文章编号:1674-7070(2010)06-0495-08

不确定奇异系统的时滞相关鲁棒 H_∞ 滤波器设计

海泉[1]　包俊东[1]

摘要

为了设计出全阶的滤波器,同时使得滤波误差动态系统是正则的、无脉冲的,并且满足一定的 H_∞ 性能指标,研究了连续奇异系统的全阶 H_∞ 滤波器的设计问题. 利用 Lyapunov-Krasovskii 泛函及二次型的积分不等式方法获得了滤波误差动态系统的 H_∞ 性能时滞相关的判据,给出了奇异系统的鲁棒 H_∞ 滤波器存在的时滞依赖的充分条件. 最后的数值例子说明了该方法的有效性.

关键词

奇异系统;不确定性;时滞相关判据;鲁棒 H_∞ 滤波

中图分类号 TP13

文献标志码 A

0　引言

Introduction

近年来,系统的 H_∞ 滤波器设计问题被许多研究者关注[1-3]. H_∞ 滤波器对于干扰噪声没有统计特性上的要求,而且拥有更好的鲁棒性[4-5],这些良好的特性也使得它在实际应用中有着很好的前景. 许多文献中都提出了有关 H_∞ 滤波器的研究[6-8].

由于奇异系统在电力系统的建模和控制及在经济上和其他领域中的广泛应用,使它得到了深入的研究. 关于奇异系统的滤波问题也被许多学者所研究[9-10]. 文献[11]给出了不确定时滞系统的鲁棒 H_∞ 保性能滤波,但该文中需要假定时滞变化率小于 1. 本文由文献[11]研究结果得到启发,讨论了不确定时滞奇异系统的鲁棒 H_∞ 滤波器设计问题. 通过 Lyapunov-Krasovskii 泛函及二次型的积分不等式方法获得了滤波误差动态系统的 H_∞ 性能时滞相关的判据,给出了奇异系统的鲁棒 H_∞ 滤波器存在的时滞依赖的充分条件. 值得指出的是,本文所提的方法没有对时变时滞项进行 $d_i(t) < 1(i = 1,2)$ 的限制.

本文采用以下记号:如果 $\boldsymbol{X}$ 是对称矩阵,$\boldsymbol{X} \geqslant 0$($\boldsymbol{X} > 0$,$\boldsymbol{X} < 0$,$\boldsymbol{X} \leqslant 0$)表示 $\boldsymbol{X}$ 为半正定矩阵(正定矩阵,负定矩阵,半负定矩阵);矩阵 $\boldsymbol{M}^{\mathrm{T}}$ 表示矩阵 $\boldsymbol{M}$ 的转置矩阵;"$*$" 表示对称矩阵的主对角线上块矩阵的转置;$\boldsymbol{I}$ 和 $\boldsymbol{0}$ 分别表示适当阶数的单位矩阵和零矩阵.

1　问题描述

Description of the problem

考虑下述不确定性时滞奇异系统:

$$\begin{cases} \boldsymbol{E}\dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \Delta\boldsymbol{A})\boldsymbol{x}(t) + (\boldsymbol{A}_1 + \Delta\boldsymbol{A}_1)\boldsymbol{x}(t - d_1(t)) + \\ \qquad (\boldsymbol{A}_2 + \Delta\boldsymbol{A}_2)\boldsymbol{x}(t - d_2(t)) + \boldsymbol{B}\boldsymbol{\omega}(t), \\ \boldsymbol{y}(t) = (\boldsymbol{C} + \Delta\boldsymbol{C})x(t) + (\boldsymbol{C}_1 + \Delta\boldsymbol{C}_1)\boldsymbol{x}(t - d_1(t)) + \\ \qquad (\boldsymbol{C}_2 + \Delta\boldsymbol{C}_2)\boldsymbol{x}(t - d_2(t)) + \boldsymbol{D}\boldsymbol{\omega}(t), \\ \boldsymbol{z}(t) = \boldsymbol{H}\boldsymbol{x}(t), \\ \boldsymbol{x}(t) = \boldsymbol{\varphi}_1(t), \quad \forall t \in [-d, 0]. \end{cases} \tag{1}$$

式中:$\boldsymbol{x}(t) \in \mathbf{R}^n$、$\boldsymbol{y}(t) \in \mathbf{R}^m$、$\boldsymbol{z}(t) \in \mathbf{R}^q$ 分别为系统的状态、测量控制输出和待估计的信号;$\boldsymbol{\omega}(t) \in \mathbf{R}^P$ 为 $L_2[0,\infty)$ 空间内的干扰输入;$\boldsymbol{E}$、$\boldsymbol{A}$、$\boldsymbol{A}_1$、$\boldsymbol{A}_2$、$\boldsymbol{B}$、$\boldsymbol{C}$、$\boldsymbol{C}_1$、$\boldsymbol{C}_2$、$\boldsymbol{D}$、$\boldsymbol{H}$ 为适当维数的已知实常阵,其中 $\boldsymbol{E}$ 可能是

收稿日期 2010-06-08

资助项目 内蒙古自然科学基金(2010MS0109)

作者简介

海泉,男,硕士,助教,从事奇异系统、鲁棒控制与滤波研究. baohq@ imnu. edu. cn

包俊东,男,教授,主要从事时滞系统、随机系统的稳定与控制研究. baohq@ immu. edu. cn

1　内蒙古师范大学　数学科学学院,呼和浩特,010022

奇异的,不失一般性,假设 $\mathrm{rank}\boldsymbol{E} = r \leqslant n$;$\Delta\boldsymbol{A}$、$\Delta\boldsymbol{A}_1$、$\Delta\boldsymbol{A}_2$、$\Delta\boldsymbol{C}$、$\Delta\boldsymbol{C}_1$、$\Delta\boldsymbol{C}_2$ 是具有适当维数的不确定时变矩阵,假设具有如下形式:

$$\begin{bmatrix} \Delta\boldsymbol{A} & \Delta\boldsymbol{A}_1 & \Delta\boldsymbol{A}_2 \\ \Delta\boldsymbol{C} & \Delta\boldsymbol{C}_1 & \Delta\boldsymbol{C}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{L}_1 \\ \boldsymbol{L}_2 \end{bmatrix} \boldsymbol{F}(t) [\boldsymbol{M}_1 \quad \boldsymbol{M}_2 \quad \boldsymbol{M}_3]. \tag{2}$$

其中:$\boldsymbol{L}_1$、$\boldsymbol{L}_2$、$\boldsymbol{M}_1$、$\boldsymbol{M}_2$、$\boldsymbol{M}_3$ 是具有适当维数的已知实常阵;$\boldsymbol{F}(t)$ 为有界不确定函数阵,满足

$$\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{F}(t) \leqslant \boldsymbol{I}. \tag{3}$$

$\boldsymbol{\varphi}_1(t) \in C([-d,0],\mathbf{R}^n)$ 函数,表示系统的初始状态;$d_i(t)(i = 1,2)$ 为系统的可微的有界时变时滞,且满足

$$0 < d_1(t) < d_2(t) < d,$$
$$0 < \dot{d}_i(t) < d^*, \quad i = 1,2. \tag{4}$$

首先给出关于奇异系统的几个定义. 对于奇异系统

$$\boldsymbol{E}\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) \tag{5}$$

有如下引理和定义:

引理 1[12] 系统(5)正则的、无脉冲的充分必要条件是存在矩阵 $\boldsymbol{Q} \in \mathbf{R}^{n\times n}$,使得

$$\boldsymbol{E}^{\mathrm{T}}\boldsymbol{Q} = \boldsymbol{Q}^{\mathrm{T}}\boldsymbol{E} \geqslant 0, \quad \boldsymbol{A}^{\mathrm{T}}\boldsymbol{Q} + \boldsymbol{Q}^{\mathrm{T}}\boldsymbol{A} < 0.$$

定义 1[12]

1) 奇异系统(5)是正则、无脉冲的,

2) 奇异系统(5)是 Lyapunov 渐近稳定的,

则奇异系统(5)是鲁棒稳定的.

现在考虑如下关于信号 $\boldsymbol{z}(t)$ 估计的全阶滤波器:

$$\begin{cases} \dot{\hat{\boldsymbol{x}}}(t) = \boldsymbol{G}\hat{\boldsymbol{x}}(t) + \boldsymbol{K}\boldsymbol{y}(t), \quad \hat{\boldsymbol{x}}_0 = 0; \\ \hat{\boldsymbol{z}}(t) = \hat{\boldsymbol{H}}\hat{\boldsymbol{x}}(t) + \hat{\boldsymbol{D}}\boldsymbol{y}(t). \end{cases} \tag{6}$$

其中 $\boldsymbol{G}$、$\boldsymbol{K}$、$\hat{\boldsymbol{H}}$、$\hat{\boldsymbol{D}}$ 为待定的滤波器参数矩阵.

定义增广状态向量

$$\begin{cases} \tilde{\boldsymbol{x}}^{\mathrm{T}}(t) = [\boldsymbol{x}^{\mathrm{T}}(t),\hat{\boldsymbol{x}}^{\mathrm{T}}(t)]; \\ \tilde{\boldsymbol{z}}(t) = \boldsymbol{z}(t) - \hat{\boldsymbol{z}}(t). \end{cases} \tag{7}$$

且令

$$\widetilde{\boldsymbol{A}} = \boldsymbol{A} + \Delta\boldsymbol{A}, \quad \widetilde{\boldsymbol{A}}_1 = \boldsymbol{A}_1 + \Delta\boldsymbol{A}_1,$$
$$\widetilde{\boldsymbol{A}}_2 = \boldsymbol{A}_2 + \Delta\boldsymbol{A}_2, \quad \widetilde{\boldsymbol{C}} = \boldsymbol{C} + \Delta\boldsymbol{C},$$
$$\widetilde{\boldsymbol{C}}_1 = \boldsymbol{C}_1 + \Delta\boldsymbol{C}_1, \quad \widetilde{\boldsymbol{C}}_2 = \boldsymbol{C}_2 + \Delta\boldsymbol{C}_2. \tag{8}$$

则由式(1)与式(6)得到的滤波误差动态系统就可以表示为以下形式:

$$\begin{cases} \overline{\boldsymbol{E}}\dot{\tilde{\boldsymbol{x}}}(t) = \overline{\boldsymbol{A}}\tilde{\boldsymbol{x}}(t) + \overline{\boldsymbol{A}}_1\tilde{\boldsymbol{x}}(t - d_1(t)) + \\ \qquad \overline{\boldsymbol{A}}_2\tilde{\boldsymbol{x}}(t - d_2(t)) + \overline{\boldsymbol{B}}\boldsymbol{\omega}(t); \\ \tilde{\boldsymbol{z}}(t) = \overline{\boldsymbol{C}}\tilde{\boldsymbol{x}}(t) + \overline{\boldsymbol{C}}_1\tilde{\boldsymbol{x}}(t - d_1(t)) + \\ \qquad \overline{\boldsymbol{C}}_2\tilde{\boldsymbol{x}}(t - d_2(t)) + \overline{\boldsymbol{D}}\boldsymbol{\omega}(t); \\ \tilde{\boldsymbol{x}}^{\mathrm{T}}(t) = [\boldsymbol{\varphi}^{\mathrm{T}}(t),0], \quad \forall t \in [-d,0]. \end{cases} \tag{9}$$

其中:

$$\overline{\boldsymbol{E}} = \begin{bmatrix} \boldsymbol{E} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix}; \quad \overline{\boldsymbol{A}} = \begin{bmatrix} \widetilde{\boldsymbol{A}} & \boldsymbol{0} \\ \boldsymbol{K}\widetilde{\boldsymbol{C}} & \boldsymbol{G} \end{bmatrix}; \quad \overline{\boldsymbol{A}}_1 = \begin{bmatrix} \widetilde{\boldsymbol{A}}_1 & \boldsymbol{0} \\ \boldsymbol{K}\widetilde{\boldsymbol{C}}_1 & \boldsymbol{0} \end{bmatrix};$$
$$\overline{\boldsymbol{A}}_2 = \begin{bmatrix} \widetilde{\boldsymbol{A}}_2 & \boldsymbol{0} \\ \boldsymbol{K}\widetilde{\boldsymbol{C}}_2 & \boldsymbol{0} \end{bmatrix}; \quad \overline{\boldsymbol{B}} = \begin{bmatrix} \boldsymbol{B} \\ \boldsymbol{K}\boldsymbol{D} \end{bmatrix};$$
$$\overline{\boldsymbol{C}} = [\boldsymbol{H} - \hat{\boldsymbol{D}}\widetilde{\boldsymbol{C}} \quad -\hat{\boldsymbol{H}}]; \quad \overline{\boldsymbol{C}}_1 = [-\hat{\boldsymbol{D}}\widetilde{\boldsymbol{C}}_1 \quad \boldsymbol{0}];$$
$$\overline{\boldsymbol{C}}_1 = [-\hat{\boldsymbol{D}}\widetilde{\boldsymbol{C}}_1 \quad \boldsymbol{0}]; \quad \overline{\boldsymbol{D}} = -\hat{\boldsymbol{D}}\boldsymbol{D}. \tag{10}$$

引入一个性能指标

$$\boldsymbol{J} = \int_0^\infty [\tilde{\boldsymbol{z}}^{\mathrm{T}}(t)\tilde{\boldsymbol{z}}(t) - \gamma^2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{\omega}(t)]\mathrm{d}t. \tag{11}$$

定义 2 考虑不确定时滞奇异系统(1)及性能指标式(11),如果对给定的衰减度 $\gamma > 0$ 和任意满足式(2)和(4)的参数不确定性,存在一个形如式(6)的滤波器使得

1) 滤波误差动态系统(9)是正则、无脉冲,

2) 滤波误差动态系统(9)渐近稳定,

3) H_∞ 性能指标满足 $\boldsymbol{J} < 0$,

则称滤波器(6)是系统(1)的一个鲁棒 H_∞ 滤波器.

