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一类非线性中立型系统的状态反馈时滞相关 H-infinity 控制

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摘要

研究了一类非线性中立型时滞系统的 H-infinity 控制问题. 利用了适当的参数待定的 Lyapunov-Krasovskii 泛函及 LMI 方法获得了中立型系统的稳定性时滞相关的判据,给出了 H-infinity 状态反馈控制器存在的时滞依赖的充分条件. 所采用的方法在设计、求解控制器时不需要做矩阵变换,求解简单,利用 Matlab 的 LMI 工具箱求解方便,并且给出的数值仿真实例说明了方法的有效性.

关键词

中立型系统;非线性;时滞相关;H-infinity 控制

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0 引言

Introduction

中立型系统的研究具有广泛的应用背景. 众所周知,在信息与控制理论、化学反应堆、电动力学、分布网络、热交换系统、无损传输线路、生物医学等[1-2]都有中立型系统的应用. 正因如此,近年来引起了人们的广泛关注[1-11]. 关于中立型系统的稳定与镇定问题及 H-infinity 控制问题的研究主要集中于时滞无关与时滞相关的判据的建立,人们已经认识到时滞相关判据更为有效、具有更小保守性. 因此,近来很多研究成果都是致力于保守性较小的判据的建立[5-11],得到了很多优秀的研究成果,所有这些研究成果都为进一步推动中立型系统的研究做出了贡献.

在文献[12-14]中,基于 LMI 方法,利用 Lyapunov-Krasovskii 泛函讨论了中立型方程

$$\frac{\mathrm{d}}{\mathrm{d}t}[\boldsymbol{x}(t)+p\boldsymbol{x}(t-\tau)]=-a\boldsymbol{x}(t)+b\tanh\boldsymbol{x}(t-\sigma) \tag{1}$$

的稳定性问题. 文献[14]对该方程做了研究,并改进了文献[12-13]的结果;之后,文献[15]给出了更为深刻的结论,改进了文献[14]的结果.

本文在文献[15]的基础上讨论了一类非线性中立型系统

$$\begin{cases}\dfrac{\mathrm{d}}{\mathrm{d}t}[\boldsymbol{x}(t)+\boldsymbol{P}\boldsymbol{x}(t-\tau)]=-\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{B}\boldsymbol{u}(t)+\\ \qquad \boldsymbol{B}_1\boldsymbol{f}(\boldsymbol{x}(t-\sigma))+\boldsymbol{B}_\omega\boldsymbol{\omega}(t),\\ \boldsymbol{x}(t)=\boldsymbol{\varphi}(t),t\in[-\sigma,0],\boldsymbol{\varphi}\in\boldsymbol{C}([-\sigma,0],\mathbf{R}^n),\\ \boldsymbol{z}(t)=\boldsymbol{C}\boldsymbol{x}(t)+\boldsymbol{C}_1\boldsymbol{x}(t-\tau)+\boldsymbol{D}\boldsymbol{u}(t)+\boldsymbol{D}_1\boldsymbol{\omega}(t)\end{cases} \tag{2}$$

的 H-infinity 控制问题,给出了时滞相关的判定条件.

本文通篇沿用如下的符号:$\mathbf{R}^n$,$\mathbf{R}^{n\times m}$分别表示 n 维向量空间和 $n\times m$矩阵空间;用“$*$”表示对称矩阵中的对称部分的相应元素;用矩阵$\boldsymbol{X}>0$($\boldsymbol{X}<0$)表示对称正定(负定)矩阵;$\rho(\boldsymbol{X})$表示矩阵 $\boldsymbol{X}$ 的谱范数;$L_2[0,\infty]$表示在$[0,\infty)$上平方可积的函数空间.

1 问题的描述

Description of the problem

考虑中立型系统

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$$\begin{cases}\dfrac{\mathrm{d}}{\mathrm{d}t}[\boldsymbol{x}(t)+\boldsymbol{P}\boldsymbol{x}(t-\tau)]=-\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{B}\boldsymbol{u}(t)+\\ \qquad \boldsymbol{B}_1\boldsymbol{f}(\boldsymbol{x}(t-\sigma))+\boldsymbol{B}_{\omega}\boldsymbol{\omega}(t),\\ \boldsymbol{x}(t)=\boldsymbol{\varphi}(t),t\in[-\sigma,0],\boldsymbol{\varphi}\in\boldsymbol{C}([-\sigma,0],\mathbf{R}^n),\\ \boldsymbol{z}(t)=\boldsymbol{C}\boldsymbol{x}(t)+\boldsymbol{C}_1\boldsymbol{x}(t-\tau)+\boldsymbol{D}\boldsymbol{u}(t)+\boldsymbol{D}_1\boldsymbol{\omega}(t).\end{cases}\tag{2}$$

其中:$\boldsymbol{x}(t)\in\mathbf{R}^n$ 为系统状态向量;$\boldsymbol{u}(t)\in\mathbf{R}^m$ 为控制输入向量;$\boldsymbol{f}(\boldsymbol{x})\in\mathbf{R}^n$ 是满足 $\|\boldsymbol{f}(\boldsymbol{x})\|\leqslant\|\boldsymbol{x}^2(t)\|$ 的非线性向量函数;τ,σ 为常时滞,且 $0\leqslant\tau\leqslant\sigma$;$\boldsymbol{\omega}(t)\in L_2[0,\infty)$ 为 p 维扰动输入向量;$\boldsymbol{z}(t)\in\mathbf{R}^q$ 为控制输出向量;$\boldsymbol{P}\in\mathbf{R}^{n\times n}$ 且 $\rho(\boldsymbol{P})<1$;$\boldsymbol{A},\boldsymbol{B},\boldsymbol{B}_1,\boldsymbol{B}_{\omega},\boldsymbol{C},\boldsymbol{C}_1,\boldsymbol{D}$ 为维数适当的已知常数矩阵.

对系统(2),考虑以下的无记忆控制器

$$\boldsymbol{u}(t)=\boldsymbol{K}\boldsymbol{x}(t),\tag{3}$$

其中:增益矩阵 $\boldsymbol{K}\in\mathbf{R}^{m\times n}$.

