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Modified explicit formulas for optimal LC prototype filter design

XUAN Xiuwei[1] DONG Jian[1] TENG Jianfu[1]

Abstract

The modified explicit formulas for Chebyshev and Butterworth LC low-pass filter design are given in this paper. These formulas fully utilize the passband tolerance obtained by rounding off the order and reduce the passband ripple without any change of the filter order, greatly facilitating the realization of the integrated filter. A design example for integrated active high-pass filters is presented and the effectiveness of the above-mentioned modified explicit formulas is verified.

Keywords

Chebyshev; LC filter design; explicit formulas; optimization

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作者简介

轩秀巍,女,硕士生,主要研究集成滤波器设计. xuanxiuwei@163.com

滕建辅(通信作者),男,教授,博士生导师,主要研究方向为滤波器设计和数字信号处理. jfteng@tju.edu.cn.

1 天津大学 电子信息工程学院,天津,300072

0 Introduction

Filters are important circuit modules in systems. They can be used in extensive aspects such as DSP systems, PLL, satellite communications, etc. With the development of system on chip, low sensitivity filter prototypes are required. Due to the low sensitivity characteristic of doubly terminated LC filters, they are chosen as design prototypes, especially for fully integrated active filters or RF filters. Performance demands for fully integrated active-RC filter have increased significantly during the last several years[1]. The active-RC filter for frequencies beyond 300 MHz has been successfully implemented[2]. Due to the element tolerance during the integration process, the LC prototype filter is normally overdesigned by reducing the passband ripple greatly; however, this will increase the order of the filter and hence the cost and complexity of the chip. In this paper, the modified explicit formulas for Chebyshev and Butterworth low-pass LC prototype filter design are given in such a way as to obtain passband optimized filters. The specifications can be satisfied at no order cost with the minimum passband tolerance.

The doubly terminated LC filter prototype can be designed by means of the filter design handbook and table method[3], or by using the CAD software, such as Filter Solutions[4]. However, for Butterworth and Chebyshev LC filter design, the explicit formulas has been derived, which simplifies the design process significantly.

The filter design can be divided into two phases, approximation and realization. The optimal use of some classical approximations in filter design has recently been proposed by Dimopoulos[5], and the design for optimum classical filters has been presented by Corral[6].

In low-pass filter design, given the maximum passband ripple $A_{\max}$, the minimum attenuation $A_{\min}$ in stopband, the passband edge frequency ω_p, the stopband edge frequency ω_s, the required order is firstly calculated. The evaluated number is normally a noninteger, it is rounded up to next higher integer as filter order. This number is used to approximate the transfer function. In the rounding up process, some surplus has been produced. Dimopoulos[5] uses such a surplus for optimum transfer function approximation. Ivan W. Selesnick once described a modification of the

Parks-McClellan algorithm where either the passband or the stopband ripple size is specified and the other is minimized[7]. In this paper, the modified explicit formulas for optimum Butterworth and Chebyshev LC filter design are given. At no order cost under the formulas presented, the specifications can be improved with the minimum passband ripple. Lastly, this proposed method is used to design integrated filters with Operational Amplifier AD8138[8].

1 Explicit formulas for optimal low-pass filter

In Chebyshev's case, filter attenuation can be calculated by

$$A = 10\ \lg|H(j\omega)|^2 = 1 - \lg\left[1 + \varepsilon^2 C_n^2\left(\frac{\omega}{\omega_p}\right)\right]. \quad (1)$$

where C_n is nth-order Chebyshev polynomial of the first kind.

Given $A_{\max}, A_{\min}, \omega_p, \omega_s$, the required order can be calculated as

$$n = \frac{\operatorname{arcosh}[(10^{0.1A_{\min}} - 1)/(10^{0.1A_{\max}} - 1)]^{1/2}}{\operatorname{arcosh}(\omega_s/\omega_p)}. \quad (2)$$

Then, the degree of the filter is obtained by rounding up n to next higher integer, so-called filter order N.

