol{f}^{\mathrm T}(\boldsymbol{x}(t-\sigma)))^{\mathrm T};$$

$$\boldsymbol{\Omega} = \begin{bmatrix} \boldsymbol{Q}_{11} & \boldsymbol{Q}_{12} & \boldsymbol{B}_1 & \boldsymbol{Q}_{14} & \boldsymbol{Q}_{15} & 0 \\ * & -\boldsymbol{E} - 2\alpha\boldsymbol{P} & \boldsymbol{P}^{\mathrm T}\boldsymbol{B}_1 & -\alpha\boldsymbol{I} & -\alpha\boldsymbol{B}_1 & 0 \\ * & * & \boldsymbol{Q}_{33} & \boldsymbol{B}_1^{\mathrm T} & \boldsymbol{B}_1^{\mathrm T}\boldsymbol{B}_1 & 0 \\ * & * & * & -\boldsymbol{\Gamma} & 0 & 0 \\ * & * & * & * & -\boldsymbol{\Theta} & 0 \\ * & * & * & * & * & -\boldsymbol{\Delta} \end{bmatrix}.$$

这里

$$\boldsymbol{Q}_{11} = 2\alpha\boldsymbol{I} - \boldsymbol{A} - \boldsymbol{A}^{\mathrm T} + \boldsymbol{B}\boldsymbol{K} + \boldsymbol{K}^{\mathrm T}\boldsymbol{B}^{\mathrm T} + \boldsymbol{E} + \beta\boldsymbol{I} + \tau^2\boldsymbol{\Gamma};$$

$$\boldsymbol{Q}_{12} = (\alpha\boldsymbol{I} - \boldsymbol{A}^{\mathrm T} + \boldsymbol{K}^{\mathrm T}\boldsymbol{B}^{\mathrm T})\boldsymbol{P} - \alpha\boldsymbol{I};$$

$$\boldsymbol{Q}_{14} = \alpha\boldsymbol{I} - \boldsymbol{A}^{\mathrm T} + \boldsymbol{K}^{\mathrm T}\boldsymbol{B}^{\mathrm T};$$

$$\boldsymbol{Q}_{15} = (\alpha\boldsymbol{I} - \boldsymbol{A}^{\mathrm T} + \boldsymbol{K}^{\mathrm T}\boldsymbol{B}^{\mathrm T})\boldsymbol{B}_1;$$

$$\boldsymbol{Q}_{33} = \boldsymbol{\Delta} - \beta\boldsymbol{I} + (\sigma-\tau)^2\boldsymbol{\Theta}.$$

根据定理条件知,矩阵 $\boldsymbol{\Omega} < 0$ 是负定的. 则由上述结论知,必存在正常数 $\lambda > 0$ 使得

$$\frac{\mathrm dV_1}{\mathrm dt}\Big|_{(7)} \leqslant -\lambda\boldsymbol{\zeta}^{\mathrm T}(t)\boldsymbol{\zeta}(t), \tag{14}$$

再由式(12),有

$$\begin{aligned}\frac{\mathrm d}{\mathrm dt}V_2(t,\boldsymbol{x}_t)\Big|_{(7)} =\ & 2\varepsilon[\boldsymbol{x}(t) + \boldsymbol{P}\boldsymbol{x}(t-\tau)]^{\mathrm T}[(-\boldsymbol{A} + \boldsymbol{B}\boldsymbol{K})\boldsymbol{x}(t) + \boldsymbol{B}_1\boldsymbol{f}(\boldsymbol{x}(t-\sigma))] = \\ & 2\varepsilon\boldsymbol{x}^{\mathrm T}(t)(-\boldsymbol{A} + \boldsymbol{B}\boldsymbol{K})\boldsymbol{x}(t) + 2\varepsilon\boldsymbol{x}^{\mathrm T}(t-\tau)\boldsymbol{P}^{\mathrm T}(-\boldsymbol{A} + \boldsymbol{B}\boldsymbol{K})\boldsymbol{x}(t) + 2\varepsilon\boldsymbol{x}^{\mathrm T}(t)\boldsymbol{B}_1\boldsymbol{f}(\boldsymbol{x}(t-\sigma)) + \\ & 2\varepsilon\boldsymbol{x}^{\mathrm T}(t-\tau)\boldsymbol{P}^{\mathrm T}\boldsymbol{B}_1\boldsymbol{f}(\boldsymbol{x}(t-\sigma)) \triangleq \varepsilon\boldsymbol{\zeta}^{\mathrm T}(t)\boldsymbol{\Sigma}\boldsymbol{\zeta}(t)\end{aligned}$$