定义算子 $\mathscr{P}_1:\boldsymbol{C}([-\sigma,0],\mathbf{R}^n)$ 为

$$\mathscr{P}_1(\boldsymbol{x}_t)=\boldsymbol{x}(t)+\boldsymbol{P}\boldsymbol{x}(t-\tau),\quad \boldsymbol{x}(t)\in\mathbf{R}^n.\tag{4}$$

算子 $\mathscr{P}_2:\boldsymbol{C}([-\sigma,0],\mathbf{R}^n)$ 为

$$\mathscr{P}_2(\boldsymbol{x}_t)=\boldsymbol{x}(t)+\boldsymbol{P}\boldsymbol{x}(t-\tau)+\alpha\int_{t-\tau}^{t}\boldsymbol{x}(s)\mathrm{d}s+\boldsymbol{B}_1\int_{t-\sigma}^{t-\tau}\boldsymbol{f}(\boldsymbol{x}(s))\mathrm{d}s,\quad \boldsymbol{x}\in\mathbf{R}^n.\tag{5}$$

这里 α 是个待定的参数.

本文的目的就是要设计形如式(3)的无记忆控制器,使得闭环系统为

1) 内部稳定;

2) 在零初始条件下,具有给定的 H_{∞} 扰动抑制水平 γ,即 $\|z\|_2\leqslant\gamma\|\boldsymbol{\omega}\|_2$.

在控制器(3)的作用下,系统(2)的闭环系统为

$$\begin{cases}\dfrac{\mathrm{d}}{\mathrm{d}t}[\boldsymbol{x}(t)+\boldsymbol{P}\boldsymbol{x}(t-\tau)]=[-\boldsymbol{A}+\boldsymbol{B}\boldsymbol{K}]\boldsymbol{x}(t)+\\ \qquad \boldsymbol{B}_1\boldsymbol{f}(\boldsymbol{x}(t-\sigma))+\boldsymbol{B}_{\omega}\boldsymbol{\omega}(t),\\ \boldsymbol{x}(t)=\boldsymbol{\varphi}(t),t\in[-\sigma,0],\boldsymbol{\varphi}\in\boldsymbol{C}([-\sigma,0],\mathbf{R}^n),\\ \boldsymbol{z}(t)=[\boldsymbol{C}+\boldsymbol{D}\boldsymbol{K}]\boldsymbol{x}(t)+\boldsymbol{C}_1\boldsymbol{x}(t-\tau)+\boldsymbol{D}_1\boldsymbol{\omega}(t).\end{cases}\tag{6}$$

由式(5)及式(6),得到系统方程的等价状态方程

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathscr{P}_2(\boldsymbol{x}_t)=[\alpha\boldsymbol{I}-\boldsymbol{A}+\boldsymbol{B}\boldsymbol{K}]\boldsymbol{x}(t)-\alpha\boldsymbol{x}(t-\tau)+\boldsymbol{B}_1\boldsymbol{f}(\boldsymbol{x}(t-\tau))+\boldsymbol{B}_{\omega}\boldsymbol{\omega}(t).\tag{7}$$

引理 1[16] 对于任意给定的正常数 $\gamma>0$ 和正定矩阵 $\boldsymbol{M}=\boldsymbol{M}^{\mathrm{T}}>0$ 及向量函数 $\boldsymbol{\omega}:[0,\gamma]\to\mathbf{R}^n$,有下面的积分不等式成立

$$\gamma\int_0^{\gamma}\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{M}\boldsymbol{\omega}(t)\mathrm{d}t\geqslant\left(\int_0^{\gamma}\boldsymbol{\omega}^{\mathrm{T}}(t)\mathrm{d}t\right)\boldsymbol{M}\left(\int_0^{\gamma}\boldsymbol{\omega}(t)\mathrm{d}t\right).$$

2 闭环系统的内部稳定性

Stability inside the closed-loop system

下面考虑在控制器(3)的作用下,闭环系统(6)的内部稳定性问题. 并给出如下定理.

定理 1 对于给定的正常数 $\tau>0,\sigma>0$,且 $\tau\leqslant\sigma$,系统(7)内部是可镇定的,如果矩阵 $\boldsymbol{P}$ 的谱范数 $\rho(\boldsymbol{P})<1$,且存在正常数 $\beta>0,\varepsilon>0$ 和常数 α 且 $|\alpha|\leqslant1$,以及矩阵 $\boldsymbol{K}$ 和正定矩阵 $\boldsymbol{E}>0,\boldsymbol{\Theta}>0,\boldsymbol{\Gamma}>0,\boldsymbol{\Delta}>0$ 使得以下矩阵不等式成立.

$$\boldsymbol{\Xi}=\begin{bmatrix}\boldsymbol{Q}_{11}&\boldsymbol{Q}_{12}&\boldsymbol{B}_1&\boldsymbol{Q}_{14}&\boldsymbol{Q}_{15}&0\\ *&-\boldsymbol{E}-2\alpha\boldsymbol{P}&\boldsymbol{P}^{\mathrm{T}}\boldsymbol{B}_1&-\alpha\boldsymbol{I}&-\alpha\boldsymbol{B}_1&0\\ *&*&\boldsymbol{Q}_{33}&\boldsymbol{B}_1^{\mathrm{T}}&\boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{B}_1&0\\ *&*&*&-\boldsymbol{\Gamma}&0&0\\ *&*&*&*&-\boldsymbol{\Theta}&0\\ *&*&*&*&*&-\boldsymbol{\Delta}\end{bmatrix}<0.\tag{8}$$

其中:

$$\boldsymbol{Q}_{11}=2\alpha\boldsymbol{I}-\boldsymbol{A}-\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{B}\boldsymbol{K}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}+\boldsymbol{E}+\beta\boldsymbol{I}+\tau^2\boldsymbol{\Gamma};$$

$$\boldsymbol{Q}_{12}=(\alpha\boldsymbol{I}-\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}})\boldsymbol{P}-\alpha\boldsymbol{I};$$

$$\boldsymbol{Q}_{14}=\alpha\boldsymbol{I}-\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}};$$

$$\boldsymbol{Q}_{15}=(\alpha\boldsymbol{I}-\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}})\boldsymbol{B}_1;$$

$$\boldsymbol{Q}_{33}=\boldsymbol{\Delta}-\beta\boldsymbol{I}+(\sigma-\tau)^2\boldsymbol{\Theta}.$$