The optimal idea in this paper arises from the difference of $N - n$, which can be changed into advantage of lower passband ripple, i. e., reducing the passband ripple. Using smaller $A_{\max}^{\text{new}}$ instead of $A_{\max}$ yields

$$N = \frac{\operatorname{arcosh}[(10^{0.1A_{\min}} - 1)/(10^{0.1A_{\max}^{\text{new}}} - 1)]^{1/2}}{\operatorname{arcosh}(\omega_s/\omega_p)}. \quad (3)$$

Thus, reducing ripple parameter $A_{\max}^{\text{new}}$ is achieved at no order cost.

$$A_{\max}^{\text{new}} = 10\ \lg\left(\frac{10^{0.1A_{\min}} - 1}{\{\cosh[N\operatorname{arcosh}(\omega_s/\omega_p)]\}^2} + 1\right). \quad (4)$$

So the optimized filter parameters are obtained as $A_{\max}^{\text{new}}, A_{\min}, \omega_p, \omega_s$.

Once the parameter $A_{\max}^{\text{new}}$ has been derived, the proposed explicit formulas[9] for the optimal Chebyshev LC low-pass filter design can be modified as follows:

$$\varepsilon = \sqrt{10^{0.1A_{\max}^{\text{new}}} - 1}, \quad h = \sqrt[N]{\frac{1}{\varepsilon} + \sqrt{1 + \frac{1}{\varepsilon^2}}},$$

$$\xi = \left(h - \frac{1}{h}\right), \quad C_1 = \frac{4\sin(\pi/2N)}{\xi R_1}.$$

$$C_{2k-1}L_{2k} = \frac{16\sin\left(\frac{4k-3}{2N}\pi\right)\sin\left(\frac{4k-1}{2N}\pi\right)}{\xi^2 + 4\sin^2\left(\frac{2k-1}{N}\pi\right)}.$$

$$C_{2k+1}L_{2k} = \frac{16\sin\left(\frac{4k-1}{2N}\pi\right)\sin\left(\frac{4k+1}{2N}\pi\right)}{\xi^2 + 4\sin^2\left(\frac{2k}{N}\pi\right)}. \quad (5)$$

For $k = 1, 2, \cdots, N/2$.

Where $A_{\max}^{\text{new}}$ is used instead of $A_{\max}$.

The end elements (L or C) are also given by (5).

$$C_N = \frac{4\sin(\pi/2N)}{\xi R_2}, \quad (N \quad \text{odd}),$$

$$L_N = \frac{4R_2\sin(\pi/2N)}{\xi}, \quad (N \quad \text{even}). \quad (6)$$

Therefore solving it for R_2 yields:

$$R_2 = \frac{4\sin(\pi/2N)}{\xi C_N} \equiv R_1, \quad (N \quad \text{odd}),$$

$$R_2 = \frac{\xi L_N}{4\sin(\pi/2N)} \neq R_1, \quad (N \quad \text{even}). \quad (7)$$

The magnitude and frequency scaling can be performed as follows:

Let $k_m = R_1$ (Given), $k_f = \omega_p = 2\pi \times f_p$, then

$$L_{\text{new}} = \frac{k_m}{k_f}L_{\text{old}} \text{ and } C_{\text{new}} = \frac{C_{\text{old}}}{k_f k_m}. \quad (8)$$

Similarly, for Butterworth filter design, $A_{\max}^{\text{new}}$ can be calculated as:

$$A_{\max}^{\text{new}} = 10\ \lg[1 + (10^{0.1A_{\min}} - 1)/(\omega_s/\omega_p)^{2N}]$$

where N is the order of the filter. Explicit formulas for optimal Butterworth LC low-pass filter design can be modified as follows:

$$\varepsilon = \sqrt{10^{0.1A_{\max}^{\text{new}}} - 1},\ L_k \text{ or } C_k = 2\varepsilon^{\frac{1}{N}}\sin\frac{2k-1}{2N}\pi.$$

Here, $k = 1, 2, \ldots, N$.

As to denormalization, let $k_m = R_1$, $k_f = \omega_p$, then L_{new} and C_{new} can be calculated as (8).