本文的研究目的是为系统(1)设计一个形如(6)的全阶鲁棒 H_∞ 滤波器.

引理 2[13] 给定适当维数矩阵 $\boldsymbol{\Omega}$、$\boldsymbol{\Gamma}$ 和 $\boldsymbol{\Xi}$,其中 $\boldsymbol{\Omega}$ 是对称的,则

$$\boldsymbol{\Omega} + \boldsymbol{\Gamma}\boldsymbol{F}(t)\boldsymbol{\Xi} + \boldsymbol{\Xi}^{\mathrm{T}}\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{\Gamma}^{\mathrm{T}} < 0$$

对所有满足 $\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{F}(t) \leqslant \boldsymbol{I}$ 的矩阵 $\boldsymbol{F}(t)$ 成立,当且仅当存在一个常数 $\varepsilon > 0$,使得

$$\boldsymbol{\Omega} + \varepsilon^{-1}\boldsymbol{\Gamma}\boldsymbol{\Gamma}^{\mathrm{T}} + \varepsilon\boldsymbol{\Xi}^{\mathrm{T}}\boldsymbol{\Xi} < 0.$$

引理 3[13](Schur 补引理) 对给定的对称矩阵 $\boldsymbol{S} = \begin{bmatrix} \boldsymbol{S}_{11} & \boldsymbol{S}_{12} \\ \boldsymbol{S}_{21} & \boldsymbol{S}_{22} \end{bmatrix}$,其中 $\boldsymbol{S}_{11} \in \mathbf{R}^{r\times r}$,以下 3 个条件是等价的:

1) $\boldsymbol{S} < 0$;

2) $\boldsymbol{S}_{11} < 0, \boldsymbol{S}_{22} - \boldsymbol{S}_{12}^{\mathrm{T}}\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{12} < 0$;

3) $\boldsymbol{S}_{22} < 0, \boldsymbol{S}_{11} - \boldsymbol{S}_{12}\boldsymbol{S}_{22}^{-1}\boldsymbol{S}_{12}^{\mathrm{T}} < 0$.

2 鲁棒 H_∞ 性能分析

Robust H_∞ performance analysis

首先给出滤波误差动态系统的鲁棒 H_∞ 性能时滞相关的判据. 为此,引入如下的变量:

$\boldsymbol{\eta}^{\mathrm{T}}(t) = [\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\tilde{\boldsymbol{x}}^{\mathrm{T}}(t - d_1(t))\tilde{\boldsymbol{x}}^{\mathrm{T}}(t - d_2(t))\boldsymbol{\omega}^{\mathrm{T}}(t)]$,

那么

$$\overline{\boldsymbol{E}}\dot{\tilde{\boldsymbol{x}}}(t) = [\overline{\boldsymbol{A}} \quad \overline{\boldsymbol{A}}_1 \quad \overline{\boldsymbol{A}}_2 \quad \overline{\boldsymbol{B}}]\boldsymbol{\eta}(t).$$

对向量 $\boldsymbol{\eta}(t)$ 和 $\dot{\tilde{\boldsymbol{x}}}(t)$,引入下面的积分不等式,该积分不等式在后面的定理证明中起到重要的作用.

引理 4[14](积分不等式) 设 $\tilde{\boldsymbol{x}}(t) \in \mathbf{R}^{2n}$ 具有一阶连续导数,则对任意已知的常数矩阵 $\boldsymbol{N}_1 \in \mathbf{R}^{n\times n}$,$\boldsymbol{N}_2 \in \mathbf{R}^{n\times n}$,$\boldsymbol{N}_3 \in \mathbf{R}^{n\times n}$,$\boldsymbol{W} \in \boldsymbol{R}^{n\times p}$,时变时滞 $d_1(t) > 0$,$d_2(t) > 0$,有以下积分不等式成立:

$$-\int_{t-d_2(t)}^{t-d_1(t)} \dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}(s)\overline{\boldsymbol{E}}^{\mathrm{T}}\overline{\boldsymbol{E}}\dot{\tilde{\boldsymbol{x}}}(s)\,\mathrm{d}s \leqslant$$

$$\boldsymbol{\eta}^{\mathrm{T}}(t)[\boldsymbol{\Pi} + (d_2(t) - d_1(t))\boldsymbol{Y}^{\mathrm{T}}\boldsymbol{Y}]\boldsymbol{\eta}(t).$$

其中:

$$\boldsymbol{Y} = [\overline{\boldsymbol{N}}_1 \quad \overline{\boldsymbol{N}}_2 \quad \overline{\boldsymbol{N}}_3 \quad \overline{\boldsymbol{W}}];\quad \overline{\boldsymbol{N}}_1 = \begin{bmatrix} \boldsymbol{N}_1 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix};$$

$$\overline{\boldsymbol{N}}_2 = \begin{bmatrix} \boldsymbol{N}_2 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix};\quad \overline{\boldsymbol{N}}_3 = \begin{bmatrix} \boldsymbol{N}_3 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix};\quad \overline{\boldsymbol{W}} = \begin{bmatrix} \boldsymbol{W} \\ \boldsymbol{0} \end{bmatrix};$$

$$\boldsymbol{\Pi} = \begin{bmatrix} \boldsymbol{0} & \overline{\boldsymbol{N}}_1^{\mathrm{T}}\overline{\boldsymbol{E}} & -\overline{\boldsymbol{N}}_1^{\mathrm{T}}\overline{\boldsymbol{E}} & \boldsymbol{0} \\ * & \overline{\boldsymbol{N}}_2^{\mathrm{T}}\overline{\boldsymbol{E}} + \overline{\boldsymbol{E}}^{\mathrm{T}}\overline{\boldsymbol{N}}_2 & \overline{\boldsymbol{E}}^{\mathrm{T}}\overline{\boldsymbol{N}}_3 - \overline{\boldsymbol{N}}_2^{\mathrm{T}}\overline{\boldsymbol{E}} & \overline{\boldsymbol{E}}^{\mathrm{T}}\overline{\boldsymbol{W}} \\ * & * & -\overline{\boldsymbol{N}}_3^{\mathrm{T}}\overline{\boldsymbol{E}} - \overline{\boldsymbol{E}}^{\mathrm{T}}\overline{\boldsymbol{N}}_3 & -\overline{\boldsymbol{E}}^{\mathrm{T}}\overline{\boldsymbol{W}} \\ * & * & * & \boldsymbol{0} \end{bmatrix}.$$

根据引理4,以下定理给出了滤波误差动态系统(9)鲁棒 H_∞ 性能分析的充分条件.

定理 1 对任意满足(4)的时变时滞 $d_i(t)(i = 1,2)$ 和任意给定的常数 $\gamma > 0$,如果存在正定矩阵 $\boldsymbol{P} > 0$、$\boldsymbol{Q}_1 > 0$、$\boldsymbol{Q}_2 > 0$ 和矩阵 $\overline{\boldsymbol{S}}$、$\overline{\boldsymbol{N}}_1$、$\overline{\boldsymbol{N}}_2$、$\overline{\boldsymbol{N}}_3$、$\overline{\boldsymbol{W}}$ 使得:

$$\boldsymbol{\Omega} = \begin{bmatrix} \boldsymbol{\Omega}_{11} & \boldsymbol{\Omega}_{12} & \boldsymbol{\Omega}_{13} & \boldsymbol{\Omega}_{14} & \overline{\boldsymbol{N}}_1^{\mathrm{T}} & \overline{\boldsymbol{A}}^{\mathrm{T}} & \overline{\boldsymbol{C}}^{\mathrm{T}} \\ * & \boldsymbol{\Omega}_{22} & \boldsymbol{\Omega}_{23} & \overline{\boldsymbol{E}}^{\mathrm{T}}\overline{\boldsymbol{W}} & \overline{\boldsymbol{N}}_2^{\mathrm{T}} & \overline{\boldsymbol{A}}_1^{\mathrm{T}} & \overline{\boldsymbol{C}}_1^{\mathrm{T}} \\ * & * & \boldsymbol{\Omega}_{33} & -\overline{\boldsymbol{E}}^{\mathrm{T}}\overline{\boldsymbol{W}} & \overline{\boldsymbol{N}}_3^{\mathrm{T}} & \overline{\boldsymbol{A}}_2^{\mathrm{T}} & \overline{\boldsymbol{C}}_2^{\mathrm{T}} \\ * & * & * & -\gamma^2\boldsymbol{I} & \overline{\boldsymbol{W}}^{\mathrm{T}} & \overline{\boldsymbol{B}}^{\mathrm{T}} & \overline{\boldsymbol{D}}^{\mathrm{T}} \\ * & * & * & * & -\bar{d}\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & * & * & -\bar{d}\boldsymbol{I} & \boldsymbol{0} \\ * & * & * & * & * & * & -\boldsymbol{I} \end{bmatrix} < \boldsymbol{0}, \quad (12)$$

其中:

$$\boldsymbol{\Omega}_{11} = \overline{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}\overline{\boldsymbol{E}} + \overline{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\overline{\boldsymbol{A}} + \overline{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{R}\overline{\boldsymbol{S}}^{\mathrm{T}} + \overline{\boldsymbol{S}}\boldsymbol{R}^{\mathrm{T}}\overline{\boldsymbol{A}} + \boldsymbol{Q}_1 + \boldsymbol{Q}_2,$$

$$\boldsymbol{\Omega}_{12} = \overline{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\overline{\boldsymbol{A}}_1 + \overline{\boldsymbol{S}}\boldsymbol{R}^{\mathrm{T}}\overline{\boldsymbol{A}}_1 + \overline{\boldsymbol{N}}_1^{\mathrm{T}}\overline{\boldsymbol{E}},$$

$$\boldsymbol{\Omega}_{13} = \overline{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\overline{\boldsymbol{A}}_2 + \overline{\boldsymbol{S}}\boldsymbol{R}^{\mathrm{T}}\overline{\boldsymbol{A}}_2 - \overline{\boldsymbol{N}}_1^{\mathrm{T}}\overline{\boldsymbol{E}},$$

$$\boldsymbol{\Omega}_{14} = \overline{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\overline{\boldsymbol{B}} + \overline{\boldsymbol{S}}\boldsymbol{R}^{\mathrm{T}}\overline{\boldsymbol{B}},$$

$$\boldsymbol{\Omega}_{22} = -(1 - d^*)\boldsymbol{Q}_1 + \overline{\boldsymbol{N}}_2^{\mathrm{T}}\overline{\boldsymbol{E}} + \overline{\boldsymbol{E}}^{\mathrm{T}}\overline{\boldsymbol{N}}_2,$$

$$\boldsymbol{\Omega}_{23} = \overline{\boldsymbol{E}}^{\mathrm{T}}\overline{\boldsymbol{N}}_3 - \overline{\boldsymbol{N}}_2^{\mathrm{T}}\overline{\boldsymbol{E}},$$

$$\boldsymbol{\Omega}_{33} = -(1 - d^*)\boldsymbol{Q}_2 - \overline{\boldsymbol{E}}^{\mathrm{T}}\overline{\boldsymbol{N}}_3 - \overline{\boldsymbol{N}}_3^{\mathrm{T}}\overline{\boldsymbol{E}},$$

且 $\overline{\boldsymbol{R}} \in \overline{\mathbf{R}}^{2n\times(n-r)}$ 为任意满足 $\overline{\boldsymbol{E}}^{\mathrm{T}}\overline{\boldsymbol{R}} = 0$ 的列满秩矩阵,则滤波误差动态系统(9)为鲁棒稳定且满足 H_∞ 性能 γ.