其中,

$$\boldsymbol{\Sigma} = \begin{bmatrix} -2\boldsymbol{A} + 2\boldsymbol{B}\boldsymbol{K} & (-\boldsymbol{A}^{\mathrm T} + \boldsymbol{K}^{\mathrm T}\boldsymbol{B}^{\mathrm T})\boldsymbol{P} & 0 & 0 & 0 & \boldsymbol{B}_1 \\ * & 0 & 0 & 0 & 0 & \boldsymbol{P}^{\mathrm T}\boldsymbol{B}_1 \\ * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{bmatrix}.$$

令 $\lambda_1 = \lambda_{\max}(\Sigma)$, 选取 ε 如下

$$\varepsilon = \begin{cases} \dfrac{\lambda}{2\lambda_1}, & \lambda_1 > 0; \\ 1, & \lambda_1 \leqslant 0. \end{cases} \tag{16}$$

则有

$$\frac{\mathrm dV}{\mathrm dt}\Big|_{(7)} = \frac{\mathrm d}{\mathrm dt}(V_1 + V_2) < -\frac{\lambda}{2}\|\boldsymbol{\zeta}(t)\|^2 \leqslant -\frac{\lambda}{2}\|\boldsymbol{x}(t)\|^2. \tag{17}$$

另外,显然看到 $V(t,\boldsymbol{x}_t) \geqslant \varepsilon\mathscr{P}_1{}^{\mathrm T}(\boldsymbol{x}_t)\mathscr{P}_1(\boldsymbol{x}_t)$, 又由于 $\rho(\boldsymbol{P}) < 1$, 故算子 $\mathscr{P}_1(\boldsymbol{x}_t)$ 是稳定的. 因此,由文献[1]定理 8.1 知系统(7)或者闭环系统(6)是渐近稳定的. 故只要不等式(8)成立,便可由式(16)确定 ε, 从而由式(8)可以确定反馈增益矩阵 $\boldsymbol{K}$, 进而求得反馈镇定控制器(3).

3 H_∞ 反馈控制器的设计

Design of H_∞ feedback controller

定理 2 对于给定的正常数 $\tau > 0, \sigma > 0$, 且 $\tau \leqslant \sigma$ 及 $\gamma > 0$ 如果矩阵 $\boldsymbol{P}$ 的谱范数 $\rho(\boldsymbol{P}) < 1$, 且存在正常数 $\beta > 0$, 和常数 α, 且 $|\alpha| \leqslant 1$. 以及正定矩阵 $\boldsymbol{E} > 0, \boldsymbol{\Theta} > 0, \boldsymbol{\Gamma} > 0, \boldsymbol{\Delta} > 0$ 使得以下矩阵不等式成立.

$$\boldsymbol{\Omega}=\begin{bmatrix}\boldsymbol{\Omega}_{11} & \boldsymbol{\Omega}_{12} & \boldsymbol{W}^{\mathrm{T}} & \boldsymbol{B}_1 & \boldsymbol{\Omega}_{15} & \boldsymbol{\Omega}_{16} & 0 & \boldsymbol{W}^{\mathrm{T}}\boldsymbol{D}_1+\boldsymbol{B}_{\boldsymbol{\omega}} \\ * & \boldsymbol{\Omega}_{22} & \boldsymbol{C}_1^{\mathrm{T}} & \boldsymbol{P}^{\mathrm{T}}\boldsymbol{B}_1 & -\alpha\boldsymbol{I}_n & -\alpha\boldsymbol{B}_1 & 0 & \boldsymbol{P}^{\mathrm{T}}\boldsymbol{B}_{\boldsymbol{\omega}}+\boldsymbol{C}_1^{\mathrm{T}} \\ * & * & -\boldsymbol{I}_q & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \boldsymbol{\Omega}_{44} & \boldsymbol{B}_1 & \boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{B}_1 & 0 & 0 \\ * & * & * & * & -\boldsymbol{\Gamma} & 0 & 0 & \boldsymbol{B}_{\boldsymbol{\omega}} \\ * & * & * & * & * & -\boldsymbol{\Theta} & 0 & \boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{B}_{\boldsymbol{\omega}} \\ * & * & * & * & * & * & -\boldsymbol{\Delta} & 0 \\ * & * & * & * & * & * & * & \boldsymbol{D}_1^{\mathrm{T}}\boldsymbol{D}_1-\gamma^2 I_p\end{bmatrix}<0. \tag{18}$$