证明 选取待定 Lyapunov-Krasovskii 泛函

$$V(t,\boldsymbol{x}_t)=V_1(t,\boldsymbol{x}_t)+V_2(t,\boldsymbol{x}_t),$$

其中,

$$\begin{aligned}V_1(t,\boldsymbol{x}_t)=&\mathscr{P}_2^{\mathrm{T}}(\boldsymbol{x}_t)\mathscr{P}_2(\boldsymbol{x}_t)+\int_{t-\tau}^{t}\boldsymbol{x}_t^{\mathrm{T}}(s)\boldsymbol{E}\boldsymbol{x}_t(s)\mathrm{d}s+\\ &\tau\int_{t-\tau}^{t}(\tau-t+s)(\alpha\boldsymbol{x}(s))^{\mathrm{T}}\boldsymbol{\Gamma}(\alpha\boldsymbol{x}(s))\mathrm{d}s+\\ &(\sigma-\tau)\int_{t-\sigma}^{t-\tau}(\sigma-t+s)\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(s))\boldsymbol{\Theta}\boldsymbol{f}(\boldsymbol{x}(s))\mathrm{d}s+\\ &\beta\int_{t-\tau}^{t}\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(s))\boldsymbol{f}(\boldsymbol{x}(s))\mathrm{d}s+\\ &\int_{t-\sigma}^{t-\tau}\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(s))\boldsymbol{\Delta}\boldsymbol{f}(\boldsymbol{x}(s))\mathrm{d}s,\end{aligned}\tag{9}$$

$$V_2(t,\boldsymbol{x}_t)=\varepsilon\mathscr{P}_1^{\mathrm{T}}(\boldsymbol{x}_t)\mathscr{P}_1(\boldsymbol{x}_t).\tag{10}$$

首先,求关于 V_1,V_2 沿着系统(7)轨线的导数,有

$$\begin{aligned}\left.\frac{\mathrm{d}V_1}{\mathrm{d}t}\right|_{(7)}=&2\left[\boldsymbol{x}(t)+\boldsymbol{P}\boldsymbol{x}(t-\tau)+\alpha\int_{t-\tau}^{t}\boldsymbol{x}(s)\mathrm{d}s+\boldsymbol{B}_1\int_{t-\sigma}^{t-\tau}\boldsymbol{f}(\boldsymbol{x}(s))\mathrm{d}s\right]^{\mathrm{T}}\left[(\alpha\boldsymbol{I}-\boldsymbol{A}+\boldsymbol{B}\boldsymbol{K})\boldsymbol{x}(t)-\right.\\ &\left.\alpha\boldsymbol{x}(t-\tau)+\boldsymbol{B}_1\boldsymbol{f}(\boldsymbol{x}(t-\tau))\right]+\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{E}\boldsymbol{x}(t)-\\ &\boldsymbol{x}^{\mathrm{T}}(t-\tau)\boldsymbol{E}\boldsymbol{x}(t-\tau)+\tau^2\alpha^2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{\Gamma}\boldsymbol{x}(t)-\\ &\tau\int_{t-\tau}^{t}(\alpha\boldsymbol{x}(s))^{\mathrm{T}}\boldsymbol{\Gamma}(\alpha\boldsymbol{x}(s))\mathrm{d}s+\\ &(\sigma-\tau)^2\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau))\boldsymbol{\Theta}\boldsymbol{f}(\boldsymbol{x}(t-\tau))-\\ &(\sigma-\tau)\int_{t-\sigma}^{t-\tau}\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(s))\boldsymbol{\Theta}\boldsymbol{f}(\boldsymbol{x}(s))\mathrm{d}s+\end{aligned}$$

$$\beta \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t))\boldsymbol{f}(\boldsymbol{x}(t)) - \beta \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau))\boldsymbol{f}(\boldsymbol{x}(t-\tau)) + \beta \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t))\boldsymbol{f}(\boldsymbol{x}(t)) + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau))\boldsymbol{\Delta}\boldsymbol{f}(\boldsymbol{x}(t-\tau)) - \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\sigma))\boldsymbol{\Delta}\boldsymbol{f}(\boldsymbol{x}(t-\sigma)), \tag{11}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}V_2(t,\boldsymbol{x}_t)\Big|_{(7)} = 2\varepsilon[\boldsymbol{x}(t) + \boldsymbol{P}\boldsymbol{x}(t-\tau)]^{\mathrm{T}}[(-\boldsymbol{A} + \boldsymbol{B}\boldsymbol{K})\boldsymbol{x}(t) + \boldsymbol{B}_1\boldsymbol{f}(\boldsymbol{x}(t-\sigma))]. \tag{12}$$

记

$$\boldsymbol{u}(t) = \alpha\int_{t-\tau}^{t}\boldsymbol{x}(s)\mathrm{d}s, \boldsymbol{v}(t) = \int_{t-\sigma}^{t-\tau}\boldsymbol{f}(\boldsymbol{x}(s))\mathrm{d}s.$$

再应用引理 1 及条件 $\|\boldsymbol{f}(\boldsymbol{x}(t)\| \leqslant \|\boldsymbol{x}(t)\|$,可以得到