证明 先证明滤波误差动态系统(9)的正则、无脉冲性. 令 $\boldsymbol{T} = \boldsymbol{P}\overline{\boldsymbol{E}} + \overline{\boldsymbol{R}}\overline{\boldsymbol{S}}^{\mathrm{T}}$,由 $\overline{\boldsymbol{E}}^{\mathrm{T}}\overline{\boldsymbol{R}} = 0$ 得

$$\overline{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{T} = \boldsymbol{T}^{\mathrm{T}}\overline{\boldsymbol{E}} \geqslant 0.$$

由于 $\boldsymbol{\Omega} < 0$ 且 $\boldsymbol{Q}_1 > 0$、$\boldsymbol{Q}_2 > 0$ 很容易得到以下不等式成立:

$$\overline{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}\overline{\boldsymbol{E}} + \overline{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\overline{\boldsymbol{A}} + \overline{\boldsymbol{A}}^{\mathrm{T}}\overline{\boldsymbol{R}}\overline{\boldsymbol{S}}^{\mathrm{T}} + \overline{\boldsymbol{S}}\overline{\boldsymbol{R}}^{\mathrm{T}}\overline{\boldsymbol{A}} = \overline{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{T} + \boldsymbol{T}^{\mathrm{T}}\overline{\boldsymbol{A}} < 0.$$

由引理 1 得到滤波误差动态系统(9)的正则、无脉冲性.

下面,证明滤波误差动态系统(9)渐近稳定.

取如下 Lyapunov-Krasovskii 泛函

$$V(\tilde{\boldsymbol{x}}(t)) = V_1(\tilde{\boldsymbol{x}}(t)) + V_2(\tilde{\boldsymbol{x}}(t)) + V_3(\tilde{\boldsymbol{x}}(t)) + V_4(\tilde{\boldsymbol{x}}(t)). \quad (13)$$

其中:

$$V_1(\tilde{\boldsymbol{x}}(t)) = \tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\overline{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\overline{\boldsymbol{E}}\tilde{\boldsymbol{x}}(t);$$

$$V_2(\tilde{\boldsymbol{x}}(t)) = \int_{t-d_1(t)}^{t} \tilde{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{Q}_1\tilde{\boldsymbol{x}}(s)\,\mathrm{d}s;$$

$$V_3(\tilde{\boldsymbol{x}}(t)) = \int_{t-d_2(t)}^{t} \tilde{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{Q}_2\tilde{\boldsymbol{x}}(s)\,\mathrm{d}s;$$

$$V_4(\tilde{\boldsymbol{x}}(t)) = \int_{-d}^{0}\int_{t+\theta}^{t} \dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}(s)\overline{\boldsymbol{E}}^{\mathrm{T}}\overline{\boldsymbol{E}}\dot{\tilde{\boldsymbol{x}}}(s)\,\mathrm{d}s\mathrm{d}\theta.$$

则 $V(\tilde{\boldsymbol{x}}(t))$ 沿着滤波误差动态系统(9)的导数为

$$\dot{V}(\tilde{\boldsymbol{x}}(t)) = \dot{V}_1(\tilde{\boldsymbol{x}}(t)) + \dot{V}_2(\tilde{\boldsymbol{x}}(t)) + \dot{V}_3(\tilde{\boldsymbol{x}}(t)) + \dot{V}_4(\tilde{\boldsymbol{x}}(t)). \quad (14)$$

其中:

$$\dot{V}_1(\tilde{\boldsymbol{x}}(t)) =$$

$$\dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}(t)\overline{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\overline{\boldsymbol{E}}\tilde{\boldsymbol{x}}(t) + \tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\overline{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\overline{\boldsymbol{E}}\dot{\tilde{\boldsymbol{x}}}(t) =$$

$$\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)(\overline{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}\overline{\boldsymbol{E}} + \overline{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\overline{\boldsymbol{A}})\tilde{\boldsymbol{x}}(t) + 2\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\overline{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\overline{\boldsymbol{A}}_1\tilde{\boldsymbol{x}}(t - d_1(t)) +$$

$$2\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\overline{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\overline{\boldsymbol{A}}_2\tilde{\boldsymbol{x}}(t - d_2(t)) + 2\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\overline{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\overline{\boldsymbol{B}}\boldsymbol{\omega}(t);$$

$$\dot{V}_2(\tilde{\boldsymbol{x}}(t)) =$$

$$\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Q}_1\tilde{\boldsymbol{x}}(t) - (1 - \dot{d}_1(t))\tilde{\boldsymbol{x}}^{\mathrm{T}}(t - d_1(t))\boldsymbol{Q}_1\tilde{\boldsymbol{x}}(t - d_1(t));$$

$$\dot{V}_3(\tilde{\boldsymbol{x}}(t)) =$$

$$\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Q}_2\tilde{\boldsymbol{x}}(t) - (1 - \dot{d}_2(t))\tilde{\boldsymbol{x}}^{\mathrm{T}}(t - d_2(t))\boldsymbol{Q}_2\tilde{\boldsymbol{x}}(t - d_2(t));$$

$$\dot{V}_4(\tilde{\boldsymbol{x}}(t)) =$$

$$d\dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}(t)\overline{\boldsymbol{E}}^{\mathrm{T}}\overline{\boldsymbol{E}}\dot{\tilde{\boldsymbol{x}}}(t) - \int_{t-d}^{t} \dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}(s)\overline{\boldsymbol{E}}^{\mathrm{T}}\overline{\boldsymbol{E}}\dot{\tilde{\boldsymbol{x}}}(s)\,\mathrm{d}s \leqslant$$

$$d\dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}(t)\overline{\boldsymbol{E}}^{\mathrm{T}}\overline{\boldsymbol{E}}\dot{\tilde{\boldsymbol{x}}}(t) - \int_{t-d_2(t)}^{t-d_1(t)} \dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}(s)\overline{\boldsymbol{E}}^{\mathrm{T}}\overline{\boldsymbol{E}}\dot{\tilde{\boldsymbol{x}}}(s)\,\mathrm{d}s =$$

$$d\dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}(t)\overline{\boldsymbol{A}}^{\mathrm{T}}\overline{\boldsymbol{A}}\tilde{\boldsymbol{x}}(t) + 2d\dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}(t)\overline{\boldsymbol{A}}^{\mathrm{T}}\overline{\boldsymbol{A}}_1\tilde{\boldsymbol{x}}(t - d_1(t)) +$$

$$2d\dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}(t)\overline{\boldsymbol{A}}^{\mathrm{T}}\overline{\boldsymbol{A}}_2\tilde{\boldsymbol{x}}(t - d_2(t)) + 2d\dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}(t)\overline{\boldsymbol{A}}^{\mathrm{T}}\overline{\boldsymbol{B}}\boldsymbol{\omega}(t) +$$

$$d\dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}(t - d_1(t))\overline{\boldsymbol{A}}_1^{\mathrm{T}}\overline{\boldsymbol{A}}_1\tilde{\boldsymbol{x}}(t - d_1(t)) +$$

$$2d\dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}(t - d_1(t))\overline{\boldsymbol{A}}_1^{\mathrm{T}}\overline{\boldsymbol{A}}_2\tilde{\boldsymbol{x}}(t - d_2(t)) +$$

$$2d\dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}(t - d_1(t))\overline{\boldsymbol{A}}_1^{\mathrm{T}}\overline{\boldsymbol{B}}\boldsymbol{\omega}(t) +$$

$$d\dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}(t - d_2(t))\overline{\boldsymbol{A}}_2^{\mathrm{T}}\overline{\boldsymbol{A}}_2\tilde{\boldsymbol{x}}(t - d_2(t)) +$$

$$2d\dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}(t - d_2(t))\overline{\boldsymbol{A}}_2^{\mathrm{T}}\overline{\boldsymbol{B}}\boldsymbol{\omega}(t) + d\boldsymbol{\omega}^{\mathrm{T}}(t)\overline{\boldsymbol{B}}^{\mathrm{T}}\overline{\boldsymbol{B}}\boldsymbol{\omega}(t) -$$

$$\int_{t-d_2(t)}^{t-d_1(t)} \dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}(s)\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{E}}\dot{\tilde{\boldsymbol{x}}}(s)\,\mathrm{d}s.$$

另外,注意到 $\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{R}} = 0$, 容易得到

$$0 = 2\dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}(t)\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{R}}\bar{\boldsymbol{S}}^{\mathrm{T}}\tilde{\boldsymbol{x}}(t) = 2[\bar{\boldsymbol{A}}\tilde{\boldsymbol{x}}(t) + \bar{\boldsymbol{A}}_1\tilde{\boldsymbol{x}}(t-d_1(t)) + \bar{\boldsymbol{A}}_2\tilde{\boldsymbol{x}}(t-d_2(t)) + \bar{\boldsymbol{B}}\boldsymbol{\omega}(t)]^{\mathrm{T}}\bar{\boldsymbol{R}}\bar{\boldsymbol{S}}^{\mathrm{T}}\tilde{\boldsymbol{x}}(t). \tag{15}$$

因此,根据式(15)和引理(4)可得

$$\begin{aligned}
&\dot{V}(\tilde{\boldsymbol{x}}(t)) = \dot{V}_1(\tilde{\boldsymbol{x}}(t)) + \dot{V}_2(\tilde{\boldsymbol{x}}(t)) + \dot{V}_3(\tilde{\boldsymbol{x}}(t)) + \dot{V}_4(\tilde{\boldsymbol{x}}(t)) \leqslant\\
&\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)[\bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{E}} + \bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{A}} + \bar{\boldsymbol{A}}^{\mathrm{T}}\bar{\boldsymbol{R}}\bar{\boldsymbol{S}}^{\mathrm{T}} + \bar{\boldsymbol{S}}\bar{\boldsymbol{R}}^{\mathrm{T}}\bar{\boldsymbol{A}} + \boldsymbol{Q}_1 + \boldsymbol{Q}_2 +\\
&\quad d\bar{\boldsymbol{N}}_1^{\mathrm{T}}\bar{\boldsymbol{N}}_1 + d\bar{\boldsymbol{A}}^{\mathrm{T}}\bar{\boldsymbol{A}}]\tilde{\boldsymbol{x}}(t) +\\
&\quad 2\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)[\bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{A}}_1 + \bar{\boldsymbol{S}}\bar{\boldsymbol{R}}^{\mathrm{T}}\bar{\boldsymbol{A}}_1 + \bar{\boldsymbol{N}}_1^{\mathrm{T}}\bar{\boldsymbol{E}} + d\bar{\boldsymbol{A}}^{\mathrm{T}}\bar{\boldsymbol{A}}_1 +\\
&\quad d\bar{\boldsymbol{N}}_1^{\mathrm{T}}\bar{\boldsymbol{N}}_2]\tilde{\boldsymbol{x}}(t-d_1(t)) +\\
&\quad 2\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)[\bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{A}}_2 + \bar{\boldsymbol{S}}\bar{\boldsymbol{R}}^{\mathrm{T}}\bar{\boldsymbol{A}}_2 - \bar{\boldsymbol{N}}_1^{\mathrm{T}}\bar{\boldsymbol{E}} + d\bar{\boldsymbol{A}}^{\mathrm{T}}\bar{\boldsymbol{A}}_2 +\\
&\quad d\bar{\boldsymbol{N}}_1^{\mathrm{T}}\bar{\boldsymbol{N}}_3]\tilde{\boldsymbol{x}}(t-d_2(t)) +\\
&\quad 2\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)[\bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{B}} + \bar{\boldsymbol{S}}\bar{\boldsymbol{R}}^{\mathrm{T}}\bar{\boldsymbol{B}} + d\bar{\boldsymbol{A}}^{\mathrm{T}}\bar{\boldsymbol{B}} + d\bar{\boldsymbol{N}}_1^{\mathrm{T}}\bar{\boldsymbol{W}}]\boldsymbol{\omega}(t) +\\
&\quad \tilde{\boldsymbol{x}}^{\mathrm{T}}(t-d_1(t))[-(1-d^*)\boldsymbol{Q}_1 + \bar{\boldsymbol{N}}_2^{\mathrm{T}}\bar{\boldsymbol{E}} + \bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{N}}_2 +\\
&\quad d\bar{\boldsymbol{A}}_1^{\mathrm{T}}\bar{\boldsymbol{A}}_1 + d\bar{\boldsymbol{N}}_2^{\mathrm{T}}\bar{\boldsymbol{N}}_2]\tilde{\boldsymbol{x}}(t-d_1(t)) +\\
&\quad 2\tilde{\boldsymbol{x}}^{\mathrm{T}}(t-d_1(t))[\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{N}}_3 - \bar{\boldsymbol{N}}_2^{\mathrm{T}}\bar{\boldsymbol{E}} + d\bar{\boldsymbol{A}}_1^{\mathrm{T}}\bar{\boldsymbol{A}}_2 +\\
&\quad d\bar{\boldsymbol{N}}_2^{\mathrm{T}}\bar{\boldsymbol{N}}_3]\tilde{\boldsymbol{x}}^{\mathrm{T}}(t-d_2(t)) +\\
&\quad 2\tilde{\boldsymbol{x}}^{\mathrm{T}}(t-d_1(t))[\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{W}} + d\bar{\boldsymbol{A}}_1^{\mathrm{T}}\bar{\boldsymbol{B}} + d\bar{\boldsymbol{N}}_2^{\mathrm{T}}\bar{\boldsymbol{W}}]\boldsymbol{\omega}(t) +\\
&\quad \tilde{\boldsymbol{x}}^{\mathrm{T}}(t-d_2(t))[-(1-d^*)\boldsymbol{Q}_2 - \bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{N}}_3 - \bar{\boldsymbol{N}}_3^{\mathrm{T}}\bar{\boldsymbol{E}} +\\
&\quad d\bar{\boldsymbol{A}}_2^{\mathrm{T}}\bar{\boldsymbol{A}}_2 + d\bar{\boldsymbol{N}}_3^{\mathrm{T}}\bar{\boldsymbol{N}}_3]\tilde{\boldsymbol{x}}(t-d_2(t)) +\\
&\quad 2\tilde{\boldsymbol{x}}^{\mathrm{T}}(t-d_2(t))[-\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{W}} + d\bar{\boldsymbol{A}}_2^{\mathrm{T}}\bar{\boldsymbol{B}} +\\
&\quad d\bar{\boldsymbol{N}}_3^{\mathrm{T}}\bar{\boldsymbol{W}}]\boldsymbol{\omega}(t) + \boldsymbol{\omega}^{\mathrm{T}}(t)[d\bar{\boldsymbol{B}}^{\mathrm{T}}\bar{\boldsymbol{B}} +\\
&\quad d\bar{\boldsymbol{W}}^{\mathrm{T}}\bar{\boldsymbol{W}}]\boldsymbol{\omega}(t).
\end{aligned} \tag{16}$$