2 A design example

Design an integrated high-pass filter; the specifications are as follows:

Passband $f_p = 2$ MHz, $A_{\max} = 1$ dB,

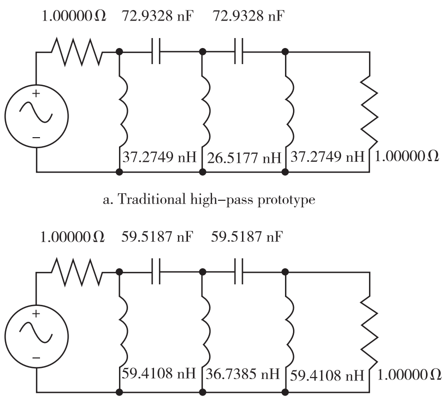
Stopband $f_s = 1$ MHz, $A_{\min} = 35$ dB,

Load resistor $R_S = R_L = 1$ kΩ.

Select Chebyshev approximation; from (2), it can be calculated: $n = 41$, and then n is rounded up to $N = 5$. From (3),

$$A_{\max}^{\text{new}} = 10\ \lg\left(\frac{10^{0.1A_{\min}} - 1}{\left\{\cosh\left[N\operatorname{arcosh}\left(\frac{\omega_s}{\omega_p}\right)\right]\right\}^2} + 1\right) = 0.102\ 535.$$

Considering element tolerance during the integration, let $A_{max}^{new} = 0.2$ dB. By the use of (5), (6) and (7), the normalized element values can be easily calculated. Then through frequency transform formulas, $L_i = 1/(2\pi f_p C_i)$ and $C_i = 1/(2\pi f_p L_i)$, we can get the optimal LC circuit for high-pass filter which is shown in Fig. 1b, and for comparison purpose, the traditional LC circuit is shown in Fig. 1a.



a. Traditional high-pass prototype

b. Optimal high-pass prototype

Fig. 1 LC circuit of high-pass prototype

Figure 2 indicates the passband simulation of the high-pass prototype filter. The line with a large ripple is for the traditional filter with the passband ripple A_{max}, and the line with a small ripple is for the optimal filter with the obtained passband ripple A_{max}^{new}. The simulation shows that the passband ripple is much smaller by using modified explicit formulas.

The high-pass filter can be converted into the integrated active filter. Here, we let $k_m = 1\ 000$ and utilize $R_{new} = k_m R_{old}$, $C_{new} = C_{old}/k_m$, for denormalization. After integrator scaling and denormalization, the high-pass integrated active circuits are shown in Figure 3.

The magnitude response of the integrated high-pass filter circuits is shown in Figure 4. It can be seen that up to 10 MHz, the ripple is not more than 1dB. The detailed passband response is shown in Figure 5. The line with a larger ripple is the simulation of the integrated filter based on Figure 1a, while the line with a smaller ripple is the simulation of the integrated filter based on Figure 1b. From Figure 5, we can see that the modified explicit formulas can achieve the much lower passband ripple.