其中:$\boldsymbol{W}=\boldsymbol{C}+\boldsymbol{DK}$;

$$\boldsymbol{\Omega}_{11}=2\alpha\boldsymbol{I}-\boldsymbol{A}-\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{BK}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}+\boldsymbol{E}+\beta\boldsymbol{I}+\tau^2\boldsymbol{\Gamma};$$

$$\boldsymbol{\Omega}_{12}=(\alpha\boldsymbol{I}-\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}})\boldsymbol{P}-\alpha\boldsymbol{I};$$

$$\boldsymbol{\Omega}_{22}=-\boldsymbol{E}-2\alpha\boldsymbol{P};$$

$$\boldsymbol{\Omega}_{15}=\alpha\boldsymbol{I}-\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}};$$

$$\boldsymbol{\Omega}_{16}=(\alpha\boldsymbol{I}-\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}})\boldsymbol{B}_1;$$

$$\boldsymbol{\Omega}_{44}=\boldsymbol{\Delta}-\beta\boldsymbol{I}+(\sigma-\tau)^2\boldsymbol{\Theta}.$$

则(3)是具有给定的 H_∞ 扰动抑制水平 γ 的控制器.

利用 Lyapunov-Krasovskii 泛函(9),定义泛函

$$J_{z\omega}=\int_0^\infty[\boldsymbol{z}^{\mathrm{T}}(t)\boldsymbol{z}(t)-\gamma^2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{\omega}(t)]\mathrm{d}t, \tag{19}$$

则由于在零初值条件下有 $V_1(t,\boldsymbol{x}_t)|_{t=0}=0$,且 $V_1(t,\boldsymbol{x}_t)|_\infty\geqslant0$. 因此

$$J_{z\omega}\leqslant\int_0^\infty[\boldsymbol{z}^{\mathrm{T}}(t)\boldsymbol{z}(t)-\gamma^2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{\omega}(t)+\dot{V}_1(t,\boldsymbol{x}_t)]\mathrm{d}t$$