当 $\boldsymbol{\omega}(t) = 0$ 时,可得

$$\dot{\boldsymbol{V}}(\boldsymbol{\xi}(t)) \leqslant \boldsymbol{v}^{\mathrm{T}}(t)\boldsymbol{\Sigma}\boldsymbol{v}(t). \tag{17}$$

其中:

$$\boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} & \boldsymbol{\Sigma}_{13} \\ * & \boldsymbol{\Sigma}_{22} & \boldsymbol{\Sigma}_{23} \\ * & * & \boldsymbol{\Sigma}_{33} \end{bmatrix};$$

$$\boldsymbol{v}(t) = [\tilde{\boldsymbol{x}}^{\mathrm{T}}(t), \tilde{\boldsymbol{x}}^{\mathrm{T}}(t-d_1(t)), \tilde{\boldsymbol{x}}^{\mathrm{T}}(t-d_2(t))]^{\mathrm{T}};$$

$$\boldsymbol{\Sigma}_{11} = \bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{E}} + \bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{A}} + \bar{\boldsymbol{A}}^{\mathrm{T}}\bar{\boldsymbol{R}}\bar{\boldsymbol{S}}^{\mathrm{T}} + \bar{\boldsymbol{S}}\bar{\boldsymbol{R}}^{\mathrm{T}}\bar{\boldsymbol{A}} + \boldsymbol{Q}_1 + \boldsymbol{Q}_2 + d\bar{\boldsymbol{N}}_1^{\mathrm{T}}\bar{\boldsymbol{N}}_1 + d\bar{\boldsymbol{A}}^{\mathrm{T}}\bar{\boldsymbol{A}};$$

$$\boldsymbol{\Sigma}_{12} = \bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{A}}_1 + \bar{\boldsymbol{S}}\bar{\boldsymbol{R}}^{\mathrm{T}}\bar{\boldsymbol{A}}_1 + \bar{\boldsymbol{N}}_1^{\mathrm{T}}\bar{\boldsymbol{E}} + d\bar{\boldsymbol{A}}^{\mathrm{T}}\bar{\boldsymbol{A}}_1 + d\bar{\boldsymbol{N}}_1^{\mathrm{T}}\bar{\boldsymbol{N}}_2;$$

$$\boldsymbol{\Sigma}_{13} = \bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{A}}_2 + \bar{\boldsymbol{S}}\bar{\boldsymbol{R}}^{\mathrm{T}}\bar{\boldsymbol{A}}_2 - \bar{\boldsymbol{N}}_1^{\mathrm{T}}\bar{\boldsymbol{E}} + d\bar{\boldsymbol{A}}^{\mathrm{T}}\bar{\boldsymbol{A}}_2 + d\bar{\boldsymbol{N}}_1^{\mathrm{T}}\bar{\boldsymbol{N}}_3;$$

$$\boldsymbol{\Sigma}_{22} = -(1-d^*)\boldsymbol{Q}_1 + \bar{\boldsymbol{N}}_2^{\mathrm{T}}\bar{\boldsymbol{E}} + \bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{N}}_2 + d\bar{\boldsymbol{A}}_1^{\mathrm{T}}\bar{\boldsymbol{A}}_1 + d\bar{\boldsymbol{N}}_2^{\mathrm{T}}\bar{\boldsymbol{N}}_2;$$

$$\boldsymbol{\Sigma}_{23} = \bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{N}}_3 - \bar{\boldsymbol{N}}_2^{\mathrm{T}}\bar{\boldsymbol{E}} + d\bar{\boldsymbol{A}}_1^{\mathrm{T}}\bar{\boldsymbol{A}}_2 + d\bar{\boldsymbol{N}}_2^{\mathrm{T}}\bar{\boldsymbol{N}}_3;$$

$$\boldsymbol{\Sigma}_{33} = -(1-d^*)\boldsymbol{Q}_2 - \bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{N}}_3 - \bar{\boldsymbol{N}}_3^{\mathrm{T}}\bar{\boldsymbol{E}} + d\bar{\boldsymbol{A}}_2^{\mathrm{T}}\bar{\boldsymbol{A}}_2 + d\bar{\boldsymbol{N}}_3^{\mathrm{T}}\bar{\boldsymbol{N}}_3.$$

且 $\bar{\boldsymbol{R}} \in \mathbf{R}^{2n\times(n-r)}$ 为任意满足 $\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{R}} = 0$ 的列满秩矩阵.

如果 $\boldsymbol{\Sigma} < 0$, 则由 Schur 补引理得到等价的不等式:

$$\boldsymbol{\Gamma} = \begin{bmatrix} \boldsymbol{\Gamma}_{11} & \boldsymbol{\Gamma}_{12} & \boldsymbol{\Gamma}_{13} & \bar{\boldsymbol{N}}_1^{\mathrm{T}} & \bar{\boldsymbol{A}}^{\mathrm{T}} \\ * & \boldsymbol{\Gamma}_{22} & \boldsymbol{\Gamma}_{23} & \bar{\boldsymbol{N}}_2^{\mathrm{T}} & \bar{\boldsymbol{A}}_1^{\mathrm{T}} \\ * & * & \boldsymbol{\Gamma}_{33} & \bar{\boldsymbol{N}}_2^{\mathrm{T}} & \bar{\boldsymbol{A}}_2^{\mathrm{T}} \\ * & * & * & -\bar{d}\boldsymbol{I} & \boldsymbol{0} \\ * & * & * & * & -\bar{d}\boldsymbol{I} \end{bmatrix} < 0. \tag{18}$$

其中:

$$\begin{aligned}
\boldsymbol{\Gamma}_{11} &= \bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{E}} + \bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{A}} + \bar{\boldsymbol{A}}^{\mathrm{T}}\bar{\boldsymbol{R}}\bar{\boldsymbol{S}}^{\mathrm{T}} + \bar{\boldsymbol{S}}\bar{\boldsymbol{R}}^{\mathrm{T}}\bar{\boldsymbol{A}} + \boldsymbol{Q}_1 + \boldsymbol{Q}_2;\\
\boldsymbol{\Gamma}_{12} &= \bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{A}}_1 + \bar{\boldsymbol{S}}\bar{\boldsymbol{R}}^{\mathrm{T}}\bar{\boldsymbol{A}}_1 + \bar{\boldsymbol{N}}_1^{\mathrm{T}}\bar{\boldsymbol{E}};\\
\boldsymbol{\Gamma}_{13} &= \bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{A}}_2 + \bar{\boldsymbol{S}}\bar{\boldsymbol{R}}^{\mathrm{T}}\bar{\boldsymbol{A}}_2 - \bar{\boldsymbol{N}}_1^{\mathrm{T}}\bar{\boldsymbol{E}};\\
\boldsymbol{\Gamma}_{22} &= -(1-d^*)\boldsymbol{Q}_1 + \bar{\boldsymbol{N}}_2^{\mathrm{T}}\bar{\boldsymbol{E}} + \bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{N}}_2;\\
\boldsymbol{\Gamma}_{23} &= \bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{N}}_3 - \bar{\boldsymbol{N}}_2^{\mathrm{T}}\bar{\boldsymbol{E}};\\
\boldsymbol{\Gamma}_{33} &= -(1-d^*)\boldsymbol{Q}_2 - \bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{N}}_3 - \bar{\boldsymbol{N}}_3^{\mathrm{T}}\bar{\boldsymbol{E}};\quad \bar{d} = d^{-1}.
\end{aligned}$$

且 $\bar{\boldsymbol{R}} \in \mathbf{R}^{2n\times(n-r)}$ 为任意满足 $\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{R}} = 0$ 的列满秩矩阵. 由高等代数的知识可知,如果 $\boldsymbol{\Omega} < 0$,则 $\boldsymbol{\Gamma} < 0$. 即式(14)是滤波误差动态系统(9)渐近稳定的充分条件.

下面证明滤波误差动态系统(9)在零初始条件 $\left(V(\tilde{\boldsymbol{x}}(t))\Big|_{t=0} = 0\right)$ 满足 $\boldsymbol{J} < 0$.

考虑到滤波误差动态系统(9)是鲁棒稳定的 $\left(V(\tilde{\boldsymbol{x}}(t))\Big|_{t\to\infty} \geqslant 0\right)$,那么对于任意非零的 $\boldsymbol{\omega}(t) \in L_2[0,\infty)$ 有

$$\begin{aligned}
\boldsymbol{J} &= \int_0^\infty [\tilde{\boldsymbol{z}}^{\mathrm{T}}(t)\tilde{z}(t) - \gamma^2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{\omega}(t)]\,\mathrm{d}t =\\
&\int_0^\infty [\tilde{\boldsymbol{z}}^{\mathrm{T}}(t)\tilde{\boldsymbol{z}}(t) - \gamma^2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{\omega}(t) + \dot{V}(\tilde{\boldsymbol{x}}(t))]\,\mathrm{d}t +\\
&\quad V(\tilde{\boldsymbol{x}}(t))\Big|_{t=0} - V(\tilde{\boldsymbol{x}}(t))\Big|_{t=\infty} \leqslant\\
&\int_0^\infty [\tilde{\boldsymbol{z}}^{\mathrm{T}}(t)\tilde{\boldsymbol{z}}(t) - \gamma^2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{\omega}(t) + \dot{V}(\tilde{\boldsymbol{x}}(t))]\,\mathrm{d}t.
\end{aligned} \tag{19}$$

其中:

$$\begin{aligned}
&\tilde{\boldsymbol{z}}^{\mathrm{T}}(t)\tilde{\boldsymbol{z}}(t) =\\
&\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\bar{\boldsymbol{C}}^{\mathrm{T}}\bar{\boldsymbol{C}}\tilde{\boldsymbol{x}}(t) + 2\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\bar{\boldsymbol{C}}^{\mathrm{T}}\bar{\boldsymbol{C}}_1\tilde{\boldsymbol{x}}(t-d_1(t)) +\\
&\quad 2\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\bar{\boldsymbol{C}}^{\mathrm{T}}\bar{\boldsymbol{C}}_2\tilde{\boldsymbol{x}}(t-d_2(t)) + 2\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\bar{\boldsymbol{C}}^{\mathrm{T}}\bar{\boldsymbol{D}}\boldsymbol{\omega}(t) +\\
&\quad \tilde{\boldsymbol{x}}^{\mathrm{T}}(t-d_1(t))\bar{\boldsymbol{C}}_1^{\mathrm{T}}\bar{\boldsymbol{C}}_1\tilde{\boldsymbol{x}}(t-d_1(t)) +\\
&\quad 2\tilde{\boldsymbol{x}}^{\mathrm{T}}(t-d_1(t))\bar{\boldsymbol{C}}_1^{\mathrm{T}}\bar{\boldsymbol{C}}_2\tilde{\boldsymbol{x}}(t-d_2(t)) +\\
&\quad 2\tilde{\boldsymbol{x}}^{\mathrm{T}}(t-d_1(t))\bar{\boldsymbol{C}}_1^{\mathrm{T}}\bar{\boldsymbol{D}}\boldsymbol{\omega}(t) + \tilde{\boldsymbol{x}}^{\mathrm{T}}(t-d_2(t))\bar{\boldsymbol{C}}_2^{\mathrm{T}}\bar{\boldsymbol{C}}_2\tilde{\boldsymbol{x}}(t-d_2(t)) +\\
&\quad 2\tilde{\boldsymbol{x}}^{\mathrm{T}}(t-d_2(t))\bar{\boldsymbol{C}}_2^{\mathrm{T}}\bar{\boldsymbol{D}}\boldsymbol{\omega}(t) + \boldsymbol{\omega}^{\mathrm{T}}(t)\bar{\boldsymbol{D}}^{\mathrm{T}}\bar{\boldsymbol{D}}\boldsymbol{\omega}(t).
\end{aligned} \tag{20}$$