$$\frac{\mathrm{d}V_1}{\mathrm{d}t}\Big|_{(7)} \leqslant \boldsymbol{x}^{\mathrm{T}}(t)[2\alpha\boldsymbol{I} - \boldsymbol{A} - \boldsymbol{A}^{\mathrm{T}} + \boldsymbol{B}\boldsymbol{K} + \boldsymbol{K}^{\mathrm{T}}\boldsymbol{B} + \boldsymbol{E} + \beta\boldsymbol{I} + \tau^2\boldsymbol{\Gamma}]\boldsymbol{x}(t) + 2\boldsymbol{x}^{\mathrm{T}}(t)[(\alpha\boldsymbol{I} - \boldsymbol{A} + \boldsymbol{B}\boldsymbol{K})^{\mathrm{T}}\boldsymbol{P} - \alpha\boldsymbol{I}]\boldsymbol{x}(t-\tau) + 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{B}_1\boldsymbol{f}(\boldsymbol{x}(t-\tau)) + 2\boldsymbol{x}^{\mathrm{T}}(t)[\alpha\boldsymbol{I} - \boldsymbol{A} + \boldsymbol{B}\boldsymbol{K}]^{\mathrm{T}}\boldsymbol{u}(t) + 2\boldsymbol{x}^{\mathrm{T}}(t)(\alpha\boldsymbol{I} - \boldsymbol{A} + \boldsymbol{B}\boldsymbol{K})^{\mathrm{T}}\boldsymbol{B}_1\boldsymbol{v}(t) - \boldsymbol{x}^{\mathrm{T}}(t-\tau)[\boldsymbol{E} + 2\alpha\boldsymbol{P}]\boldsymbol{x}(t-\tau) + 2\boldsymbol{x}^{\mathrm{T}}(t-\tau)\boldsymbol{P}^{\mathrm{T}}\boldsymbol{B}_1\boldsymbol{f}(\boldsymbol{x}(t-\tau)) - 2\alpha\boldsymbol{x}^{\mathrm{T}}(t-\tau)\boldsymbol{u}(t) - 2a\boldsymbol{x}^{\mathrm{T}}(t-\tau)\boldsymbol{B}_1\boldsymbol{v}(t) + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau))[(\sigma-\tau)^2\boldsymbol{\Theta} - \beta\boldsymbol{I} + \boldsymbol{\Delta}]\cdot\boldsymbol{f}(\boldsymbol{x}(t-\tau)) + 2\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau))\boldsymbol{B}_1{}^{\mathrm{T}}\boldsymbol{u}(t) + 2\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau))\boldsymbol{B}_1{}^{\mathrm{T}}\boldsymbol{B}_1\boldsymbol{v}(t) - \boldsymbol{u}^{\mathrm{T}}(t)\boldsymbol{\Gamma}\boldsymbol{u}(t) - \boldsymbol{v}^{\mathrm{T}}(t)\boldsymbol{\Theta}\boldsymbol{v}(t) - \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\sigma))\boldsymbol{\Delta}\boldsymbol{f}(\boldsymbol{x}(t-\sigma)) \triangleq \boldsymbol{\zeta}^{\mathrm{T}}(t)\boldsymbol{\Omega}\boldsymbol{\zeta}(t). \tag{13}$$

其中:

$$\boldsymbol{\zeta}(t) = (\boldsymbol{x}^{\mathrm{T}}(t), \boldsymbol{x}^{\mathrm{T}}(t-\tau), \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau)), \boldsymbol{u}^{\mathrm{T}}(t), \boldsymbol{v}^{\mathrm{T}}(t), \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\sigma)))^{\mathrm{T}};$$

$$\boldsymbol{\Omega} = \begin{bmatrix} \boldsymbol{Q}_{11} & \boldsymbol{Q}_{12} & \boldsymbol{B}_1 & \boldsymbol{Q}_{14} & \boldsymbol{Q}_{15} & 0 \\ * & -\boldsymbol{E} - 2\alpha\boldsymbol{P} & \boldsymbol{P}^{\mathrm{T}}\boldsymbol{B}_1 & -\alpha\boldsymbol{I} & -\alpha\boldsymbol{B}_1 & 0 \\ * & * & \boldsymbol{Q}_{33} & \boldsymbol{B}_1^{\mathrm{T}} & \boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{B}_1 & 0 \\ * & * & * & -\boldsymbol{\Gamma} & 0 & 0 \\ * & * & * & * & -\boldsymbol{\Theta} & 0 \\ * & * & * & * & * & -\boldsymbol{\Delta} \end{bmatrix}.$$

这里

$$\boldsymbol{Q}_{11} = 2\alpha\boldsymbol{I} - \boldsymbol{A} - \boldsymbol{A}^{\mathrm{T}} + \boldsymbol{B}\boldsymbol{K} + \boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}} + \boldsymbol{E} + \beta\boldsymbol{I} + \tau^2\boldsymbol{\Gamma};$$

$$\boldsymbol{Q}_{12} = (\alpha\boldsymbol{I} - \boldsymbol{A}^{\mathrm{T}} + \boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}})\boldsymbol{P} - \alpha\boldsymbol{I};$$

$$\boldsymbol{Q}_{14} = \alpha\boldsymbol{I} - \boldsymbol{A}^{\mathrm{T}} + \boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}};$$

$$\boldsymbol{Q}_{15} = (\alpha\boldsymbol{I} - \boldsymbol{A}^{\mathrm{T}} + \boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}})\boldsymbol{B}_1;$$

$$\boldsymbol{Q}_{33} = \boldsymbol{\Delta} - \beta\boldsymbol{I} + (\sigma-\tau)^2\boldsymbol{\Theta}.$$

根据定理条件知,矩阵 $\boldsymbol{\Omega} < 0$ 是负定的. 则由上述结论知,必存在正常数 $\lambda > 0$ 使得

$$\frac{\mathrm{d}V_1}{\mathrm{d}t}\Big|_{(7)} \leqslant -\lambda\boldsymbol{\zeta}^{\mathrm{T}}(t)\boldsymbol{\zeta}(t), \tag{14}$$

再由式(12),有

$$\frac{\mathrm{d}}{\mathrm{d}t}V_2(t,\boldsymbol{x}_t)\Big|_{(7)} = 2\varepsilon[\boldsymbol{x}(t) + \boldsymbol{P}\boldsymbol{x}(t-\tau)]^{\mathrm{T}}[(-\boldsymbol{A} + \boldsymbol{B}\boldsymbol{K})\boldsymbol{x}(t) + \boldsymbol{B}_1\boldsymbol{f}(\boldsymbol{x}(t-\sigma))] = 2\varepsilon\boldsymbol{x}^{\mathrm{T}}(t)(-\boldsymbol{A} + \boldsymbol{B}\boldsymbol{K})\boldsymbol{x}(t) + 2\varepsilon\boldsymbol{x}^{\mathrm{T}}(t-\tau)\boldsymbol{P}^{\mathrm{T}}(-\boldsymbol{A} + \boldsymbol{B}\boldsymbol{K})\boldsymbol{x}(t) + 2\varepsilon\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{B}_1\boldsymbol{f}(\boldsymbol{x}(t-\sigma)) + 2\varepsilon\boldsymbol{x}^{\mathrm{T}}(t-\tau)\boldsymbol{P}^{\mathrm{T}}\boldsymbol{B}_1\boldsymbol{f}(\boldsymbol{x}(t-\sigma)) \triangleq \varepsilon\boldsymbol{\zeta}^{\mathrm{T}}(t)\boldsymbol{\Sigma}\boldsymbol{\zeta}(t)$$