又

$$\begin{aligned}&\boldsymbol{z}^{\mathrm{T}}(t)\boldsymbol{z}(t)-\gamma^2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{\omega}(t)+\dot{V}_1(t,\boldsymbol{x}_t)= \\ &[(\boldsymbol{C}+\boldsymbol{DK})\boldsymbol{x}(t)+\boldsymbol{C}_1\boldsymbol{x}(t-\tau)+ \\ &\boldsymbol{D}_1\boldsymbol{\omega}(t)]^{\mathrm{T}}\times[(\boldsymbol{C}+\boldsymbol{DK})\boldsymbol{x}(t)+ \\ &\boldsymbol{C}_1\boldsymbol{x}(t-\tau)+\boldsymbol{D}_1\boldsymbol{\omega}(t)]- \\ &\gamma^2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{\omega}(t)+\dot{V}_1(t,\boldsymbol{x}_t)\leqslant \\ &\boldsymbol{x}^{\mathrm{T}}(t)[2\alpha\boldsymbol{I}-\boldsymbol{A}-\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{BK}+ \\ &\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}+\boldsymbol{E}+\beta\boldsymbol{I}+\tau^2\boldsymbol{\Gamma}]\boldsymbol{x}(t)+ \\ &2\boldsymbol{x}^{\mathrm{T}}(t)[(\alpha\boldsymbol{I}-\boldsymbol{A}+\boldsymbol{BK})^{\mathrm{T}}\boldsymbol{P}- \\ &\alpha\boldsymbol{I}]\boldsymbol{x}(t-\tau)+2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{B}_1\boldsymbol{f}(\boldsymbol{x}(t-\tau))+ \\ &2\boldsymbol{x}^{\mathrm{T}}(t)[\alpha\boldsymbol{I}-\boldsymbol{A}+\boldsymbol{BK}]^{\mathrm{T}}\boldsymbol{u}(t)+ \\ &2\boldsymbol{x}^{\mathrm{T}}(t)(\alpha\boldsymbol{I}-\boldsymbol{A}+\boldsymbol{BK})^{\mathrm{T}}\boldsymbol{B}_1\boldsymbol{v}(t)- \\ &\boldsymbol{x}^{\mathrm{T}}(t-\tau)[\boldsymbol{E}+2\alpha\boldsymbol{P}]\boldsymbol{x}(t-\tau)+ \\ &2\boldsymbol{x}^{\mathrm{T}}(t-\tau)\boldsymbol{P}^{\mathrm{T}}\boldsymbol{B}_1\boldsymbol{f}(\boldsymbol{x}(t-\tau))- \\ &2\alpha\boldsymbol{x}^{\mathrm{T}}(t-\tau)\boldsymbol{u}(t)- \\ &2\alpha\boldsymbol{x}^{\mathrm{T}}(t-\tau)\boldsymbol{B}_1\boldsymbol{v}(t)+ \\ &\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau))[(\sigma-\tau)^2\boldsymbol{\Theta}- \\ &\beta\boldsymbol{I}+\boldsymbol{\Delta}]\boldsymbol{f}(\boldsymbol{x}(t-\tau))+ \\ &2\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau))\boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{u}(t)+ \\ &2\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau))\boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{B}_1\boldsymbol{v}(t)- \\ &\boldsymbol{u}^{\mathrm{T}}(t)\boldsymbol{\Gamma}\boldsymbol{u}(t)-\boldsymbol{v}^{\mathrm{T}}(t)\boldsymbol{\Theta}\boldsymbol{v}(t)- \\ &\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\sigma))\boldsymbol{\Delta}\boldsymbol{f}(\boldsymbol{x}(t-\sigma))+ \\ &2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{B}_{\boldsymbol{\omega}}\boldsymbol{\omega}(t)+2\boldsymbol{x}^{\mathrm{T}}(t-\tau)\boldsymbol{P}^{\mathrm{T}}\boldsymbol{B}_{\boldsymbol{\omega}}\boldsymbol{\omega}(t)+ \\ &2\boldsymbol{u}^{\mathrm{T}}(t)\boldsymbol{B}_{\boldsymbol{\omega}}\boldsymbol{\omega}(t)+2\boldsymbol{v}^{\mathrm{T}}(t)\boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{B}_{\boldsymbol{\omega}}\boldsymbol{\omega}(t)+ \\ &[(\boldsymbol{C}+\boldsymbol{DK})\boldsymbol{x}(t)+\boldsymbol{C}_1\boldsymbol{x}(t-\tau)+ \\ &\boldsymbol{D}_1\boldsymbol{\omega}(t)]^{\mathrm{T}}[(\boldsymbol{C}+\boldsymbol{DK})\boldsymbol{x}(t)+ \\ &\boldsymbol{C}_1\boldsymbol{x}(t-\tau)+\boldsymbol{D}_1\boldsymbol{\omega}(t)]- \\ &\gamma^2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{\omega}(t),\end{aligned} \tag{20}$$

其中,

$$\boldsymbol{\zeta}_1(t)=(\boldsymbol{x}^{\mathrm{T}}(t),\boldsymbol{x}^{\mathrm{T}}(t-\tau),\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau)),\boldsymbol{u}^{\mathrm{T}}(t),\boldsymbol{v}^{\mathrm{T}}(t),\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\sigma)),\boldsymbol{\omega}^{\mathrm{T}}(t))^{\mathrm{T}},$$

所以

$$J_{z\omega}\leqslant\int_0^\infty\boldsymbol{\zeta}_1^{\mathrm{T}}(t)[\Xi^*+\Xi_1^*]\boldsymbol{\zeta}(t)\mathrm{d}t, \tag{21}$$

其中:

$$\Xi^*=\begin{bmatrix}\bar{\boldsymbol{Q}}_{11} & \boldsymbol{Q}_{12} & \boldsymbol{B}_1 & \boldsymbol{Q}_{14} & \boldsymbol{Q}_{15} & 0 & \boldsymbol{B}_{\boldsymbol{\omega}} \\ * & \boldsymbol{Q}_{22} & \boldsymbol{P}^{\mathrm{T}}\boldsymbol{B}_1 & -\alpha\boldsymbol{I}_n & -\alpha\boldsymbol{B}_1 & 0 & \boldsymbol{P}^{\mathrm{T}}\boldsymbol{B}_{\boldsymbol{\omega}} \\ * & * & \boldsymbol{Q}_{33} & \boldsymbol{B}_1^{\mathrm{T}} & \boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{B}_1 & 0 & 0 \\ * & * & * & -\boldsymbol{\Gamma} & 0 & 0 & \boldsymbol{B}_{\boldsymbol{\omega}} \\ * & * & * & * & -\boldsymbol{\Theta} & 0 & \boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{B}_{\boldsymbol{\omega}} \\ * & * & * & * & * & -\boldsymbol{\Delta} & 0 \\ * & * & * & * & * & * & 0\end{bmatrix};$$