由此得

$$\tilde{\boldsymbol{z}}^{\mathrm{T}}(t)\tilde{\boldsymbol{z}}(t) - \gamma^2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{\omega}(t) + \dot{V}(\boldsymbol{\xi}(t)) = \boldsymbol{\eta}^{\mathrm{T}}(t)\boldsymbol{\Theta}\boldsymbol{\eta}(t).$$

其中:

$$\boldsymbol{\Theta}=\begin{bmatrix}\boldsymbol{\Theta}_{11}&\boldsymbol{\Theta}_{12}&\boldsymbol{\Theta}_{13}&\boldsymbol{\Theta}_{14}\\ *&\boldsymbol{\Theta}_{22}&\boldsymbol{\Theta}_{23}&\boldsymbol{\Theta}_{24}\\ *&*&\boldsymbol{\Theta}_{33}&\boldsymbol{\Theta}_{34}\\ *&*&*&\boldsymbol{\Theta}_{44}\end{bmatrix};\tag{21}$$

$$\begin{aligned}\boldsymbol{\Theta}_{11}&=\overline{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}\overline{\boldsymbol{E}}+\overline{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\overline{\boldsymbol{A}}+\overline{\boldsymbol{A}}^{\mathrm{T}}\overline{\boldsymbol{R}}\overline{\boldsymbol{S}}^{\mathrm{T}}+\overline{\boldsymbol{S}}\overline{\boldsymbol{R}}^{\mathrm{T}}\overline{\boldsymbol{A}}+\boldsymbol{Q}_1+\\ &\quad\boldsymbol{Q}_2+d\overline{\boldsymbol{N}}_1^{\mathrm{T}}\overline{\boldsymbol{N}}_1+d\overline{\boldsymbol{A}}^{\mathrm{T}}\overline{\boldsymbol{A}}+\overline{\boldsymbol{C}}^{\mathrm{T}}\overline{\boldsymbol{C}};\\ \boldsymbol{\Theta}_{12}&=\overline{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\overline{\boldsymbol{A}}_1+\overline{\boldsymbol{S}}\overline{\boldsymbol{R}}^{\mathrm{T}}\overline{\boldsymbol{A}}_1+\overline{\boldsymbol{N}}_1^{\mathrm{T}}\overline{\boldsymbol{E}}+d\overline{\boldsymbol{A}}^{\mathrm{T}}\overline{\boldsymbol{A}}_1+d\overline{\boldsymbol{N}}_1^{\mathrm{T}}\overline{\boldsymbol{N}}_2+\overline{\boldsymbol{C}}^{\mathrm{T}}\overline{\boldsymbol{C}}_1;\\ \boldsymbol{\Theta}_{13}&=\overline{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\overline{\boldsymbol{A}}_2+\overline{\boldsymbol{S}}\overline{\boldsymbol{R}}^{\mathrm{T}}\overline{\boldsymbol{A}}_2-\overline{\boldsymbol{N}}_1^{\mathrm{T}}\overline{\boldsymbol{E}}+d\overline{\boldsymbol{A}}^{\mathrm{T}}\overline{\boldsymbol{A}}_2+d\overline{\boldsymbol{N}}_1^{\mathrm{T}}\overline{\boldsymbol{N}}_3+\overline{\boldsymbol{C}}^{\mathrm{T}}\overline{\boldsymbol{C}}_2;\\ \boldsymbol{\Theta}_{14}&=\overline{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\overline{\boldsymbol{B}}+\overline{\boldsymbol{S}}\overline{\boldsymbol{R}}^{\mathrm{T}}\overline{\boldsymbol{B}}+d\overline{\boldsymbol{A}}^{\mathrm{T}}\overline{\boldsymbol{A}}_2+d\overline{\boldsymbol{N}}_1^{\mathrm{T}}\overline{\boldsymbol{N}}_3+\overline{\boldsymbol{C}}^{\mathrm{T}}\overline{\boldsymbol{D}};\\ \boldsymbol{\Theta}_{22}&=-(1-d^*)\boldsymbol{Q}_1+\overline{\boldsymbol{N}}_2^{\mathrm{T}}\overline{\boldsymbol{E}}+\overline{\boldsymbol{E}}^{\mathrm{T}}\overline{\boldsymbol{N}}_2+\\ &\quad d\overline{\boldsymbol{A}}_1^{\mathrm{T}}\overline{\boldsymbol{A}}_1+d\overline{\boldsymbol{N}}_2^{\mathrm{T}}\overline{\boldsymbol{N}}_2+\overline{\boldsymbol{C}}_1^{\mathrm{T}}\overline{\boldsymbol{C}}_1;\\ \boldsymbol{\Theta}_{23}&=\overline{\boldsymbol{E}}^{\mathrm{T}}\overline{\boldsymbol{N}}_3-\overline{\boldsymbol{N}}_2^{\mathrm{T}}\overline{\boldsymbol{E}}+d\overline{\boldsymbol{A}}_1^{\mathrm{T}}\overline{\boldsymbol{A}}_2+d\overline{\boldsymbol{N}}_2^{\mathrm{T}}\overline{\boldsymbol{N}}_3+\overline{\boldsymbol{C}}_1^{\mathrm{T}}\overline{\boldsymbol{C}}_2;\\ \boldsymbol{\Theta}_{24}&=\overline{\boldsymbol{E}}^{\mathrm{T}}\overline{\boldsymbol{W}}+d\overline{\boldsymbol{A}}_1^{\mathrm{T}}\overline{\boldsymbol{B}}+d\overline{\boldsymbol{N}}_2^{\mathrm{T}}\overline{\boldsymbol{W}}+\overline{\boldsymbol{C}}_1^{\mathrm{T}}\overline{\boldsymbol{D}};\\ \boldsymbol{\Theta}_{33}&=-(1-d^*)\boldsymbol{Q}_2-\overline{\boldsymbol{E}}^{\mathrm{T}}\overline{\boldsymbol{N}}_3-\overline{\boldsymbol{N}}_3^{\mathrm{T}}\overline{\boldsymbol{E}}+\\ &\quad d\overline{\boldsymbol{A}}_2^{\mathrm{T}}\overline{\boldsymbol{A}}_2+d\overline{\boldsymbol{N}}_3^{\mathrm{T}}\overline{\boldsymbol{N}}_3+\overline{\boldsymbol{C}}_2^{\mathrm{T}}\overline{\boldsymbol{C}}_2;\\ \boldsymbol{\Theta}_{34}&=-\overline{\boldsymbol{E}}^{\mathrm{T}}\overline{\boldsymbol{W}}+d\overline{\boldsymbol{A}}_2^{\mathrm{T}}\overline{\boldsymbol{B}}+d\overline{\boldsymbol{N}}_3^{\mathrm{T}}\overline{\boldsymbol{W}}+\overline{\boldsymbol{C}}_2^{\mathrm{T}}\overline{\boldsymbol{D}};\\ \boldsymbol{\Theta}_{44}&=-\gamma^2\boldsymbol{I}+d\overline{\boldsymbol{B}}^{\mathrm{T}}\overline{\boldsymbol{B}}+d\overline{\boldsymbol{W}}^{\mathrm{T}}\overline{\boldsymbol{W}}+\overline{\boldsymbol{D}}^{\mathrm{T}}\overline{\boldsymbol{D}}.\end{aligned}$$

且 $\overline{\boldsymbol{R}}\in\overline{\mathbf{R}}^{2n\times(n-r)}$ 为任意满足 $\overline{\boldsymbol{E}}^{\mathrm{T}}\overline{\boldsymbol{R}}=0$ 的列满秩矩阵.

如果 $\boldsymbol{\Theta}<0$,则由 Schur 补引理得到式(12).因此,滤波误差动态系统(9)在零初始条件 $\left(V(\tilde{\boldsymbol{x}}(t))\Big|_{t=0}=0\right)$ 满足 $\boldsymbol{J}<0$. 证毕.

3 鲁棒 H_∞ 滤波器设计

Design of robust H_∞ filter

上述定理 1 只能从理论上说明鲁棒 H_∞ 滤波器存在的充分条件,具体的滤波器无法求出.下面的定理给出了求具体滤波器的方法.

定理 2 对任意满足(4)的时变时滞 $d_i(t)(i=1,2)$,不确定时滞奇异系统(1)和性能指标式(11).给定标量 $\gamma>0$,如果存在正定矩阵

$$\boldsymbol{P}=\begin{bmatrix}\boldsymbol{P}_1&-\boldsymbol{P}_2\\ -\boldsymbol{P}_2&\boldsymbol{P}_2\end{bmatrix}>0,$$

$$\boldsymbol{Q}_1=\begin{bmatrix}\boldsymbol{Q}_{1,11}&\boldsymbol{Q}_{1,12}\\ *&\boldsymbol{Q}_{1,22}\end{bmatrix}>0,$$

$$\boldsymbol{Q}_2=\begin{bmatrix}\boldsymbol{Q}_{2,11}&\boldsymbol{Q}_{2,12}\\ *&\boldsymbol{Q}_{2,22}\end{bmatrix}>0$$

和矩阵 $\boldsymbol{S}$、$\boldsymbol{N}_1$、$\boldsymbol{N}_2$、$\boldsymbol{N}_3$、$\boldsymbol{W}$、$\boldsymbol{X}$、$\boldsymbol{Z}$ 以及标量 $\varepsilon_1>0$、$\varepsilon_2>0$ 使得

$$\boldsymbol{\Xi}=\begin{vmatrix}(\boldsymbol{\Xi}_1)_{6\times6}&(\boldsymbol{\Xi}_2)_{6\times7}\\ (\boldsymbol{\Xi}_2)_{6\times7}^{\mathrm{T}}&(\boldsymbol{\Xi}_3)_{7\times7}\end{vmatrix}<0.\tag{22}$$

其中:

$$(\boldsymbol{\Xi}_1)_{6\times6}=\begin{vmatrix}\boldsymbol{\Xi}_{11}&\boldsymbol{\Xi}_{12}&\boldsymbol{\Xi}_{13}&\boldsymbol{0}&\boldsymbol{\Xi}_{15}&\boldsymbol{0}\\ *&\boldsymbol{\Xi}_{22}&\boldsymbol{\Xi}_{23}&\boldsymbol{0}&\boldsymbol{\Xi}_{25}&\boldsymbol{0}\\ *&*&\boldsymbol{\Xi}_{33}&\boldsymbol{\Xi}_{34}&\boldsymbol{\Xi}_{35}&\boldsymbol{0}\\ *&*&*&\boldsymbol{\Xi}_{44}&\boldsymbol{0}&\boldsymbol{0}\\ *&*&*&*&\boldsymbol{\Xi}_{55}&\boldsymbol{\Xi}_{56}\\ *&*&*&*&*&\boldsymbol{\Xi}_{66}\end{vmatrix};$$

$$(\boldsymbol{\Xi}_2)_{6\times7}=\begin{vmatrix}\boldsymbol{\Xi}_{17}&\boldsymbol{N}_1^{\mathrm{T}}\boldsymbol{A}^{\mathrm{T}}&\boldsymbol{C}^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}}&(\boldsymbol{H}-\hat{\boldsymbol{D}}\boldsymbol{C})^{\mathrm{T}}&\boldsymbol{\Xi}_{112}&\boldsymbol{0}\\ \boldsymbol{\Xi}_{27}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{G}^{\mathrm{T}}&-\hat{\boldsymbol{H}}^{\mathrm{T}}&\boldsymbol{0}&\boldsymbol{\Xi}_{213}\\ \boldsymbol{E}^{\mathrm{T}}\boldsymbol{W}&\boldsymbol{N}_2^{\mathrm{T}}\boldsymbol{A}_1^{\mathrm{T}}&\boldsymbol{C}_1^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}}&-(\hat{\boldsymbol{D}}\boldsymbol{C}_1)^{\mathrm{T}}&\boldsymbol{0}&\boldsymbol{0}\\ \boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}\\ -\boldsymbol{E}^{\mathrm{T}}\boldsymbol{W}&\boldsymbol{N}_3^{\mathrm{T}}\boldsymbol{A}_2^{\mathrm{T}}&\boldsymbol{C}_2^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}}&-(\hat{\boldsymbol{D}}\boldsymbol{C}_2)^{\mathrm{T}}&\boldsymbol{0}&\boldsymbol{0}\\ \boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}\end{vmatrix};$$

$$(\boldsymbol{\Xi}_3)_{7\times7}=\begin{vmatrix}-\gamma^2\boldsymbol{I}&\boldsymbol{W}^{\mathrm{T}}&\boldsymbol{B}^{\mathrm{T}}&\boldsymbol{D}^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}}&-(\hat{\boldsymbol{D}}\boldsymbol{D})^{\mathrm{T}}&\boldsymbol{0}&\boldsymbol{0}\\ *&-\bar{d}\boldsymbol{I}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}\\ *&*&-\bar{d}\boldsymbol{I}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{L}_1&\boldsymbol{0}\\ *&*&*&-\bar{d}\boldsymbol{I}&\boldsymbol{0}&\boldsymbol{K}\boldsymbol{L}_2&\boldsymbol{0}\\ *&*&*&*&-\boldsymbol{I}&-\hat{\boldsymbol{D}}\boldsymbol{L}_2&\boldsymbol{0}\\ *&*&*&*&*&-\varepsilon_1\boldsymbol{I}&\boldsymbol{0}\\ *&*&*&*&*&*&-\varepsilon_2\boldsymbol{I}\end{vmatrix}.$$