其中,

$$\boldsymbol{\Sigma} = \begin{bmatrix} -2\boldsymbol{A} + 2\boldsymbol{B}\boldsymbol{K} & (-\boldsymbol{A}^{\mathrm{T}} + \boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}})\boldsymbol{P} & 0 & 0 & 0 & \boldsymbol{B}_1 \\ * & 0 & 0 & 0 & 0 & \boldsymbol{P}^{\mathrm{T}}\boldsymbol{B}_1 \\ * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{bmatrix}.$$

令 $\lambda_1 = \lambda_{\max}(\Sigma)$,选取 ε 如下

$$\varepsilon = \begin{cases} \dfrac{\lambda}{2\lambda_1}, & \lambda_1 > 0; \\ 1, & \lambda_1 \leqslant 0. \end{cases} \tag{16}$$

则有

$$\frac{\mathrm{d}V}{\mathrm{d}t}\Big|_{(7)} = \frac{\mathrm{d}}{\mathrm{d}t}(V_1 + V_2) < -\frac{\lambda}{2}\|\boldsymbol{\zeta}(t)\|^2 \leqslant -\frac{\lambda}{2}\|\boldsymbol{x}(t)\|^2. \tag{17}$$

另外,显然看到 $V(t,\boldsymbol{x}_t) \geqslant \varepsilon\mathscr{P}_1^{\mathrm{T}}(\boldsymbol{x}_t)\mathscr{P}_1(\boldsymbol{x}_t)$,又由于 $\rho(\boldsymbol{P}) < 1$,故算子 $\mathscr{P}_1(\boldsymbol{x}_t)$ 是稳定的. 因此,由文献[1]定理 8.1 知系统(7)或者闭环系统(6)是渐近稳定的. 故只要不等式(8)成立,便可由式(16)确定 ε,从而由式(8)可以确定反馈增益矩阵 $\boldsymbol{K}$,进而求得反馈镇定控制器(3).

3 H_∞ 反馈控制器的设计

Design of H_∞ feedback controller

定理 2 对于给定的正常数 $\tau > 0, \sigma > 0$,且 $\tau \leqslant \sigma$ 及 $\gamma > 0$ 如果矩阵 $\boldsymbol{P}$ 的谱范数 $\rho(\boldsymbol{P}) < 1$,且存在正常数 $\beta > 0$,和常数 α,且 $|\alpha| \leqslant 1$. 以及正定矩阵 $\boldsymbol{E} > 0, \boldsymbol{\Theta} > 0, \boldsymbol{\Gamma} > 0, \boldsymbol{\Delta} > 0$ 使得以下矩阵不等式成立.

$$\boldsymbol{\Omega}=\begin{bmatrix}\boldsymbol{\Omega}_{11} & \boldsymbol{\Omega}_{12} & \boldsymbol{W}^{\mathrm{T}} & \boldsymbol{B}_1 & \boldsymbol{\Omega}_{15} & \boldsymbol{\Omega}_{16} & 0 & \boldsymbol{W}^{\mathrm{T}}\boldsymbol{D}_1+\boldsymbol{B}_{\omega}\\ * & \boldsymbol{\Omega}_{22} & \boldsymbol{C}_1^{\mathrm{T}} & \boldsymbol{P}^{\mathrm{T}}\boldsymbol{B}_1-\alpha\boldsymbol{I}_n & -\alpha\boldsymbol{B}_1 & 0 & \boldsymbol{P}^{\mathrm{T}}\boldsymbol{B}_{\omega}+\boldsymbol{C}_1^{\mathrm{T}}\\ * & * & -\boldsymbol{I}_q & 0 & 0 & 0 & 0 & 0\\ * & * & * & \boldsymbol{\Omega}_{44} & \boldsymbol{B}_1 & \boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{B}_1 & 0 & 0\\ * & * & * & * & -\boldsymbol{\Gamma} & 0 & 0 & \boldsymbol{B}_{\omega}\\ * & * & * & * & * & -\boldsymbol{\Theta} & 0 & \boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{B}_{\omega}\\ * & * & * & * & * & * & -\boldsymbol{\Delta} & 0\\ * & * & * & * & * & * & * & \boldsymbol{D}_1^{\mathrm{T}}\boldsymbol{D}_1-\gamma^2\boldsymbol{I}_p\end{bmatrix}<0. \tag{18}$$

其中:$\boldsymbol{W}=\boldsymbol{C}+\boldsymbol{DK}$;

$\boldsymbol{\Omega}_{11}=2\alpha\boldsymbol{I}-\boldsymbol{A}-\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{BK}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}+\boldsymbol{E}+\beta\boldsymbol{I}+\tau^2\boldsymbol{\Gamma}$;

$\boldsymbol{\Omega}_{12}=(\alpha\boldsymbol{I}-\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}})\boldsymbol{P}-\alpha\boldsymbol{I}$;

$\boldsymbol{\Omega}_{22}=-\boldsymbol{E}-2\alpha\boldsymbol{P}$;

$\boldsymbol{\Omega}_{15}=\alpha\boldsymbol{I}-\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}$;

$\boldsymbol{\Omega}_{16}=(\alpha\boldsymbol{I}-\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}})\boldsymbol{B}_1$;

$\boldsymbol{\Omega}_{44}=\boldsymbol{\Delta}-\beta\boldsymbol{I}+(\sigma-\tau)^2\boldsymbol{\Theta}$.

则(3)是具有给定的 H_∞ 扰动抑制水平 γ 的控制器.

利用 Lyapunov-Krasovskii 泛函(9),定义泛函

$$J_{z\omega}=\int_0^{\infty}[\boldsymbol{z}^{\mathrm{T}}(t)\boldsymbol{z}(t)-\gamma^2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{\omega}(t)]\mathrm{d}t, \tag{19}$$

则由于在零初值条件下有 $V_1(t,\boldsymbol{x}_t)\mid_{t=0}=0$,且 $V_1(t,\boldsymbol{x}_t)\mid_{\infty}\geqslant 0$. 因此

$$J_{z\omega}\leqslant\int_0^{\infty}[\boldsymbol{z}^{\mathrm{T}}(t)\boldsymbol{z}(t)-\gamma^2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{\omega}(t)+\dot{V}_1(t,\boldsymbol{x}_t)]\mathrm{d}t$$