$$\Xi_1^*=\begin{bmatrix}\boldsymbol{W}^{\mathrm{T}}\boldsymbol{W} & \boldsymbol{W}^{\mathrm{T}}\boldsymbol{C}_1 & 0 & 0 & 0 & 0 & \boldsymbol{W}^{\mathrm{T}}\boldsymbol{D}_1 \\ * & \boldsymbol{C}_1^{\mathrm{T}}\boldsymbol{C}_1 & 0 & 0 & 0 & 0 & \boldsymbol{C}_1^{\mathrm{T}}\boldsymbol{D}_1 \\ * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & \boldsymbol{D}_1^{\mathrm{T}}\boldsymbol{D}_1-\gamma^2\boldsymbol{I}_p\end{bmatrix}.$$

其中:$\boldsymbol{W}=\boldsymbol{C}+\boldsymbol{DK}$;

$$\bar{\boldsymbol{Q}}_{11}=2\alpha\boldsymbol{I}-\boldsymbol{A}-\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{BK}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}+\boldsymbol{E}+\beta\boldsymbol{I}+\tau^2\boldsymbol{\Gamma};$$

$$\boldsymbol{Q}_{12}=(\alpha\boldsymbol{I}-\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}})\boldsymbol{P}-\alpha\boldsymbol{I};$$

$$Q_{14} = \alpha I - A^{T} + K^{T}B^{T};$$
$$Q_{15} = (\alpha I - A^{T} + K^{T}B^{T})B_1;$$
$$Q_{22} = -E - 2\alpha P;$$
$$Q_{33} = \Delta - \beta I + (\sigma - \tau)^2\Theta.$$

而

$$\Xi^{*} + \Xi_1^{*} =$$
$$\begin{bmatrix} \bar{Q}_{11}+W^{T}W & Q_{12}+W^{T}C_1 & B_1 & Q_{14} & Q_{15} & 0 & W^{T}D_1+B_{\omega} \\ * & Q_{22}+C_1^{T}C_1 & P^{T}B_1-\alpha I_n-\alpha B_1 & 0 & P^{T}B_{\omega}+C_1^{T}D_1 \\ * & * & Q_{33} & B_1^{T} & B_1^{T}B_1 & 0 & 0 \\ * & * & * & -\Gamma & 0 & 0 & B_{\omega} \\ * & * & * & * & -\Theta & 0 & B_1^{T}B_{\omega} \\ * & * & * & * & * & -\Delta & 0 \\ * & * & * & * & * & * & D_1^{T}D_1-\gamma^2 I_p \end{bmatrix}, \quad (22)$$

其中：$W = C + DK$.

如果 $\Xi^{*} + \Xi_1^{*} < 0$，则由 Shur 补得到等价的不等式：

$$\begin{bmatrix} \bar{Q}_{11} & Q_{12} & W^{T} & B_1 & Q_{14} & Q_{15} & 0 & W^{T}D_1+B_{\omega} \\ * & Q_{22} & C_1^{T} & P^{T}B_1-\alpha I_n-\alpha B_1 & 0 & P^{T}B_{\omega}+C_1^{T} \\ * & * & -I_q & 0 & 0 & 0 & 0 & 0 \\ * & * & * & Q_{44} & B_1^{T} & B_1^{T}B_1 & 0 & 0 \\ * & * & * & * & -\Gamma & 0 & 0 & B_{\omega} \\ * & * & * & * & * & -\Theta & 0 & B_1^{T}B_{\omega} \\ * & * & * & * & * & * & -\Delta & 0 \\ * & * & * & * & * & * & * & D_1^{T}D_1-\gamma^2 I_p \end{bmatrix} < 0, \quad (23)$$