其中:

$$\begin{aligned}\boldsymbol{\Xi}_{11}&=\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{E}-\boldsymbol{C}^{\mathrm{T}}\boldsymbol{Z}^{\mathrm{T}}\boldsymbol{E}+\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{A}-\boldsymbol{E}^{\mathrm{T}}\boldsymbol{Z}\boldsymbol{C}+\boldsymbol{A}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{S}^{\mathrm{T}}+\\ &\quad\boldsymbol{S}\boldsymbol{R}^{\mathrm{T}}\boldsymbol{A}+\boldsymbol{Q}_{1,11}+\boldsymbol{Q}_{2,11}+\varepsilon_1\boldsymbol{M}_1^{\mathrm{T}}\boldsymbol{M}_1+\varepsilon_2\boldsymbol{M}_1^{\mathrm{T}}\boldsymbol{M}_1;\\ \boldsymbol{\Xi}_{12}&=\boldsymbol{C}^{\mathrm{T}}\boldsymbol{Z}^{\mathrm{T}}-\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}_2-\boldsymbol{E}^{\mathrm{T}}\boldsymbol{X}+\boldsymbol{Q}_{1,12}+\boldsymbol{Q}_{2,12};\\ \boldsymbol{\Xi}_{13}&=\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{A}_1+\boldsymbol{S}\boldsymbol{R}^{\mathrm{T}}\boldsymbol{A}_1+\boldsymbol{N}_1^{\mathrm{T}}\boldsymbol{E}-\boldsymbol{E}^{\mathrm{T}}\boldsymbol{Z}\boldsymbol{C}_1+\\ &\quad\varepsilon_1\boldsymbol{M}_1^{\mathrm{T}}\boldsymbol{M}_2+\varepsilon_2\boldsymbol{M}_1^{\mathrm{T}}\boldsymbol{M}_2;\\ \boldsymbol{\Xi}_{15}&=\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{A}_2+\boldsymbol{S}\boldsymbol{R}^{\mathrm{T}}\boldsymbol{A}_2-\boldsymbol{E}^{\mathrm{T}}\boldsymbol{Z}\boldsymbol{C}_2-\boldsymbol{N}_1^{\mathrm{T}}\boldsymbol{E}+\\ &\quad\varepsilon_1\boldsymbol{M}_1^{\mathrm{T}}\boldsymbol{M}_3+\varepsilon_2\boldsymbol{M}_1^{\mathrm{T}}\boldsymbol{M}_3;\\ \boldsymbol{\Xi}_{17}&=\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{B}+\boldsymbol{S}\boldsymbol{R}^{\mathrm{T}}\boldsymbol{B}-\boldsymbol{E}^{\mathrm{T}}\boldsymbol{Z}\boldsymbol{D};\\ \boldsymbol{\Xi}_{112}&=(\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}_1+\boldsymbol{S}\boldsymbol{R}^{\mathrm{T}})\boldsymbol{L}_1-\boldsymbol{E}^{\mathrm{T}}\boldsymbol{Z}\boldsymbol{L}_2;\\ \boldsymbol{\Xi}_{22}&=\boldsymbol{X}^{\mathrm{T}}+\boldsymbol{X}+\boldsymbol{Q}_{1,22}+\boldsymbol{Q}_{2,22};\\ \boldsymbol{\Xi}_{23}&=\boldsymbol{Z}\boldsymbol{C}_1-\boldsymbol{P}_2\boldsymbol{A}_1;\boldsymbol{\Xi}_{25}=\boldsymbol{Z}\boldsymbol{C}_2-\boldsymbol{P}_2\boldsymbol{A}_2;\\ \boldsymbol{\Xi}_{27}&=\boldsymbol{Z}\boldsymbol{D}-\boldsymbol{P}_2\boldsymbol{B};\boldsymbol{\Xi}_{213}=\boldsymbol{Z}\boldsymbol{L}_2-\boldsymbol{P}_2\boldsymbol{L}_1;\\ \boldsymbol{\Xi}_{33}&=-(1-d^*)\boldsymbol{Q}_{1,11}+\boldsymbol{N}_2^{\mathrm{T}}\boldsymbol{E}+\boldsymbol{E}^{\mathrm{T}}\boldsymbol{N}_2+\\ &\quad\varepsilon_1\boldsymbol{M}_2^{\mathrm{T}}\boldsymbol{M}_2+\varepsilon_2\boldsymbol{M}_2^{\mathrm{T}}\boldsymbol{M}_2;\\ \boldsymbol{\Xi}_{34}&=-(1-d^*)\boldsymbol{Q}_{1,12};\\ \boldsymbol{\Xi}_{35}&=\boldsymbol{E}^{\mathrm{T}}\boldsymbol{N}_3-\boldsymbol{N}_2^{\mathrm{T}}\boldsymbol{E}+\varepsilon_1\boldsymbol{M}_2^{\mathrm{T}}\boldsymbol{M}_3+\varepsilon_2\boldsymbol{M}_2^{\mathrm{T}}\boldsymbol{M}_3;\\ \boldsymbol{\Xi}_{44}&=-(1-d^*)\boldsymbol{Q}_{1,22};\\ \boldsymbol{\Xi}_{55}&=-(1-d^*)\boldsymbol{Q}_{2,11}-\boldsymbol{N}_3^{\mathrm{T}}\boldsymbol{E}-\boldsymbol{E}^{\mathrm{T}}\boldsymbol{N}_3+\end{aligned}$$

$$\varepsilon_1 \boldsymbol{M}_3^{\mathrm{T}}\boldsymbol{M}_3 + \varepsilon_2 \boldsymbol{M}_3^{\mathrm{T}}\boldsymbol{M}_3.$$

$$\boldsymbol{\Xi}_{56} = -(1-d^*)\boldsymbol{Q}_{2,12}; \quad \boldsymbol{\Xi}_{66} = -(1-d^*)\boldsymbol{Q}_{2,22}.$$

且 $\boldsymbol{R} \in \mathbf{R}^{n\times(n-r)}$ 为任意满足 $\boldsymbol{E}^{\mathrm{T}}\boldsymbol{R} = 0$ 的列满秩矩阵,则式(6)为系统(1)的鲁棒 H_∞ 滤波器,既有 $\boldsymbol{J} < 0$ 且滤波器的参数矩阵为 $\boldsymbol{G} = \boldsymbol{P}_2^{-1}\boldsymbol{X}, \boldsymbol{K} = \boldsymbol{P}_2^{-1}\boldsymbol{Z}$,其他的两个参数矩阵 $\hat{\boldsymbol{H}}$ 和 $\hat{\boldsymbol{D}}$ 由式(22)可以解出.

证明 在定理1的基础上,式(21)中取

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{P}_1 & -\boldsymbol{P}_2 \\ -\boldsymbol{P}_2 & \boldsymbol{P}_2 \end{bmatrix} > 0, \quad \boldsymbol{Q}_1 = \begin{bmatrix} \boldsymbol{Q}_{1,11} & \boldsymbol{Q}_{1,12} \\ * & \boldsymbol{Q}_{1,22} \end{bmatrix} > 0,$$

$$\boldsymbol{Q}_2 = \begin{bmatrix} \boldsymbol{Q}_{2,12} & \boldsymbol{Q}_{2,12} \\ * & \boldsymbol{Q}_{2,22} \end{bmatrix} > 0, \quad \bar{\boldsymbol{R}} = \begin{bmatrix} \boldsymbol{R} \\ 0 \end{bmatrix}, \quad \bar{\boldsymbol{S}} = \begin{bmatrix} \boldsymbol{S} \\ 0 \end{bmatrix}. \tag{23}$$

并考虑定义式(10),如果令

$$\boldsymbol{P}_2\boldsymbol{G} = \boldsymbol{X}, \quad \boldsymbol{P}_2\boldsymbol{K} = \boldsymbol{Z}. \tag{24}$$

则有

$$\boldsymbol{\Delta} = \begin{bmatrix} \boldsymbol{\Delta}_{11} & \boldsymbol{\Delta}_{12} & \boldsymbol{\Delta}_{13} & \boldsymbol{0} & \boldsymbol{\Delta}_{15} & 0 & \boldsymbol{\Delta}_{17} \\ * & \boldsymbol{\Delta}_{22} & \boldsymbol{\Delta}_{23} & \boldsymbol{0} & \boldsymbol{\Delta}_{25} & \boldsymbol{0} & \boldsymbol{\Delta}_{27} \\ * & * & \boldsymbol{\Delta}_{33} & \boldsymbol{\Xi}_{34} & \boldsymbol{\Delta}_{35} & \boldsymbol{0} & \boldsymbol{\Delta}_{37} \\ * & * & * & \boldsymbol{\Xi}_{44} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & * & \boldsymbol{\Delta}_{55} & \boldsymbol{\Xi}_{56} & \boldsymbol{\Delta}_{47} \\ * & * & * & * & * & \boldsymbol{\Xi}_{66} & 0 \\ * & * & * & * & * & * & \boldsymbol{\Delta}_{77} \end{bmatrix}. \tag{25}$$

式中:

$$\boldsymbol{\Delta}_{11} = \tilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{E} - \tilde{\boldsymbol{C}}^{\mathrm{T}}\boldsymbol{Z}^{\mathrm{T}}\boldsymbol{E} + \boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}_1\tilde{\boldsymbol{A}} - \boldsymbol{E}^{\mathrm{T}}\boldsymbol{Z}\tilde{\boldsymbol{C}} + \boldsymbol{S}\boldsymbol{R}^{\mathrm{T}}\tilde{\boldsymbol{A}} + \tilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{S}^{\mathrm{T}} + \boldsymbol{Q}_{1,11} + \boldsymbol{Q}_{2,11} + d\boldsymbol{N}_1^{\mathrm{T}}\boldsymbol{N}_1 + d(\tilde{\boldsymbol{A}}^{\mathrm{T}}\tilde{\boldsymbol{A}} + \tilde{\boldsymbol{C}}^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}}\boldsymbol{K}\tilde{\boldsymbol{C}}) + (\boldsymbol{H} - \hat{\boldsymbol{C}}\tilde{\boldsymbol{C}})^{\mathrm{T}}(\boldsymbol{H} - \hat{\boldsymbol{C}}\tilde{\boldsymbol{C}});$$

$$\boldsymbol{\Delta}_{12} = \tilde{\boldsymbol{C}}^{\mathrm{T}}\boldsymbol{Z}^{\mathrm{T}} - \tilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}_2 - \boldsymbol{E}^{\mathrm{T}}\boldsymbol{X} + \boldsymbol{Q}_{1,12} + \boldsymbol{Q}_{2,12} + d\tilde{\boldsymbol{C}}^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}}\boldsymbol{G} - (\boldsymbol{H} - \hat{\boldsymbol{C}}\tilde{\boldsymbol{C}})^{\mathrm{T}}\hat{\boldsymbol{H}};$$

$$\boldsymbol{\Delta}_{13} = \boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}_1\tilde{\boldsymbol{A}}_1 + \boldsymbol{S}\boldsymbol{R}^{\mathrm{T}}\tilde{\boldsymbol{A}}_1 - \boldsymbol{E}^{\mathrm{T}}\boldsymbol{Z}\tilde{\boldsymbol{C}}_1 + \boldsymbol{N}_1^{\mathrm{T}}\boldsymbol{E} + d(\tilde{\boldsymbol{A}}^{\mathrm{T}}\tilde{\boldsymbol{A}}_1 + \tilde{\boldsymbol{C}}^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}}\boldsymbol{K}\tilde{\boldsymbol{C}}_1) + d\boldsymbol{N}_1^{\mathrm{T}}\boldsymbol{N}_2 - (\boldsymbol{H} - \hat{\boldsymbol{C}}\tilde{\boldsymbol{C}})^{\mathrm{T}}\hat{\boldsymbol{C}}\tilde{\boldsymbol{C}}_1;$$