又

$$\begin{aligned}&\boldsymbol{z}^{\mathrm{T}}(t)\boldsymbol{z}(t)-\gamma^2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{\omega}(t)+\dot{V}_1(t,\boldsymbol{x}_t)=\\&[(\boldsymbol{C}+\boldsymbol{DK})\boldsymbol{x}(t)+\boldsymbol{C}_1\boldsymbol{x}(t-\tau)+\\&\boldsymbol{D}_1\boldsymbol{\omega}(t)]^{\mathrm{T}}\times[(\boldsymbol{C}+\boldsymbol{DK})\boldsymbol{x}(t)+\\&\boldsymbol{C}_1\boldsymbol{x}(t-\tau)+\boldsymbol{D}_1\boldsymbol{\omega}(t)]-\\&\gamma^2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{\omega}(t)+\dot{V}_1(t,\boldsymbol{x}_t)\leqslant\\&\boldsymbol{x}^{\mathrm{T}}(t)[2\alpha\boldsymbol{I}-\boldsymbol{A}-\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{BK}+\\&\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}+\boldsymbol{E}+\beta\boldsymbol{I}+\tau^2\boldsymbol{\Gamma}]\boldsymbol{x}(t)+\\&2\boldsymbol{x}^{\mathrm{T}}(t)[(\alpha\boldsymbol{I}-\boldsymbol{A}+\boldsymbol{BK})^{\mathrm{T}}\boldsymbol{P}-\\&\alpha\boldsymbol{I}]\boldsymbol{x}(t-\tau)+2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{B}_1\boldsymbol{f}(\boldsymbol{x}(t-\tau))+\\&2\boldsymbol{x}^{\mathrm{T}}(t)[\alpha\boldsymbol{I}-\boldsymbol{A}+\boldsymbol{BK}]^{\mathrm{T}}\boldsymbol{u}(t)+\\&2\boldsymbol{x}^{\mathrm{T}}(t)(\alpha\boldsymbol{I}-\boldsymbol{A}+\boldsymbol{BK})^{\mathrm{T}}\boldsymbol{B}_1\boldsymbol{v}(t)-\\&\boldsymbol{x}^{\mathrm{T}}(t-\tau)[\boldsymbol{E}+2\alpha\boldsymbol{P}]\boldsymbol{x}(t-\tau)+\\&2\boldsymbol{x}^{\mathrm{T}}(t-\tau)\boldsymbol{P}^{\mathrm{T}}\boldsymbol{B}_1\boldsymbol{f}(\boldsymbol{x}(t-\tau))-\\&2\alpha\boldsymbol{x}^{\mathrm{T}}(t-\tau)\boldsymbol{u}(t)-\\&2a\boldsymbol{x}^{\mathrm{T}}(t-\tau)\boldsymbol{B}_1\boldsymbol{v}(t)+\\&\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau))[(\sigma-\tau)^2\boldsymbol{\Theta}-\\&\beta\boldsymbol{I}+\boldsymbol{\Delta}]\boldsymbol{f}(\boldsymbol{x}(t-\tau))+\\&2\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau))\boldsymbol{B}_1{}^{\mathrm{T}}\boldsymbol{u}(t)+\\&2\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau))\boldsymbol{B}_1{}^{\mathrm{T}}\boldsymbol{B}_1\boldsymbol{v}(t)-\\&\boldsymbol{u}^{\mathrm{T}}(t)\boldsymbol{\Gamma}\boldsymbol{u}(t)-\boldsymbol{v}^{\mathrm{T}}(t)\boldsymbol{\Theta}\boldsymbol{v}(t)-\\&\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\sigma))\boldsymbol{\Delta}\boldsymbol{f}(\boldsymbol{x}(t-\sigma))+\\&2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{B}_{\omega}\boldsymbol{\omega}(t)+2\boldsymbol{x}^{\mathrm{T}}(t-\tau)\boldsymbol{P}^{\mathrm{T}}\boldsymbol{B}_{\omega}\boldsymbol{\omega}(t)+\\&2\boldsymbol{u}^{\mathrm{T}}(t)\boldsymbol{B}_{\omega}\boldsymbol{\omega}(t)+2\boldsymbol{v}^{\mathrm{T}}(t)\boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{B}_{\omega}\boldsymbol{\omega}(t)+\\&[(\boldsymbol{C}+\boldsymbol{DK})\boldsymbol{x}(t)+\boldsymbol{C}_1\boldsymbol{x}(t-\tau)+\\&\boldsymbol{D}_1\boldsymbol{\omega}(t)]^{\mathrm{T}}[(\boldsymbol{C}+\boldsymbol{DK})\boldsymbol{x}(t)+\\&\boldsymbol{C}_1\boldsymbol{x}(t-\tau)+\boldsymbol{D}_1\boldsymbol{\omega}(t)]-\\&\gamma^2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{\omega}(t),\end{aligned} \tag{20}$$

其中,

$$\begin{aligned}\boldsymbol{\zeta}_1(t)=&(\boldsymbol{x}^{\mathrm{T}}(t),\boldsymbol{x}^{\mathrm{T}}(t-\tau),\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau)),\boldsymbol{u}^{\mathrm{T}}(t),\\&\boldsymbol{v}^{\mathrm{T}}(t),\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\sigma)),\boldsymbol{\omega}^{\mathrm{T}}(t))^{\mathrm{T}},\end{aligned}$$

所以

$$J_{z\omega}\leqslant\int_0^{\infty}\boldsymbol{\zeta}_1{}^{\mathrm{T}}(t)[\boldsymbol{\Xi}^{*}+\boldsymbol{\Xi}_1{}^{*}]\boldsymbol{\zeta}(t)\mathrm{d}t, \tag{21}$$