这里 $Q_{44} = Q_{33}$，I_r 表示 r 阶单位矩阵. 由式(23)得到 $\Omega < 0$，从而式(8)成立，故系统(7)是渐近稳定的. 另外，由 Shur 补得到式(23)等价于

$$\Omega =$$
$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & W^{T} & B_1 & \Omega_{15} & \Omega_{16} & 0 & W^{T}D_1+B_{\omega} \\ * & \Omega_{22} & C_1^{T} & P^{T}B_1-\alpha I_n-\alpha B_1 & 0 & P^{T}B_{\omega}+C_1^{T} \\ * & * & -I_q & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Omega_{44} & B_1^{T} & B_1^{T}B_1 & 0 & 0 \\ * & * & * & * & -\Gamma & 0 & 0 & B_{\omega} \\ * & * & * & * & * & -\Theta & 0 & B_1^{T}B_{\omega} \\ * & * & * & * & * & * & -\Delta & 0 \\ * & * & * & * & * & * & * & D_1^{T}D_1-\gamma^2 I_p \end{bmatrix} < 0. \quad (24)$$

其中：

$$\Omega_{11} = 2\alpha I - A - A^{T} + BK + K^{T}B^{T} + E + \beta I + \tau^2\Gamma;$$
$$\Omega_{12} = (\alpha I - A^{T} + K^{T}B^{T})P - \alpha I;$$
$$\Omega_{22} = -E - 2\alpha P;$$
$$\Omega_{15} = \alpha I - A^{T} + K^{T}B^{T};$$
$$\Omega_{16} = (\alpha I - A^{T} + K^{T}B^{T})B_1;$$
$$\Omega_{44} = \Delta - \beta I + (\sigma - \tau)^2\Theta.$$

式(24)就是式(18). 定理证毕.

4 H_∞ 反馈控制器的设计实例

An example in design of H_∞ feedback controller

考虑中立型系统

$$\begin{cases} \frac{d}{dt}[x(t) + Px(t-\tau)] = -Ax(t) + \\ \quad Bu(t) + B_1 f(x(t-\sigma)) + B_{\omega}\omega(t), \\ x(t) = \varphi(t), t \in [-\sigma, 0], \varphi \in C([-\sigma, 0], \mathbf{R}^n), \\ z(t) = Cx(t) + C_1 x(t-\tau) + Du(t) + D_1\omega(t). \end{cases} \quad (25)$$

其中：

$$P = \begin{bmatrix} -0.2 & 0.1 \\ 0.2 & -0.3 \end{bmatrix}, \quad A = \begin{bmatrix} 15 & -5 \\ 6 & -15 \end{bmatrix},$$
$$B = \begin{bmatrix} 5 \\ 50 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -0.2 & 1 \\ 0.02 & 0.1 \end{bmatrix},$$
$$B_{\omega} = \begin{bmatrix} 0.01 \\ 0.05 \end{bmatrix}, \quad C = [0.5 \quad -1.7],$$
$$C_1 = [-0.1 \quad -0.01],$$
$$D = -3.5, \quad D_1 = -1.7, \quad \tau = 0.1, \quad \sigma = 0.6.$$

通过式(18)，可以求得扰动抑制水平和 H_∞ 状态反馈控制器分别为

$$\gamma = 2.6146e^{+004}, \quad K = [-0.0572 \quad -1.6018].$$

5 结束语

Concluding remarks

本文中，讨论了一类非线性中立型时滞系统的 H-infinity 控制问题. 利用了 Lyapunov-Krasovs-kii 泛函以及 LMI 方法，获得了中立型系统的稳定性时滞相关的判据，且判据与时滞的相关性不仅依赖于哪一个单一的时滞，同时依赖于离散型时滞，也依赖于中立型时滞. 给出了 H-infinity 状态反馈控制器存在的时滞依赖的充分条件. 本文所采用的方法，在设计、求解控制器时不需要做矩阵变换，求解简单. 利用 Matlab 的 LMI 工具箱求解方便. 最后通过数值仿真实例说明了方法的有效性.

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On delay dependent criteria for H_∞ control of a class of nonlinear neutral systems

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Abstract In this paper,the problem of H-infinity control of a class of non-linear neutral delay systems is concerned. The appropriate Lyapunov-Krasovskii functional with undetermined parameters is employed. By use of LMI approach,the delay dependent criteria for asymptotic stability are obtained. The delay dependent sufficient conditions of the existence of state feedback controller for H-infinity control are given. The method used in this paper is simple and does not need the matrix transformation for designing and calculating the controller. The LMI tool box in Matlab can be conveniently employed to solve the controller. At the end of this paper a numerical example is provided to illustrate the feasibility of the conclusion.

Key words neutral type system;non-linearity;delay dependent;H-infinity control