$$\boldsymbol{\Delta}_{15} = \boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}_1\tilde{\boldsymbol{A}}_2 + \boldsymbol{S}\boldsymbol{R}^{\mathrm{T}}\tilde{\boldsymbol{A}}_2 - \boldsymbol{E}^{\mathrm{T}}\boldsymbol{Z}\tilde{\boldsymbol{C}}_2 - \boldsymbol{N}_1^{\mathrm{T}}\boldsymbol{E} + d(\tilde{\boldsymbol{A}}^{\mathrm{T}}\tilde{\boldsymbol{A}}_2 + \tilde{\boldsymbol{C}}^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}}\boldsymbol{K}\tilde{\boldsymbol{C}}_2) + d\boldsymbol{N}_1^{\mathrm{T}}\boldsymbol{N}_3 - (\boldsymbol{H} - \hat{\boldsymbol{C}}\tilde{\boldsymbol{C}})^{\mathrm{T}}\hat{\boldsymbol{C}}\tilde{\boldsymbol{C}}_2;$$

$$\boldsymbol{\Delta}_{17} = \boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{B} + \boldsymbol{S}\boldsymbol{R}^{\mathrm{T}}\boldsymbol{B} - \boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}_2\boldsymbol{K}\boldsymbol{D} - (\boldsymbol{H} - \hat{\boldsymbol{C}}\tilde{\boldsymbol{C}})^{\mathrm{T}}\hat{\boldsymbol{C}}\boldsymbol{D} + d\boldsymbol{N}_1^{\mathrm{T}} + d(\tilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{B} + \tilde{\boldsymbol{C}}^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}}\boldsymbol{K}\boldsymbol{D});$$

$$\boldsymbol{\Delta}_{22} = \boldsymbol{X}^{\mathrm{T}} + \boldsymbol{X} + \boldsymbol{Q}_{1,22} + \boldsymbol{Q}_{2,22} + d\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G} + \hat{\boldsymbol{H}}^{\mathrm{T}}\hat{\boldsymbol{H}};$$

$$\boldsymbol{\Delta}_{23} = \boldsymbol{Z}\tilde{\boldsymbol{C}}_1 - \boldsymbol{P}_2\tilde{\boldsymbol{A}}_1 + d\boldsymbol{G}^{\mathrm{T}}\boldsymbol{K}\tilde{\boldsymbol{C}}_1 + \hat{\boldsymbol{H}}^{\mathrm{T}}\hat{\boldsymbol{C}}\tilde{\boldsymbol{C}}_1;$$

$$\boldsymbol{\Delta}_{25} = \boldsymbol{Z}\tilde{\boldsymbol{C}}_2 - \boldsymbol{P}_2\tilde{\boldsymbol{A}}_2 + d\boldsymbol{G}^{\mathrm{T}}\boldsymbol{K}\tilde{\boldsymbol{C}}_2 + \hat{\boldsymbol{H}}^{\mathrm{T}}\hat{\boldsymbol{C}}\tilde{\boldsymbol{C}}_2;$$

$$\boldsymbol{\Delta}_{27} = \boldsymbol{P}_2\boldsymbol{K}\boldsymbol{D} - \boldsymbol{P}_2\boldsymbol{B} + d\boldsymbol{G}^{\mathrm{T}}\boldsymbol{K}\boldsymbol{D} + \hat{\boldsymbol{H}}^{\mathrm{T}}\hat{\boldsymbol{C}}\boldsymbol{D};$$

$$\boldsymbol{\Delta}_{33} = -(1-d^{\cdot})\boldsymbol{Q}_{1,11} + \boldsymbol{E}^{\mathrm{T}}\boldsymbol{N}_2 + \boldsymbol{N}_2^{\mathrm{T}}\boldsymbol{E} + d(\tilde{\boldsymbol{A}}_1^{\mathrm{T}}\tilde{\boldsymbol{A}}_1 + \tilde{\boldsymbol{C}}_1^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}}\boldsymbol{K}\tilde{\boldsymbol{C}}_1) + d\boldsymbol{N}_2^{\mathrm{T}}\boldsymbol{N}_2 + (\hat{\boldsymbol{C}}\tilde{\boldsymbol{C}}_1)^{\mathrm{T}}(\hat{\boldsymbol{C}}\tilde{\boldsymbol{C}}_1);$$

$$\boldsymbol{\Delta}_{35} = \boldsymbol{E}^{\mathrm{T}}\boldsymbol{N}_3 - \boldsymbol{N}_2^{\mathrm{T}}\boldsymbol{E} + d(\tilde{\boldsymbol{A}}_1^{\mathrm{T}}\tilde{\boldsymbol{A}}_2 + \tilde{\boldsymbol{C}}_1^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}}\boldsymbol{K}\tilde{\boldsymbol{C}}_2) + d\boldsymbol{N}_2^{\mathrm{T}}\boldsymbol{N}_3 + (\hat{\boldsymbol{C}}\tilde{\boldsymbol{C}}_1)^{\mathrm{T}}(\hat{\boldsymbol{C}}\tilde{\boldsymbol{C}}_2);$$

$$\boldsymbol{\Delta}_{37} = \boldsymbol{E}^{\mathrm{T}}\boldsymbol{W} + (\hat{\boldsymbol{C}}\tilde{\boldsymbol{C}}_1)^{\mathrm{T}}(\hat{\boldsymbol{C}}\boldsymbol{D}) + d\boldsymbol{N}_2^{\mathrm{T}}\boldsymbol{W} + d(\tilde{\boldsymbol{A}}_1^{\mathrm{T}}\boldsymbol{B} + \tilde{\boldsymbol{C}}_1^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}}\boldsymbol{K}\boldsymbol{D});$$

$$\boldsymbol{\Delta}_{55} = -(1-d^{\cdot})\boldsymbol{Q}_{2,11} - \boldsymbol{E}^{\mathrm{T}}\boldsymbol{N}_3 - \boldsymbol{N}_3^{\mathrm{T}}\boldsymbol{E} + d(\tilde{\boldsymbol{A}}_2^{\mathrm{T}}\tilde{\boldsymbol{A}}_2 + \tilde{\boldsymbol{C}}_2^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}}\boldsymbol{K}\tilde{\boldsymbol{C}}_2) + d\boldsymbol{N}_3^{\mathrm{T}}\boldsymbol{N}_3 + (\hat{\boldsymbol{C}}\tilde{\boldsymbol{C}}_2)^{\mathrm{T}}(\hat{\boldsymbol{C}}\tilde{\boldsymbol{C}}_2);$$

$$\boldsymbol{\Delta}_{57} = -\boldsymbol{E}^{\mathrm{T}}\boldsymbol{W} + (\hat{\boldsymbol{C}}\tilde{\boldsymbol{C}}_2)^{\mathrm{T}}(\hat{\boldsymbol{C}}\boldsymbol{D}) + d\boldsymbol{N}_3^{\mathrm{T}}\boldsymbol{W} + d(\tilde{\boldsymbol{A}}_2^{\mathrm{T}}\boldsymbol{B} + \tilde{\boldsymbol{C}}_2^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}}\boldsymbol{K}\boldsymbol{D});$$

$$\boldsymbol{\Delta}_{77} = -\boldsymbol{\gamma}^2\boldsymbol{I} + (\hat{\boldsymbol{C}}\boldsymbol{D})^{\mathrm{T}}(\hat{\boldsymbol{C}}\boldsymbol{D}) + d\boldsymbol{W}^{\mathrm{T}}\boldsymbol{W} + d(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{B} + \boldsymbol{D}^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}}\boldsymbol{K}\boldsymbol{D}).$$

且 $\boldsymbol{R} \in \mathbf{R}^{n\times(n-r)}$ 为任意满足 $\boldsymbol{E}^{\mathrm{T}}\boldsymbol{R} = 0$ 的列满秩矩阵.

如果 $\boldsymbol{\Delta} < 0$,则由 Schur 补引理得到等价的不等式:

$$\boldsymbol{\Lambda} = \begin{vmatrix} (\boldsymbol{\Lambda}_1)_{6\times6} & (\boldsymbol{\Lambda}_2)_{6\times5} \\ (\boldsymbol{\Lambda}_2)_{6\times5}^{\mathrm{T}} & (\boldsymbol{\Lambda}_3)_{5\times5} \end{vmatrix} < 0. \tag{26}$$

其中

$$(\boldsymbol{\Lambda}_1)_{6\times6} = \begin{vmatrix} \boldsymbol{\Lambda}_{11} & \boldsymbol{\Lambda}_{12} & \boldsymbol{\Lambda}_{13} & \boldsymbol{0} & \boldsymbol{\Lambda}_{15} & \boldsymbol{0} \\ * & \boldsymbol{\Lambda}_{22} & \boldsymbol{\Lambda}_{23} & \boldsymbol{0} & \boldsymbol{\Lambda}_{25} & \boldsymbol{0} \\ * & * & \boldsymbol{\Lambda}_{33} & \boldsymbol{\Xi}_{34} & \boldsymbol{\Lambda}_{35} & \boldsymbol{0} \\ * & * & * & \boldsymbol{\Xi}_{44} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & * & \boldsymbol{\Lambda}_{55} & \boldsymbol{\Xi}_{56} \\ * & * & * & * & * & \boldsymbol{\Xi}_{66} \end{vmatrix};$$

$$(\boldsymbol{\Lambda}_2)_{6\times5} = \begin{vmatrix} \boldsymbol{\Lambda}_{17} & \boldsymbol{N}_1^{\mathrm{T}} & \tilde{\boldsymbol{A}}^{\mathrm{T}} & \tilde{\boldsymbol{C}}^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}} & (\boldsymbol{H} - \hat{\boldsymbol{C}}\tilde{\boldsymbol{C}})^{\mathrm{T}} \\ \boldsymbol{\Lambda}_{27} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{G}^{\mathrm{T}} & -\hat{\boldsymbol{H}}^{\mathrm{T}} \\ \boldsymbol{E}^{\mathrm{T}}\boldsymbol{W} & \boldsymbol{N}_2^{\mathrm{T}} & \tilde{\boldsymbol{A}}_1^{\mathrm{T}} & \tilde{\boldsymbol{C}}_1^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}} & -(\hat{\boldsymbol{C}}\tilde{\boldsymbol{C}}_1)^{\mathrm{T}} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ -\boldsymbol{E}^{\mathrm{T}}\boldsymbol{W} & \boldsymbol{N}_3^{\mathrm{T}} & \tilde{\boldsymbol{A}}_2^{\mathrm{T}} & \tilde{\boldsymbol{C}}_2^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}} & -(\hat{\boldsymbol{C}}\tilde{\boldsymbol{C}}_2)^{\mathrm{T}} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{vmatrix};$$

$$(\boldsymbol{\Lambda}_3)_{5\times5} = \begin{vmatrix} -\boldsymbol{\gamma}^2\boldsymbol{I} & \boldsymbol{W}^{\mathrm{T}} & \boldsymbol{B}^{\mathrm{T}} & \boldsymbol{D}^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}} & -(\hat{\boldsymbol{C}}\boldsymbol{D})^{\mathrm{T}} \\ * & -\bar{d}\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & -\bar{d}\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & -\bar{d}\boldsymbol{I} & \boldsymbol{0} \\ * & * & * & * & \boldsymbol{I} \end{vmatrix}.$$