其中:

$$\boldsymbol{\Xi}^{*}=\begin{bmatrix}\overline{\boldsymbol{Q}}_{11} & \boldsymbol{Q}_{12} & \boldsymbol{B}_1 & \boldsymbol{Q}_{14} & \boldsymbol{Q}_{15} & 0 & \boldsymbol{B}_{\omega}\\ * & \boldsymbol{Q}_{22} & \boldsymbol{P}^{\mathrm{T}}\boldsymbol{B}_1 & -\alpha\boldsymbol{I}_n & -\alpha\boldsymbol{B}_1 & 0 & \boldsymbol{P}^{\mathrm{T}}\boldsymbol{B}_{\omega}\\ * & * & \boldsymbol{Q}_{33} & \boldsymbol{B}_1{}^{\mathrm{T}} & \boldsymbol{B}_1{}^{\mathrm{T}}\boldsymbol{B}_1 & 0 & 0\\ * & * & * & -\boldsymbol{\Gamma} & 0 & 0 & \boldsymbol{B}_{\omega}\\ * & * & * & * & -\boldsymbol{\Theta} & 0 & \boldsymbol{B}_1{}^{\mathrm{T}}\boldsymbol{B}_{\omega}\\ * & * & * & * & * & -\boldsymbol{\Delta} & 0\\ * & * & * & * & * & * & 0\end{bmatrix};$$

$$\boldsymbol{\Xi}_1{}^{*}=\begin{bmatrix}\boldsymbol{W}^{\mathrm{T}}\boldsymbol{W} & \boldsymbol{W}^{\mathrm{T}}\boldsymbol{C}_1 & 0 & 0 & 0 & 0 & \boldsymbol{W}^{\mathrm{T}}\boldsymbol{D}_1\\ * & \boldsymbol{C}_1{}^{\mathrm{T}}\boldsymbol{C}_1 & 0 & 0 & 0 & 0 & \boldsymbol{C}_1{}^{\mathrm{T}}\boldsymbol{D}_1\\ * & * & 0 & 0 & 0 & 0 & 0\\ * & * & * & 0 & 0 & 0 & 0\\ * & * & * & * & 0 & 0 & 0\\ * & * & * & * & * & 0 & 0\\ * & * & * & * & * & * & \boldsymbol{D}_1{}^{\mathrm{T}}\boldsymbol{D}_1-\gamma^2\boldsymbol{I}_p\end{bmatrix}.$$

其中:$\boldsymbol{W}=\boldsymbol{C}+\boldsymbol{DK}$;

$\overline{\boldsymbol{Q}}_{11}=2\alpha\boldsymbol{I}-\boldsymbol{A}-\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{BK}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}+\boldsymbol{E}+\beta\boldsymbol{I}+\tau^2\boldsymbol{\Gamma}$;

$\boldsymbol{Q}_{12}=(\alpha\boldsymbol{I}-\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}})\boldsymbol{P}-\alpha\boldsymbol{I}$;

$$\boldsymbol{Q}_{14} = \alpha\boldsymbol{I} - \boldsymbol{A}^{\mathrm{T}} + \boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}};$$
$$\boldsymbol{Q}_{15} = (\alpha\boldsymbol{I} - \boldsymbol{A}^{\mathrm{T}} + \boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}})\boldsymbol{B}_1;$$
$$\boldsymbol{Q}_{22} = -\boldsymbol{E} - 2\alpha\boldsymbol{P};$$
$$\boldsymbol{Q}_{33} = \boldsymbol{\Delta} - \beta\boldsymbol{I} + (\sigma - \tau)^2\boldsymbol{\Theta}.$$

而

$\boldsymbol{\Xi}^* + \boldsymbol{\Xi}_1^* =$

$$\begin{bmatrix} \bar{Q}_{11}+W^{\mathrm{T}}W & Q_{12}+W^{\mathrm{T}}C_1 & B_1 & Q_{14} & Q_{15} & 0 & W^{\mathrm{T}}D_1+B_\omega \\ * & Q_{22}+C_1^{\mathrm{T}}C_1 & P^{\mathrm{T}}B_1-\alpha I_n-\alpha B_1 & 0 & P^{\mathrm{T}}B_\omega+C_1^{\mathrm{T}}D_1 \\ * & * & Q_{33} & B_1^{\mathrm{T}} & B_1^{\mathrm{T}}B_1 & 0 & 0 \\ * & * & * & -\Gamma & 0 & 0 & B_\omega \\ * & * & * & * & -\Theta & 0 & B_1^{\mathrm{T}}B_\omega \\ * & * & * & * & * & -\Delta & 0 \\ * & * & * & * & * & * & D_1^{\mathrm{T}}D_1-\gamma^2 I_p \end{bmatrix}, \quad (22)$$

其中：$\boldsymbol{W} = \boldsymbol{C} + \boldsymbol{DK}$.

如果 $\boldsymbol{\Xi}^* + \boldsymbol{\Xi}_1^* < 0$，则由 Shur 补得到等价的不等式：

$$\begin{bmatrix} \bar{\boldsymbol{Q}}_{11} & \boldsymbol{Q}_{12} & \boldsymbol{W}^{\mathrm{T}} & \boldsymbol{B}_1 & \boldsymbol{Q}_{14} & \boldsymbol{Q}_{15} & 0 & \boldsymbol{W}^{\mathrm{T}}\boldsymbol{D}_1 + \boldsymbol{B}_\omega \\ * & \boldsymbol{Q}_{22} & \boldsymbol{C}_1^{\mathrm{T}} & \boldsymbol{P}^{\mathrm{T}}\boldsymbol{B}_1 - \alpha\boldsymbol{I}_n - \alpha\boldsymbol{B}_1 & 0 & \boldsymbol{P}^{\mathrm{T}}\boldsymbol{B}_\omega + \boldsymbol{C}_1^{\mathrm{T}} \\ * & * & -\boldsymbol{I}_q & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \boldsymbol{Q}_{44} & \boldsymbol{B}_1^{\mathrm{T}} & \boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{B}_1 & 0 & 0 \\ * & * & * & * & -\boldsymbol{\Gamma} & 0 & 0 & \boldsymbol{B}_\omega \\ * & * & * & * & * & -\boldsymbol{\Theta} & 0 & \boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{B}_\omega \\ * & * & * & * & * & * & -\boldsymbol{\Delta} & 0 \\ * & * & * & * & * & * & * & \boldsymbol{D}_1^{\mathrm{T}}\boldsymbol{D}_1 - \gamma^2\boldsymbol{I}_p \end{bmatrix} < 0, \quad (23)$$

这里 $\boldsymbol{Q}_{44} = \boldsymbol{Q}_{33}$，$\boldsymbol{I}_r$ 表示 r 阶单位矩阵. 由式(23)得到 $\boldsymbol{\Omega} < 0$，从而式(8)成立，故系统(7)是渐近稳定的. 另外，由 Shur 补得到式(23)等价于