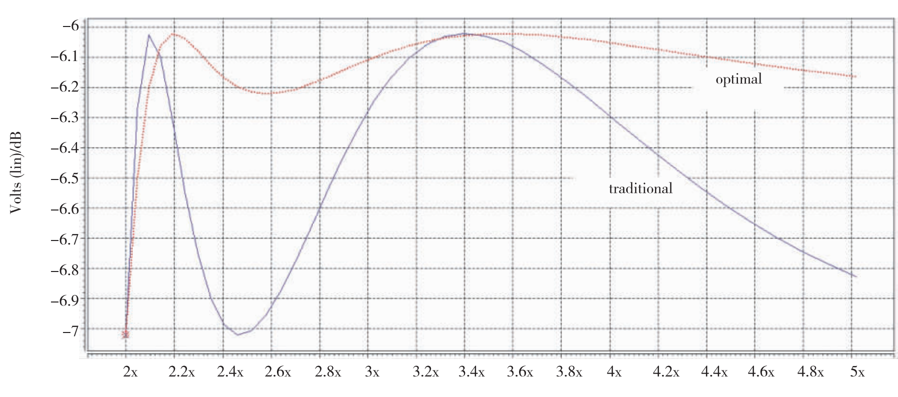


Fig. 2 Comparison of passband ripple

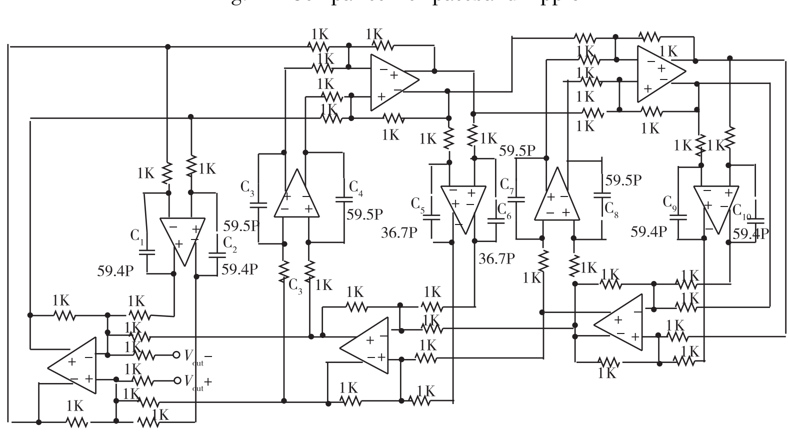


Fig. 3 Active integrated high-pass filter circuit

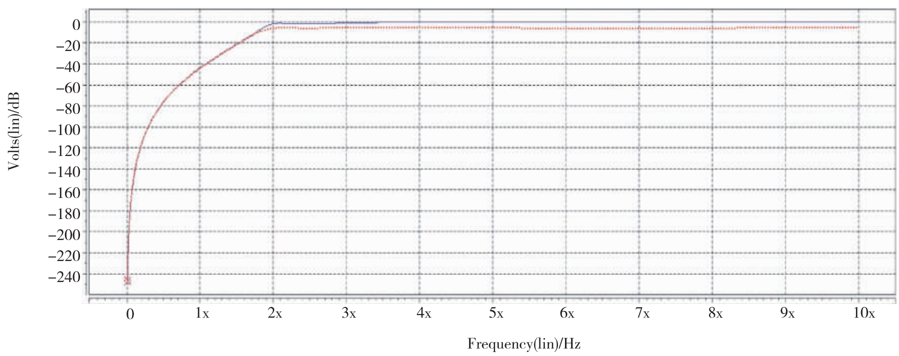


Fig. 4 Simulation of integrated filter

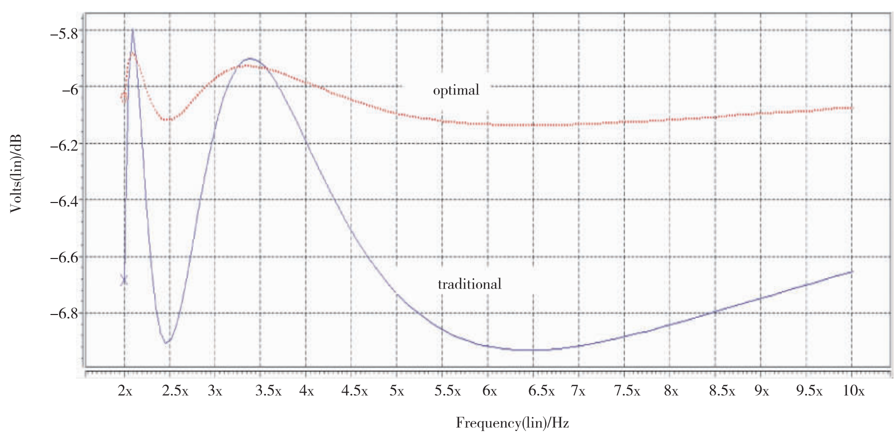


Fig. 5 Passband simulation of integrated filter

3 Conclusion

In this paper the modified explicit formulas for doubly terminated LC Chebyshev and Butterworth low-pass filter design are presented along with their optimum passband behavior, i. e. , a lower passband ripple at no order cost. The proposed method can be used to design integrated active RC filters. The simulation result indicates that the passband characteristic of the filter is improved by the optimized design.

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LC 滤波器设计的优化解析公式

轩秀巍[1] 董健[1] 滕建辅[1]
1 天津大学 电子信息工程学院,天津 300072

摘要 给出了改进的 Chebyshev 和 Butterworth 低通原型 LC 滤波器的解析设计公式. 此公式充分利用阶数取整带来的余量,在不改变滤波器阶数的前提下减小了通带波动,极大地方便了集成滤波器的实现. 给出了集成有源高通滤波器设计实例,验证了改进的 Chebyshev 和 Butterworth 低通原型 LC 滤波器解析设计公式的有效性.

关键词 Chebyshev; LC 滤波器设计; 显式; 最优化