其中

$$\boldsymbol{\Lambda}_{11} = \tilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{E} - \tilde{\boldsymbol{C}}^{\mathrm{T}}\boldsymbol{Z}^{\mathrm{T}}\boldsymbol{E} + \boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}_1\tilde{\boldsymbol{A}} - \boldsymbol{E}^{\mathrm{T}}\boldsymbol{Z}\tilde{\boldsymbol{C}} + \boldsymbol{S}\boldsymbol{R}^{\mathrm{T}}\tilde{\boldsymbol{A}} + \tilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{S}^{\mathrm{T}} + \boldsymbol{Q}_{1,11} + \boldsymbol{Q}_{2,11};$$

$$\boldsymbol{\Lambda}_{12} = \tilde{\boldsymbol{C}}^{\mathrm{T}}\boldsymbol{Z}^{\mathrm{T}} - \tilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}_2 - \boldsymbol{E}^{\mathrm{T}}\boldsymbol{X} + \boldsymbol{Q}_{1,12} + \boldsymbol{Q}_{2,12};$$

$$\boldsymbol{\Lambda}_{13} = \boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}_1\tilde{\boldsymbol{A}}_1 + \boldsymbol{S}\boldsymbol{R}^{\mathrm{T}}\tilde{\boldsymbol{A}}_1 - \boldsymbol{E}^{\mathrm{T}}\boldsymbol{Z}\tilde{\boldsymbol{C}}_1 + \boldsymbol{N}_1^{\mathrm{T}}\boldsymbol{E};$$

$$\boldsymbol{\Lambda}_{15} = \boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}_1\tilde{\boldsymbol{A}}_2 + \boldsymbol{S}\boldsymbol{R}^{\mathrm{T}}\tilde{\boldsymbol{A}}_2 - \boldsymbol{E}^{\mathrm{T}}\boldsymbol{Z}\tilde{\boldsymbol{C}}_2 - \boldsymbol{N}_1^{\mathrm{T}}\boldsymbol{E};$$

$$\boldsymbol{\Lambda}_{17} = \boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{B} + \boldsymbol{S}\boldsymbol{R}^{\mathrm{T}}\boldsymbol{B} - \boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}_2\boldsymbol{K}\boldsymbol{D};$$

$$\boldsymbol{\Lambda}_{22} = \boldsymbol{X}^{\mathrm{T}} + \boldsymbol{X} + \boldsymbol{Q}_{1,22} + \boldsymbol{Q}_{2,22}; \quad \boldsymbol{\Lambda}_{23} = \boldsymbol{Z}\widetilde{\boldsymbol{C}}_1 - \boldsymbol{P}_2\widetilde{\boldsymbol{A}}_1;$$

$$\boldsymbol{\Lambda}_{25} = \boldsymbol{Z}\widetilde{\boldsymbol{C}}_2 - \boldsymbol{P}_2\widetilde{\boldsymbol{A}}_2; \quad \boldsymbol{\Lambda}_{27} = \boldsymbol{P}_2\boldsymbol{K}\boldsymbol{D} - \boldsymbol{P}_2\boldsymbol{B};$$

$$\boldsymbol{\Lambda}_{33} = -(1-d^*)\boldsymbol{Q}_{1,11} + \boldsymbol{E}^{\mathrm{T}}\boldsymbol{N}_2 + \boldsymbol{N}_2^{\mathrm{T}}\boldsymbol{E};$$

$$\boldsymbol{\Lambda}_{35} = \boldsymbol{E}^{\mathrm{T}}\boldsymbol{N}_3 - \boldsymbol{N}_2^{\mathrm{T}}\boldsymbol{E};$$

$$\boldsymbol{\Lambda}_{55} = -(1-d^*)\boldsymbol{Q}_{2,11} - \boldsymbol{E}^{\mathrm{T}}\boldsymbol{N}_3 - \boldsymbol{N}_3^{\mathrm{T}}\boldsymbol{E}.$$

且 $\boldsymbol{R} \in \mathbf{R}^{n\times(n-r)}$ 为任意满足 $\boldsymbol{E}^{\mathrm{T}}\boldsymbol{R} = 0$ 的列满秩矩阵.

考虑到式(2)、(8),应用引理 2 得到等价的不等式(22). 这说明了所设计的滤波器具有给定的 H_∞ 干抑制度 γ.

最后,由式(24) 可得滤波器的两个参数矩阵 $\boldsymbol{G} = \boldsymbol{P}_2^{-1}\boldsymbol{X}$ 和 $\boldsymbol{K} = \boldsymbol{P}_2^{-1}\boldsymbol{Z}$,其他的两个参数矩阵 $\hat{\boldsymbol{H}}$ 和 $\hat{\boldsymbol{C}}$ 由矩阵不等式(22)可以解出.

证毕.

4 算例
Calculation example

以下通过一个数值例子来说明本文设计方法的有效性.

例 1 考虑系统(1)具有如下参数:

$$\boldsymbol{E} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}; \quad \boldsymbol{A} = \begin{bmatrix} 4 & -1.7 \\ -0.5 & 2.5 \end{bmatrix};$$

$$\boldsymbol{A}_1 = \begin{bmatrix} -0.3 & 2.5 \\ 0.1 & -0.5 \end{bmatrix}; \quad \boldsymbol{A}_2 = \begin{bmatrix} 0.1 & -0.5 \\ -0.3 & -0.8 \end{bmatrix};$$

$$\boldsymbol{C} = \begin{bmatrix} 0.1 & -0.1 \\ -0.1 & -0.2 \end{bmatrix}; \quad \boldsymbol{C}_1 = \begin{bmatrix} -0.1 & -0.1 \\ -0.5 & -0.2 \end{bmatrix};$$

$$\boldsymbol{C}_2 = \begin{bmatrix} -1 & -0.3 \\ -0.6 & -0.4 \end{bmatrix}; \quad \boldsymbol{B} = \begin{bmatrix} -0.5 \\ -0.1 \end{bmatrix};$$

$$\boldsymbol{D} = \begin{bmatrix} -4 \\ -1 \end{bmatrix}; \quad \boldsymbol{H} = [-1 \quad -0.1];$$

$$\boldsymbol{L}_1 = \boldsymbol{L}_2 = 0.02\boldsymbol{I}; \quad \boldsymbol{M}_1 = \boldsymbol{M}_2 = \boldsymbol{M}_3 = \boldsymbol{I};$$

$$\boldsymbol{\omega}(t) \in L_2[0,\infty).$$

设计鲁棒 H_∞ 滤波器.

在该例题中,令 $\boldsymbol{R} = [0 \quad 1]^{\mathrm{T}}$. 根据本文定理 2,对不同的时滞变化率 d^*,通过求解线性矩阵不等式(22) 的可行性问题,表 1 给出了系统不同的最大允许的时滞上确界 d 和衰减度 γ 及相应的滤波器矩阵 $\boldsymbol{G}$、$\boldsymbol{K}$、$\hat{\boldsymbol{H}}$、$\hat{\boldsymbol{C}}$.

表 1 不同的 d^* 值时所求得的最大时滞值和衰减度 γ 及相应的滤波器矩阵 $\boldsymbol{G}$、$\boldsymbol{K}$、$\hat{\boldsymbol{H}}$、$\hat{\boldsymbol{C}}$

Table 1 The maximal time-delay, attenuation degree γ and corresponding filter matrix $\boldsymbol{G}$、$\boldsymbol{K}$、$\hat{\boldsymbol{H}}$、$\hat{\boldsymbol{C}}$ for different d^*

d^*	d	γ	$\boldsymbol{G}$	$\boldsymbol{K}$	$\hat{\boldsymbol{H}}$	$\hat{\boldsymbol{C}}$
0.1	0.177 2	2.317 0	$\begin{pmatrix} -11.332\,0 & -0.181\,4 \\ -6.815\,0 & 15.905\,9 \end{pmatrix}$	$\begin{pmatrix} 0.563\,5 & -3.193\,6 \\ -0.364\,2 & 2.347\,7 \end{pmatrix}$	(0.129 1 0.116 2)	(0.051 1 −0.168 9)
0.3	0.201 6	2.130 9	$\begin{pmatrix} -19.889\,1 & 3.514\,0 \\ -6.651\,0 & 16.281\,2 \end{pmatrix}$	$\begin{pmatrix} 0.551\,3 & -3.145\,2 \\ -0.347\,8 & 2.062\,0 \end{pmatrix}$	(0.105 2 0.096 9)	(0.044 8 −0.141 0)
0.5	0.113 0	3.137 5	$\begin{pmatrix} -70.824\,0 & 40.056\,8 \\ 3.839\,8 & 8.181\,1 \end{pmatrix}$	$\begin{pmatrix} 1.807\,7 & -10.461\,9 \\ -0.663\,3 & 3.743\,7 \end{pmatrix}$	(0.132 4 0.121 7)	(0.047 1 −0.121 1)
1.3	0.031 4	5.693 4	$\begin{pmatrix} -1\,918.2 & 1\,658.8 \\ 971.3 & -852.3 \end{pmatrix}$	$\begin{pmatrix} -40.220\,5 & 128.002\,9 \\ 19.906\,9 & -63.203\,0 \end{pmatrix}$	(0.070 2 −0.086 0)	(0.063 4 −0.069 7)

5 结束语
Concluding remarks

本文研究了不确定奇异系统的时滞相关鲁棒 H_∞ 滤波器设计问题,通过 Lyapuno-Krasovskii 泛函及二次型的积分不等式方法获得了滤波误差动态系统的 H_∞ 性能时滞相关的判据,给出了奇异系统的鲁棒 H_∞ 滤波器存在的时滞依赖的充分条件. 所给出的结果都采用严格线性矩阵不等式形式,利用 Matlab 的 LMI 工具箱求解方便简单. 最后,通过数值仿真例子验证了本文所提出方法的有效性.

参考文献
References

[1] Nagpal K M, Khargoneker P P. Filtering and smoothing in an H_∞ setting[J]. IEEE Transactions on Automatic Control, 1991, 36(2):152-166

[2] Shaked U. H_∞ minimum error state estimation of linear stationary processes[J]. IEEE Transactions on Automatic Control, 1990, 35(5):554-558

[3] Xie L H, de Souza C E. On robust filtering for linear systems with parameter uncertainty[C]// Proceedings of the 34th IEEE Conference on Decision and Control, 1995:2087-2092

[4] de Souza C E, Palhares R M, Peres P L D. Robust H_∞ filter design for uncertain linear systems with multiple time-varing state delays [J]. IEEE Transactions on Signal Processing, 2001, 49(3):

569-576
[5] Shaked U, Theodor Y. H_∞ optimal estimation: a tutorial[C]// Proceedings of the 31st IEEE Conference on Decision and Control, 1992: 2278-2286
[6] Fu M Y. Interpolation approach to H_∞ optimal estimation and it s interconnection to loop transfer recovery[J]. Systems & Control Letters, 1991, 17(1): 29-36
[7] Gershon E, Limebeer D J N, Shaked U, et al. Robust H_∞ filtering of stationary continuous-time linear systems with stochastic uncertainties[J]. IEEE Transactions on Automatic Control, 2001, 46(11): 1788-1793
[8] Park P G, Kailath T. H_∞ filtering via convex optimization[J]. International Journal of Control, 1997, 66(1): 15-22
[9] 吴保卫,仝云旭,陈敏. 不确定多时滞奇异系统的鲁棒 H_∞ 降维滤波器的设计[J]. 陕西师范大学学报:自然科学版,2008, 36(1):7-14
WU Baowei, TONG Yunxu, CHEN Min. A design of robust H_∞ reduced-order filter design for uncertain singular system with multiple time-varying state delays[J]. Journal of Shaanxi Normal University: Natural Science Edition, 2008, 36(1): 7-14
[10] 陈凌,曾建平. 奇异系统的降阶 H_∞ 滤波器的设计[J]. 厦门大学学报:自然科学版,2008,47(4):489-494
CHEN Ling, ZENG Jianping. Reduce-order H_∞ filtering design for singular systems[J]. Journal of Xiamen University: Natural Science, 2008, 47(4): 489-494
[11] 陈云,王俊红,鲁仁全. 不确定时滞系统的鲁棒 H_∞ 保性能滤波[J]. 浙江大学学报:工学版,2007,41(10):1664-1668
CHEN Yun, WANG Junhong, LU Renquan. Robust H_∞ guaranteed cost filtering for uncertain time-delay systems[J]. Journal of Zhejiang University: Engineering Science, 2007, 41(10): 1664-1668
[12] 鲁仁全,苏宏业,薛安克,等. 奇异系统的鲁棒控制理论[M]. 北京:科学出版社,2008:11-36
LU Renquan, SU Hongye, XUE Anke, et al. Robust control theory of singular system[M]. Beijing: Science Press, 2008: 11-36
[13] 俞立. 鲁棒控制-线性矩阵不等式处理方法[M]. 北京:清华大学出版社,2002
YU Li. Robust control-the solving method of linear matrix inequalities[M]. Beijing: Tsinghua University Press, 2002
[14] 王惠姣. 不确定奇异系统的鲁棒控制研究[D]. 杭州:浙江大学信息学部,2008
WANG Huijiao. Study on robust control for uncertain singular systems[D]. Hangzhou: Faculty of Information Technology, Zhejiang University, 2008

Design of delay-dependent robust H_∞ filter for uncertain singular system

HAI Quan[1] BAO Jundong[1]
1 Mathematics Science College, Inner Mongolia Normal University, Huhhot 010022

Abstract The design problem of full order H_∞ filter for continuous singular system is addressed in this paper, in order to design satisfactory full order H_∞ filter with regular and impulse-free filter error dynamics. Lyapunov-Krasovskii functional and quadratic inequality method was used to get H_∞ time-delay criterion of filter error dynamics. A sufficient condition of robust H_∞ filter existence of singular system is provided. Finally, a numerical example is given to verify the effectiveness and practicability of the design method.

Key words singular system; uncertainly; delay-dependent criterion; robust H_∞ filter