$\boldsymbol{\Omega} =$

$$\begin{bmatrix} \boldsymbol{\Omega}_{11} & \boldsymbol{\Omega}_{12} & \boldsymbol{W}^{\mathrm{T}} & \boldsymbol{B}_1 & \boldsymbol{\Omega}_{15} & \boldsymbol{\Omega}_{16} & 0 & \boldsymbol{W}^{\mathrm{T}}\boldsymbol{D}_1 + \boldsymbol{B}_\omega \\ * & \boldsymbol{\Omega}_{22} & \boldsymbol{C}_1^{\mathrm{T}} & \boldsymbol{P}^{\mathrm{T}}\boldsymbol{B}_1 - \alpha\boldsymbol{I}_n - \alpha\boldsymbol{B}_1 & 0 & \boldsymbol{P}^{\mathrm{T}}\boldsymbol{B}_\omega + \boldsymbol{C}_1^{\mathrm{T}} \\ * & * & -\boldsymbol{I}_q & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \boldsymbol{\Omega}_{44} & \boldsymbol{B}_1^{\mathrm{T}} & \boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{B}_1 & 0 & 0 \\ * & * & * & * & -\boldsymbol{\Gamma} & 0 & 0 & \boldsymbol{B}_\omega \\ * & * & * & * & * & -\boldsymbol{\Theta} & 0 & \boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{B}_\omega \\ * & * & * & * & * & * & -\boldsymbol{\Delta} & 0 \\ * & * & * & * & * & * & * & \boldsymbol{D}_1^{\mathrm{T}}\boldsymbol{D}_1 - \gamma^2\boldsymbol{I}_p \end{bmatrix} < 0. \quad (24)$$

其中：

$$\boldsymbol{\Omega}_{11} = 2\alpha\boldsymbol{I} - \boldsymbol{A} - \boldsymbol{A}^{\mathrm{T}} + \boldsymbol{BK} + \boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}} + \boldsymbol{E} + \beta\boldsymbol{I} + \tau^2\boldsymbol{\Gamma};$$
$$\boldsymbol{\Omega}_{12} = (\alpha\boldsymbol{I} - \boldsymbol{A}^{\mathrm{T}} + \boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}})\boldsymbol{P} - \alpha\boldsymbol{I};$$
$$\boldsymbol{\Omega}_{22} = -\boldsymbol{E} - 2\alpha\boldsymbol{P};$$
$$\boldsymbol{\Omega}_{15} = \alpha\boldsymbol{I} - \boldsymbol{A}^{\mathrm{T}} + \boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}};$$
$$\boldsymbol{\Omega}_{16} = (\alpha\boldsymbol{I} - \boldsymbol{A}^{\mathrm{T}} + \boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}})\boldsymbol{B}_1;$$
$$\boldsymbol{\Omega}_{44} = \boldsymbol{\Delta} - \beta\boldsymbol{I} + (\sigma - \tau)^2\boldsymbol{\Theta}.$$

式(24)就是式(18). 定理证毕.

4 H_∞ 反馈控制器的设计实例

An example in design of H_∞ feedback controller

考虑中立型系统

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}[\boldsymbol{x}(t) + \boldsymbol{P}\boldsymbol{x}(t-\tau)] = -\boldsymbol{A}\boldsymbol{x}(t) + \\ \qquad \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{B}_1\boldsymbol{f}(\boldsymbol{x}(t-\sigma)) + \boldsymbol{B}_\omega\boldsymbol{\omega}(t), \\ \boldsymbol{x}(t) = \boldsymbol{\varphi}(t), t \in [-\sigma, 0], \boldsymbol{\varphi} \in \boldsymbol{C}([-\sigma, 0], \mathbf{R}^n), \\ \boldsymbol{z}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{C}_1\boldsymbol{x}(t-\tau) + \boldsymbol{D}\boldsymbol{u}(t) + \boldsymbol{D}_1\boldsymbol{\omega}(t). \end{cases} \quad (25)$$

其中：

$$\boldsymbol{P} = \begin{bmatrix} -0.2 & 0.1 \\ 0.2 & -0.3 \end{bmatrix}, \quad \boldsymbol{A} = \begin{bmatrix} 15 & -5 \\ 6 & -15 \end{bmatrix},$$
$$\boldsymbol{B} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}, \quad \boldsymbol{B}_1 = \begin{bmatrix} -0.2 & 1 \\ 0.02 & 0.1 \end{bmatrix},$$
$$\boldsymbol{B}_\omega = \begin{bmatrix} 0.01 \\ 0.05 \end{bmatrix}, \quad \boldsymbol{C} = [0.5 \quad -1.7],$$
$$\boldsymbol{C}_1 = [-0.1 \quad -0.01],$$
$$\boldsymbol{D} = -3.5, \quad \boldsymbol{D}_1 = -1.7, \quad \tau = 0.1, \quad \sigma = 0.6.$$

通过式(18)，可以求得扰动抑制水平和 H_∞ 状态反馈控制器分别为

$$\gamma = 2.614\,6\mathrm{e}^{+004}, \quad \boldsymbol{K} = [-0.057\,2 \quad -1.601\,8].$$

5 结束语

Concluding remarks

本文中，讨论了一类非线性中立型时滞系统的 H-infinity 控制问题. 利用了 Lyapunov-Krasovs-kii 泛函以及 LMI 方法，获得了中立型系统的稳定性时滞相关的判据，且判据与时滞的相关性不仅依赖于哪一个单一的时滞，同时依赖于离散型时滞，也依赖于中立型时滞. 给出了 H-infinity 状态反馈控制器存在的时滞依赖的充分条件. 本文所采用的方法，在设计、求解控制器时不需要做矩阵变换，求解简单. 利用 Matlab 的 LMI 工具箱求解方便. 最后通过数值仿真实例说明了方法的有效性.

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On delay dependent criteria for H_∞ control of a class of nonlinear neutral systems

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Abstract In this paper,the problem of H-infinity control of a class of non-linear neutral delay systems is concerned. The appropriate Lyapunov-Krasovskii functional with undetermined parameters is employed. By use of LMI approach,the delay dependent criteria for asymptotic stability are obtained. The delay dependent sufficient conditions of the existence of state feedback controller for H-infinity control are given. The method used in this paper is simple and does not need the matrix transformation for designing and calculating the controller. The LMI tool box in Matlab can be conveniently employed to solve the controller. At the end of this paper a numerical example is provided to illustrate the feasibility of the conclusion.

Key words neutral type system;non-linearity;delay dependent;H-infinity control